LAB SESSION 1:

A Brief Background of MATLAB and Analysis of First Order Electrical systems.

Objective:

In the first part of this lab we will look at the different commands that are frequently used to solve the mathematical expressions used in control system analysis, like Laplace transform, inverse Laplace transform, transfer function, partial fraction and matrix system etc. In the second part, we derive the mathematical output equations, with two procedures, time domain and frequency domain, for the first order electrical systems. Also implement those equations in MATLAB to plot and analyze their output responses according to their component's behavior.

Equipment Required:

PC and MATLAB® R2017b

Procedure:

Laplace Transform:

In general, the mathematical expression for laplace transform is

$$\boldsymbol{F}(\boldsymbol{S}) = \int_{\infty}^{\infty} \boldsymbol{f}(\boldsymbol{t})\boldsymbol{e}^{-\boldsymbol{st}}\boldsymbol{dt}$$

The commands we used in MATLAB for calculating the laplace transform are given in the following syntaxes:

laplace(f)

laplace (f, transVar)

laplace (f, var, transVar)

In the first syntax the independent variable is ***t*** and the transformation variable is ***s*** by default. In the second syntax the independent variable is ***t*** and the transformation variable is given by the users own choice. In the third syntax the transformation variable and the independent variable is introduced by the user itself. To use these commands firstly we introduce the symbolic variables in terms of independent variable and the transformation variables used in the equation which is in time domain. This is sown by an example

syms a t

f = exp(-a*t);

laplace(f)

The answer in the command window will be:

F(S)=1/ (a + s)

Another example is as follows, in which we introduce the transformation variable as ***y***

syms a y t

f = exp(-a*t);

laplace (f, y)

The answer in the command window will be:

F(S)=1/ (a + y)

The last syntax is left as an exercise for the students.

<u>EXERCISE 1:</u>

Find the laplace transform for the following equation and used ***x*** as transformation variable.

f(t)=1.25+3.5*t*exp(-2*t) +1.25*exp(-2*t)

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S. No	function	Matlab code	output
1	$f(t)=t^4$	syms t; f = t^4; laplace(f)	
2	$g(t) = \frac{1}{\sqrt{2}}$	syms s; g = 1/sqrt(s); laplace (g	
3	$f(t) = e^{-at}$	syms t a x; f = exp(-a*t); laplace (f, x)	

4	$g(t) = 12\frac{d^2y}{dt^2}$	G=laplace(12*diff(sym('y(t)'),2))	

Inverse Laplace Transform:

The commands we used for inverse laplace transform in MATLAB are

ilaplace(F)

ilaplace (F, transVar)

ilaplace (F, var, transVar)

In the first syntax the independent variable is ***s*** and the transformation variable is ***t,*** by default. In the second syntax the independent variable is ***s*** and the transformation variable is given by the users. In the third syntax the transformation variable and the independent variable is introduced by the user itself.

syms s
F = 1/s^2;
ilaplace(F)

The answer in the command window will be:

ans =
t

for another example

syms a s
F = 1/(s-a) ^2;
ilaplace(F)

The answer in the command window will be:

ans =
t*exp(a*t)

EXERCISE 2:

Solve the following table

S. No	Function	Matlab code	output
1	$G(s) = \frac{1}{s^2}$	syms s; f = 1/s^2; ilaplace(f)	
2	$G(s) = \frac{1}{(s-a)^2}$	syms a s; g = 1/(s-a) ^2; ilaplace(g)	
3	$F(s) = \frac{1}{u^2 - a^2}$	syms x u a; f = 1/(u^2-a^2); ilaplace (f, x) simplify (ilaplace (f, x))	
4	$G(s) = \frac{1}{s} - \frac{2}{s+4} + \frac{1}{s+5}$	ilaplace(1/s-2/(s+4) +1/(s+5))	

MATRIX ADDRESSING:

MATLAB is based on matrix and vector algebra; even scalars are treated as 1x1 matrices. Therefore, vector and matrix operations are as simple as common calculator operations. Vectors can be defined in two ways. The first method is used for arbitrary elements:

v = [1 3 5 7];

V creates a 1x4 vector with elements 1, 3, 5 and 7. Note that commas could have been used in place of spaces to separate the elements. Additional elements can be added to the vector:

v (5) = 8;

yields the vector v = [1 3 5 7 8]. Previously defined vectors can be used to define a new vector. For example, with v defined above

a = [9 10];

b = [v a];

creates the vector

b = [1 3 5 7 8 9 10].

