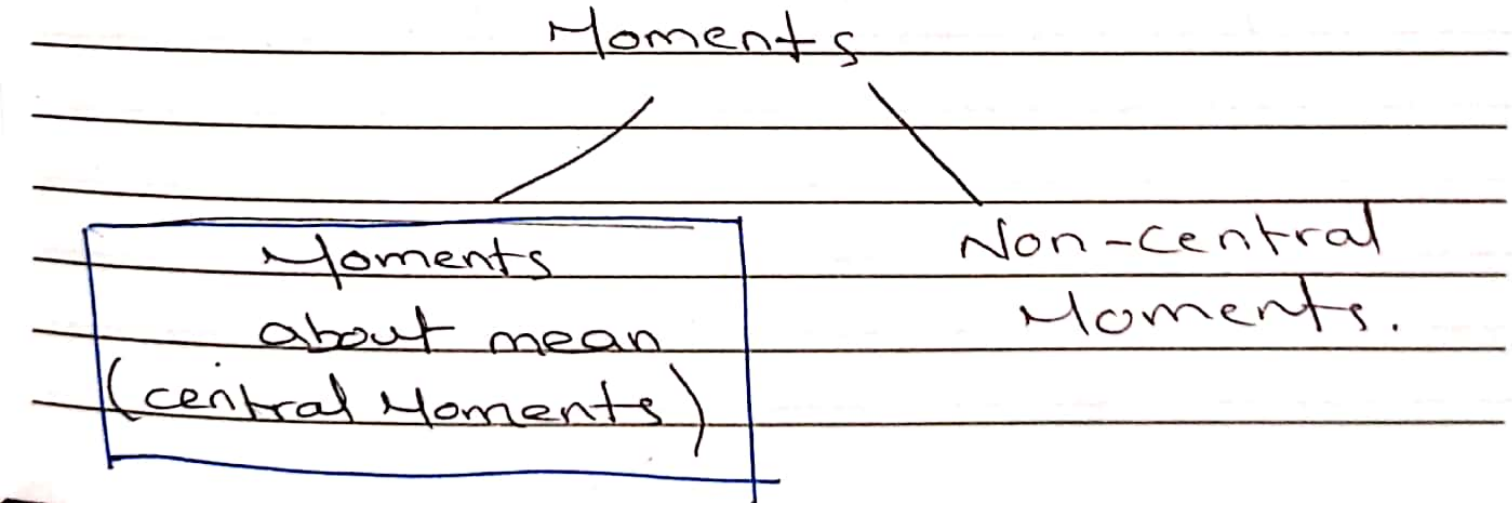
⇒ Moments:

→ The moments are defined as the average of different powers of deviations of observations from their mean.

⇒ Merits of Moments:

1→ Moments tell us about the mean, variance, skewness, kurtosis of dataset.

2→ From moments, we can find shape of any dataset.



⇒ Central Moments:

$$\mu_r = \frac{\Sigma (x-\mu)^r}{N}$$

First four moments about mean

central moments are;

$$\mu_1 = \frac{\Sigma (x-\mu)^1}{N}$$

$$\mu_2 = \frac{\Sigma (x-\mu)^2}{N}$$

$$\mu_3 = \frac{\Sigma (x-\mu)^3}{N}$$

$$\mu_4 = \frac{\Sigma (x-\mu)^4}{N}$$

→ Skewness (Moment Method):

The numerical measure that is used to know about the symmetry & Skewness of dataset is $\sqrt{\beta_1}$

$$\sqrt{\beta_1} = \frac{\mu_3}{\sqrt{\mu_2^3}}$$

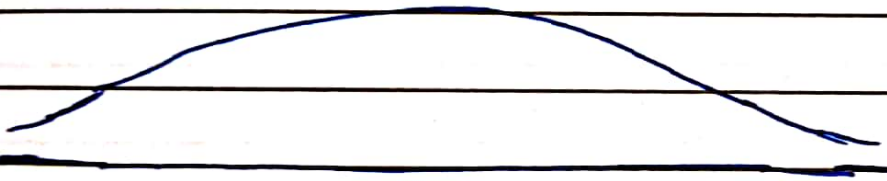
If $\sqrt{\beta_1} = 0$ → Symmetrical

If $\sqrt{\beta_1} > 0$ → Positively Skewed.

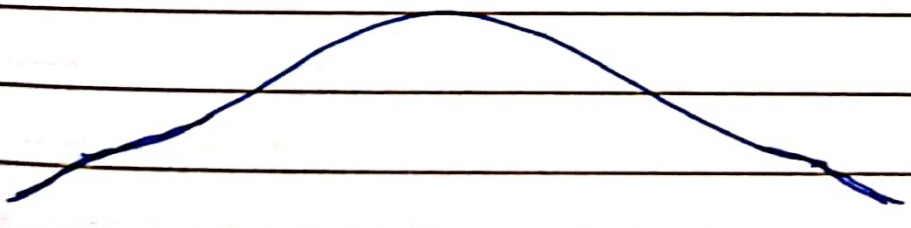
If $\sqrt{\beta_1} < 0$ → Negatively Skewed.

⇒ kurtosis :

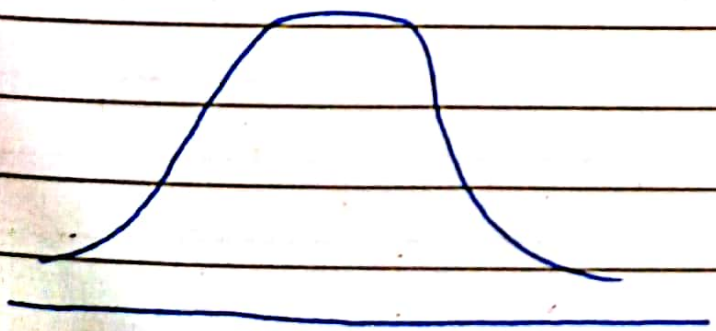
kurtosis is defined as the degree of peakness or flatness. It indicates the length of the peakedness of symmetrical distribution. Symmetrical distribution may be platykurtic, mesokurtic or leptokurtic i.e.,



platykurtic (flat)



Mesokurtic (normal)



Leptokurti (peaked)

⇒ Kurtosis (Moment Method):

The measure of kurtosis based on moments is β_2

$$\beta_2 = \frac{\mu_4}{\mu_2^2}$$

For $\beta_2 = 3$ ⟶ Mesokurtic

For $\beta_2 < 3$ ⟶ Platykurtic

For $\beta_2 > 3$ ⟶ Leptokurtic

⟹ Example:

calculate first four moments about mean for the following set of marks obtained in the examination.

45, 32, 37, 46, 39, 36, 41, 48 & 36

Also calculate $\sqrt{\beta_1}$.

$$M = \frac{\Sigma X}{N}$$

$$= \frac{45+32+37+46+39+36+41+48+36}{9}$$

$$\boxed{M = 40}$$

We know that:

$$M_1 = \frac{\Sigma (x-M)}{N}$$

$$M_2 = \frac{\Sigma (x-M)^2}{N}$$

$$M_3 = \frac{\Sigma (x-M)^3}{N}$$

$$M_4 = \frac{\Sigma (x-M)^4}{N}$$

x	$x-M$	$(x-M)^2$	$(x-M)^3$	$(x-M)^4$
45	5	25	125	625
32	-8	64	-512	4096
37	-3	9	-27	81
46	6	36	216	1296
39	-1	1	-1	1
36	-4	16	-64	256
41	1	1	1	1
48	8	64	512	4096
36	-4	16	-64	256
	0	232	186	10708

$$M_1 = \frac{0}{9} = \boxed{0}$$

$$M_2 = \frac{232}{9} = \boxed{25.78}$$

$$M_3 = \frac{186}{9} = \boxed{20.67}$$

$$M_4 = \frac{10708}{9} = \boxed{1189.78}$$

$$\sqrt{\beta_1} = \frac{\mu_3}{\sqrt{\mu_2^3}}$$

$$= \frac{20.67}{\sqrt{(25.78)^3}}$$

$$= \frac{20.67}{\sqrt{17133.60}}$$

$$= \frac{20.67}{130.8954}$$

$$\sqrt{\beta_1} = 0.16 > 0$$

Positively Skewed.

→ Example :

First four central moments are

0, 43.7220, 43.7407, 4363.0296

Find $\sqrt{\beta_1}$.

Here,

$\mu_1 = 0$

$\mu_2 = 43.7220$

$\mu_3 = 43.7407$

$\mu_4 = 4363.0296$.

So,

$$\sqrt{\beta_1} = \frac{\mu_3}{\sqrt{\mu_2^3}}$$

$$\sqrt{\beta_1} = \frac{43.7407}{\sqrt{(43.7220)^3}}$$

$$= \frac{43.7407}{\sqrt{83,579.556}}$$

$$= \frac{43.7407}{289.1013}$$

$$\sqrt{\beta_1} = 0.1513$$

As $\sqrt{\beta_1} > 0$

So, the given distribution is positively skewed.

⟹ Example:

First four central moments are:

0, 43.4988, 17.3354, 4131.1478

Find $\sqrt{\beta_1}$:

Here;

$\mu_1 = 0$

$\mu_2 = 43.4988$

$\mu_3 = 17.3354$

$\mu_4 = 4131.1478$

We know that;

$$\sqrt{\beta_1} = \frac{\mu_3}{\sqrt{\mu_2^3}}$$

$$\sqrt{\beta_1} = \frac{17.3354}{\sqrt{(43.4988)^3}}$$

$$= \frac{17.3354}{\sqrt{82,306.0631}}$$

$$= \frac{17.3354}{286.89}$$

$$\sqrt{\beta_1} = 0.0604 > 0$$

Positively Skewed.

→ Example:

First four moments about mean are 0, 11, 49, 192.