The second method is used for creating vectors with equally spaced elements:

t = 0:0.1:10;

creates a 1x101 vector with the elements 0, 0.1, 0.2, 0.3...,10. Note that the middle number defines the increment. If only two numbers are given, then the increment is set to a default of 1:

k = 0:10;

creates a 1x11 vector with the elements 0, 1, 2, ..., 10.

A particular element of a matrix can be assigned:

M (1,2) = 5;

place the number 5 in the first row, second column.

Operations and functions that were defined for scalars in the previous section can also be used on vectors and matrices. For example,

a = [1 2 3];

b = [4 5 6];

c = a + b

yields:

c =5 7 9

Functions are applied element by element. For example,

t = 0:10;

x = cos(2*t);

creates a vector x with elements equal to cos(2t) for t = 0, 1, 2, ..., 10.

Operations that need to be performed element-by-element can be accomplished by preceding the operation by a ".". For example, to obtain a vector x that contains the elements of x(t) = tcos(t) at specific points in time, you cannot simply multiply the vector t with the vector cos(t). Instead you multiply their elements together by adding a '.' In between:

t = 0:10;

x = t.*cos(t);

The result of x will be

x= [0 0.54 -0.83 -2.96 -2.61 1.4 5.76 5.27 -1.16 -8.20 -8.39]

EXERCISE 3:

Explore magic () command of MATLAB. Try A = magic (5), record your output and explain.

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PARTIAL FRACTION EXPANSION:

residue function finds the partial fraction expansion of the ratio of two polynomials. This is particularly useful for applications that represent systems in transfer function form. For polynomials b and a,

$$\frac{b(s)}{a(s)} = \frac{r_1}{s-p_1} + \frac{r_2}{s-p_2} + \cdots \frac{r_n}{s-p_n} k_s$$

If there are no multiple roots. Here r is a column vector of residues, p is a column vector of pole locations, and k is a row vector of direct terms.

b = [-4 8];

a = [1 6 8];

[r, p, k] = residue (b, a)

The output will be

r= [-12; 8]   p= [-4; -2] k= []

Given three input arguments (r, p, and k), residue converts back to polynomial form, try this out.

[b2, a2] = residue (r, p, k)

The output will be

$$F(S) = \frac{b(s)}{a(s)} = \frac{-4s+8}{s^2+6s+8}$$

EXERCISE 4:

Use MATLAB to find partial fraction of the following

$$\frac{b(s)}{a(s)} = \frac{2s^3+5s^2+3s+6}{s^3+6s^2+11s+6}$$

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First order Electrical systems

The dynamics of many systems of interest to engineers may be represented by a simple model containing one independent energy storage element. For example, the braking of an automobile, the discharge of an electronic camera flash, the flow of fluid from a tank, and the cooling of a cup of coffee may all be approximated by a first-order differential equation, which may be written in a standard form as:

$$\tau\frac{dy}{dt}+y(t)=f(t)$$

Where the system is defined by the single parameter τ, the system time constant, and $f(t)$ is a forcing function.

In this part of the experiment we deal with an electrical system containing a resistor and a capacitor as shown in the fig, for which we derive a differential equation and solve it for the capacitor voltage v_c and current i_c through it.

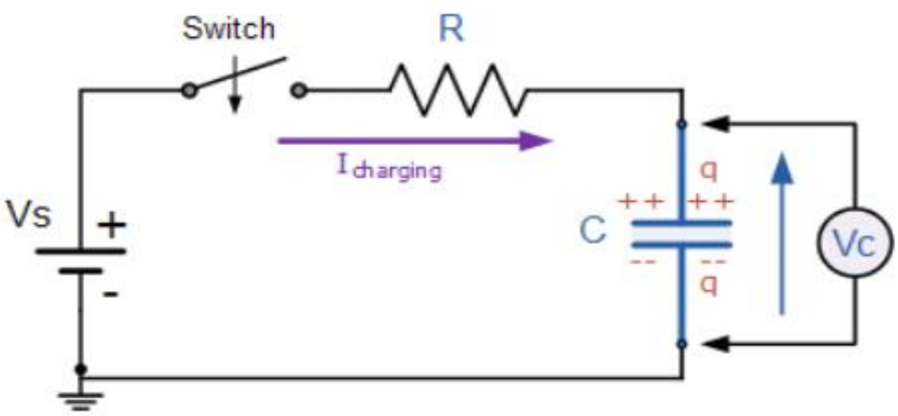

Figure 1.1: A simple RC series Circuit

By applying the KVL

$$V_R+V_C=V_S$$

$$IR+V_C=V_S$$

$$I=I_C$$

$$I_c=C\frac{dv_c}{dt}$$

$$RC\frac{dv_c}{dt}+V_C=V_S \tag{1}$$

By rearranging we have

$$\frac{dv_c}{dt}=\frac{V_S-V_C}{RC}$$

By taking the integral on both sides we have

$$\int \frac{dv_c}{V_S - V_C} = \int \frac{dt}{RC}$$

$$\ln(V_S - V_C) = \frac{-t}{RC} + K$$

Now by assuming initial conditions are zero

$$V_C = 0 \ \ when\ t = 0$$

$$K = \ln\,(V_S)$$

$$\ln(V_S - V_C) = \frac{-t}{RC} + \ln\,(V_S)$$

$$\ln\left(\frac{V_S - V_C}{V_S}\right) = \frac{-t}{RC}$$

$$e^{\ln\left(\frac{V_S - V_C}{V_S}\right)} = e^{\frac{-t}{RC}}$$

$$\frac{V_S - V_C}{V_S} = e^{\frac{-t}{RC}}$$

$$\boxed{\boldsymbol{V_C = V_S(1 - e^{\frac{-t}{RC}})}}$$

Now finding the current I_C through capacitor

$$I_c = C\frac{dv_c}{dt}$$

$$I_C = C\frac{dV_S(1 - e^{\frac{-t}{RC}})}{dt}$$

$$\boxed{\boldsymbol{I_C = \frac{V_S}{R} e^{\frac{-t}{RC}}}}$$

The second method is to use the frequency domain analysis

Now taking the equation (1)