Find $\sqrt{\beta_1}$

$\mu_1 = 0$

$\mu_2 = 11$

$\mu_3 = 49$

$\mu_4 = 192$

$$\sqrt{\beta_1} = \frac{\mu_3}{\sqrt{\mu_2^3}} = \frac{49}{\sqrt{(11)^3}}$$

$$\sqrt{\beta_1} = 1.34$$

$\sqrt{\beta_1} > 0$

Positively Skewed.

⟹ Example:

First three moments about mean are; 0, 2.78, −1.03. What can you say about skewness

$$\mu_1 = 0$$

$$\mu_2 = 2.78$$

$$\mu_3 = -1.03$$

$$\sqrt{\beta_1} = \frac{\mu_3}{\sqrt{\mu_2^3}}$$

$$= \frac{-1.03}{\sqrt{(2.78)^3}}$$

$$\sqrt{\beta_1} = -0.22 \quad < 0$$

Negatively Skewed

⇒ Example:

First three moments about mean are 0, 3 & 0. What can you say about skewness. Is the distribution symmetrical, positively skewed or negatively skewed?

$\mu_1 = 0$

$\mu_2 = 3$

$\mu_3 = 0$

$$\sqrt{\beta_1} = \frac{\mu_3}{\sqrt{\mu_2^3}}$$

$$\sqrt{\beta_1} = \frac{0}{\sqrt{3^3}} = 0$$

The distribution is symmetrical.

⟹ Example:

First four moments about mean are 0, 47, -105, 5621.

Find β_2.

$\mu_1 = 0$

$\mu_2 = 47$

$\mu_3 = -105$

$\mu_4 = 5621$

$$\beta_2 = \frac{\mu_4}{\mu_2^2}$$

$$= \frac{5621}{(47)^2}$$

$\beta_2 = 2.54 < 3$

platy-kurtic.

→ Example:

First four moments about mean are, 0, 45.81, 43.74 & 4917.37. Find β_2.

$M_1 = 0$

$M_2 = 45.81$

$M_3 = 43.74$

$M_4 = 4917.37$

$$\beta_2 = \frac{M_4}{(M_2)^2}$$

$$= \frac{4917.37}{(45.81)^2} = 2.34 < 3$$

platykurtic.

→ Example:

For a dataset the second and fourth moments about mean are 2.6364 & 28.30256. Find β_2.

$$\mu_2 = 2.6364$$

$$\mu_4 = 28.30256.$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2}$$

$$= \frac{28.30256}{(2.6364)^2}$$

$$\beta_2 = 4.07 \quad > 3$$

Leptokurtic

⇒ First four moments about mean are 0, 73.91, -21.546 & 12110.94. Find out whether the distribution is leptokurtic or platykurtic.

$\mu_1 = 0$

$\mu_2 = 73.91$

$\mu_3 = -21.546$

$\mu_4 = 12110.94$

$$\beta_2 = \frac{\mu_4}{\mu_2^2} = \frac{12110.94}{(73.91)^2}$$

$$\boxed{\beta_2 = 2.22}$$

Since $\beta_2 < 3$

The distribution is platykurtic.

⟹ Example:

Second moment about mean of two datasets are 9 & 16 while fourth moment about mean are 230 & 780 respectively. Find out which of the distribution is

i) Platykurtic.

ii) Leptokurtic.

Dataset 1	Dataset 2
$\mu_2 = 9$	$\mu_2 = 16$
$\mu_4 = 230$	$\mu_4 = 780$
$\beta_2 = \frac{\mu_4}{\mu_2^2}$	$\beta_2 = \frac{\mu_4}{\mu_2^2}$

$$\beta_2 = \frac{230}{(9)^2}$$

$$\beta_2 = 2.84$$

Since;

$$\beta_2 < 3$$

Therefore the dist. is platykurtic

$$\beta_2 = \frac{780}{(16)^2}$$

$$\beta_2 = 3.05$$

Since;

$$\beta_2 \neq 3$$

Therefore the dist. is Leptokurtic

⟹ Example:

The second moment about mean of two datasets are 13.76 and 63.0 while the fourth moments about the mean are 528.06 & 9500 respectively. which of the dist. is

a) platykurtic.

b) Leptokurtic.

c) Mesokurtic.

Dataset 1:

$$\mu_2 = 13.76$$

$$\mu_4 = 528.06$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2}$$

$$\beta_2 = \frac{528.06}{(13.76)^2}$$

$$\beta_2 = 2.79$$

$$\beta_2 < 3$$

Platykurtic.

Dataset 2:

$$\mu_2 = 63.0$$

$$\mu_4 = 9500$$

$$\beta_2 = \frac{\mu_4}{\mu_2^2}$$

$$= \frac{9500}{(63)^2}$$

$$\beta_2 = 2.39 < 3$$

Platykurtic.

→ Example:

For a dataset

$\mu_2 = 9$ 9

$\mu_4 = 243$

Find β_2 =

$$\beta_2 = \frac{\mu_4}{\mu_2^2}$$

$$= \frac{243}{(9)^2}$$

$$= \frac{243}{81}$$

$\beta_2 = 3 \Longrightarrow$ Mesokurtic