$$RC\frac{dv_c}{dt} + V_C = V_S u(t)$$

By taking the laplace transform

$$RC[SV_C(S) - V_C{}'(0)] + V_C(S) = \frac{V_S}{S}$$

Taking the initial conditions are zero

$$V_C[RCS + 1] = \frac{V_S}{S}$$

$$V_C = \frac{V_S}{S(RCS + 1)}$$

Now applying the method of partial fraction

$$\frac{V_S}{S(RCS + 1)} = \frac{A}{S} + \frac{B}{RC(S + {}^{1}/_{RC})}$$

By putting $S = 0$ and $S = -1/RC$ and finding the values of A and B

We have

$$V_C = \frac{V_S}{S} - \frac{V_S}{S + {}^{1}/_{RC}}$$

Now by taking the inverse laplace transform we have

$$\boldsymbol{V_C(t) = V_S(1 - e^{\frac{-t}{RC}})}$$

Again, takin the derivative of $V_c(t)$ we have the current through capacitor

$$I_c = C\frac{dv_c}{dt}$$

$$\boldsymbol{I_C = \frac{V_S}{R} e^{\frac{-t}{RC}}}$$

The output would be like for the above circuit

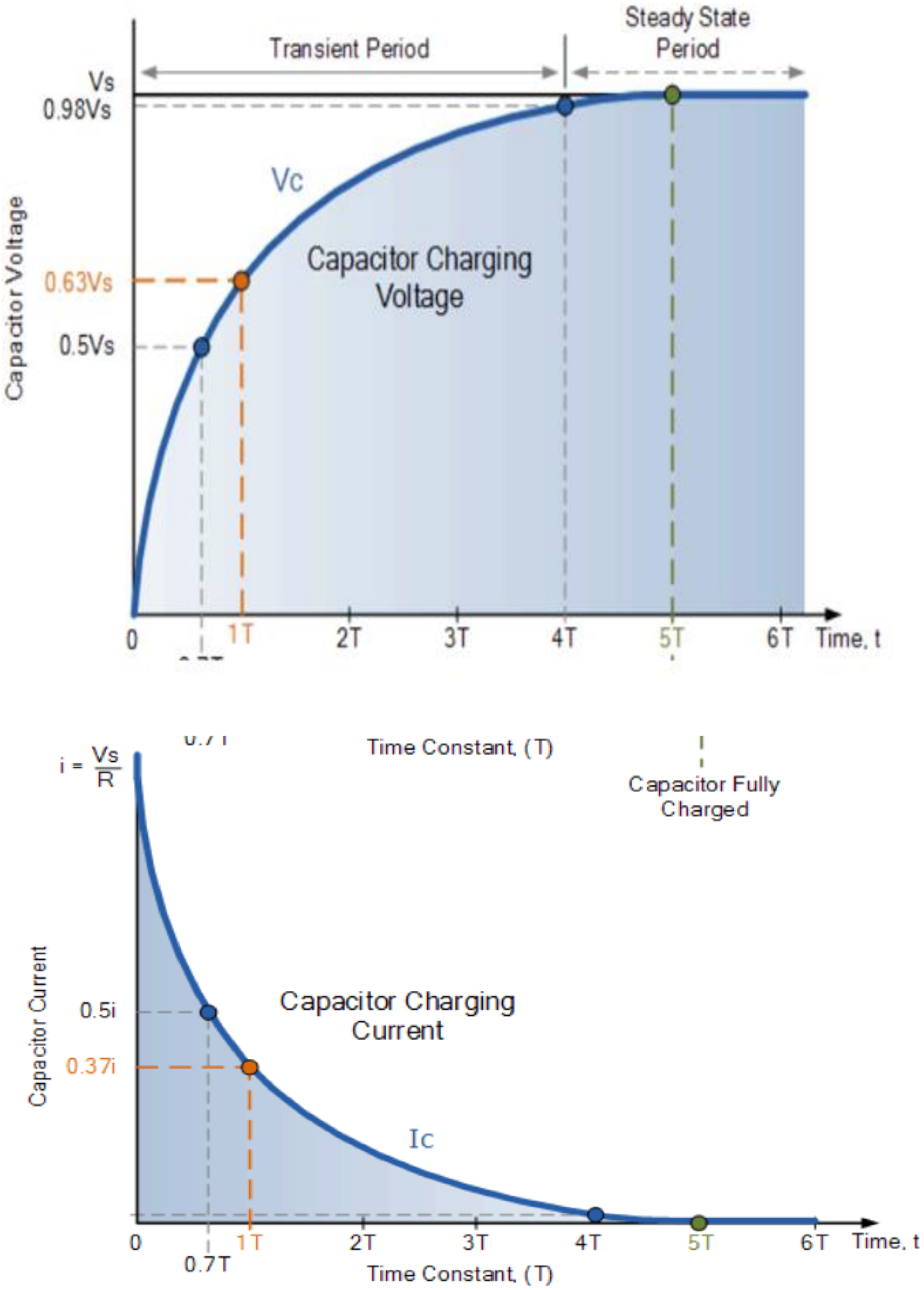

Figure 1.2: The output of the series RC circuit

<u>Matlab code:</u>

To see the response of the system in real time we implement our equations of $V_C$and I_C in MATLAB and plot the responses for analysis purposes.

EXERCISE 4:

Write the MATLAB code here

Draw or Paste the plot here

EXERCISE 5:

a) Now consider the following RL series circuit and find its differential equation and solve it for the both methods (time and frequency domain).
b) Also write the MATLAB program and plot the results for both I_L and V_L.

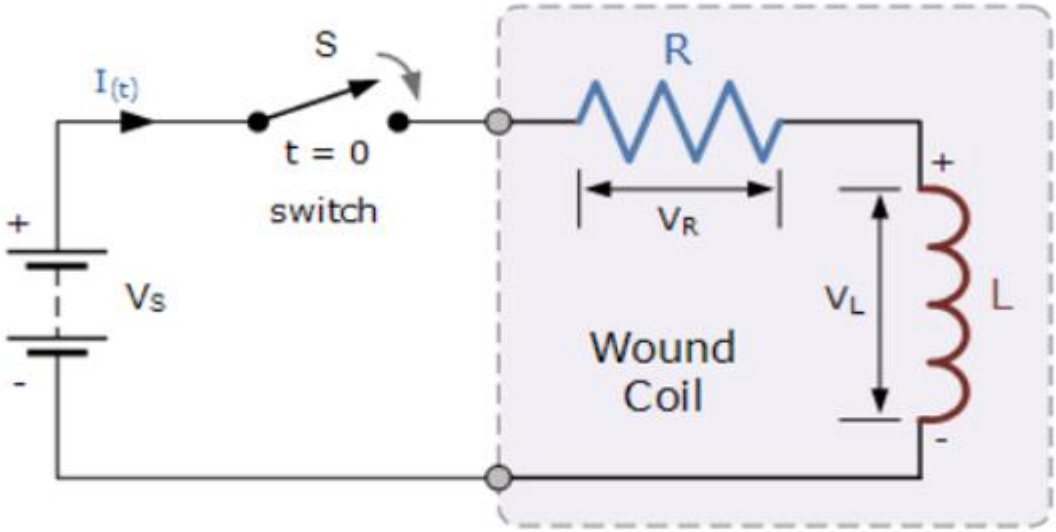

Figure 1.3: A simple RL series Circuit

The output will be

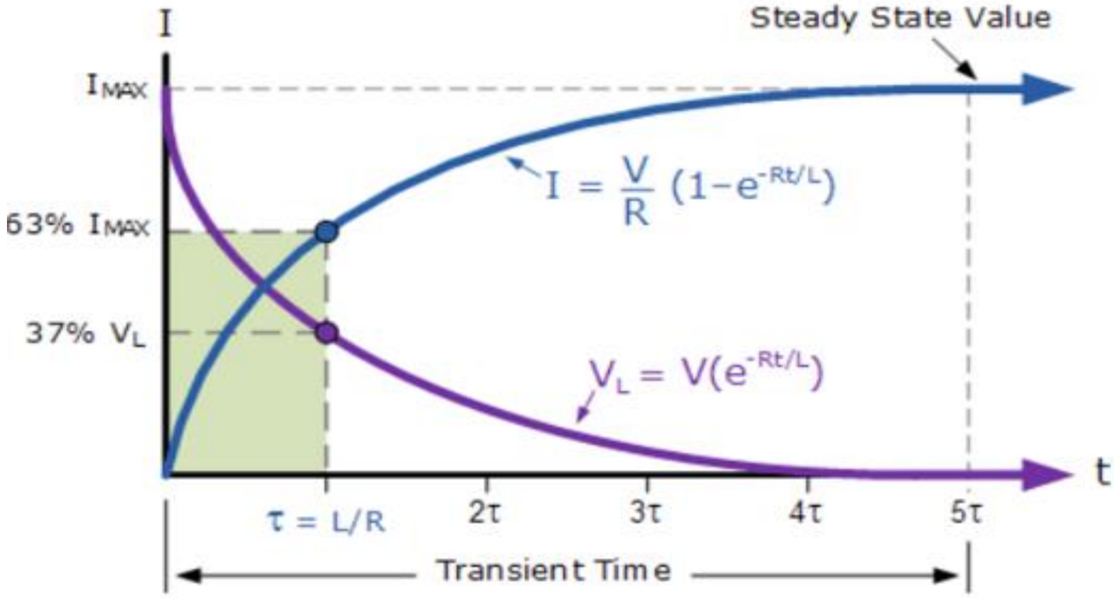

Figure 1.4: The output of the series RL circuit

<u>Solve here for the differential equation/solution (Time/Frequency domain)</u>

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Write the MATLAB code here

Draw or Paste the plot here