⇒ Two-way Analysis of Variance

When each observation is classified according to two criteria (or variables) of classification simultaneously. The classified data are recorded in a table, in which the columns represent one criterion of the classification and rows represent the other criterion.

Rows	1	2	...	j	...	c	Total	Mean
1	X_{11}	X_{12}	...	X_{1j}	...	X_{1c}	$T_{1.}$	$\bar{X}_{1.}$
2	X_{21}	X_{22}	...	X_{2j}	...	X_{2c}	$T_{2.}$	$\bar{X}_{2.}$
⋮	⋮							
i	X_{i1}	X_{i2}	...	X_{ij}	...	X_{ic}	$T_{i.}$	$\bar{X}_{i.}$
⋮								
r	X_{r1}	X_{r2}	...	X_{rj}	...	X_{rc}	$T_{r.}$	$\bar{X}_{r}$
Totals	$T_{.1}$	$T_{.2}$		$T_{.j}$		$T_{.c}$	$T_{..}$	
Mean	$\bar{X}_{.1}$	$\bar{X}_{.2}$		$\bar{X}_{.j}$		$\bar{X}_{.c}$		$\bar{X}_{..}$

⇒ There are now two null hypothesis, one corresponding to the problem that all the r-rows mean are equal and the other corresponding to the problem that all the c-column means are equal. Thus

$$H_0' : \mu_{1.} = \mu_{2.} = \cdots \mu_{r.}$$

$$H_0'' : \mu_{.1} = \mu_{.2} = \cdots = \mu_{.c}$$

H_1' : Not all $\mu_{i.}$ are equal

H_1'' : Not all $\mu_{.j}$ are equal

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ANOVA - Table

Source of variation	d.f	Sum of squares	Mean Square (MS)	F
Between Rows	$r-1$	$SSR = \sum_i \frac{T_{i.}^2}{c} - \frac{T_{..}^2}{rc}$	$S_1^2 = \frac{SSR}{r-1}$	$F_1 = \frac{S_1^2}{S_3^2}$
Between columns	$c-1$	$SSC = \sum_j \frac{T_{.j}^2}{r} - \frac{T_{..}^2}{rc}$	$S_2^2 = \frac{SSC}{c-1}$	$F_2 = \frac{S_1^2}{S_3^2}$
Error (within)	$(r-1)(c-1)$	$SSE = SST - SSR - SSC$	$S_3^2 = \frac{SSE}{(r-1)(c-1)}$	
Total	$rc-1$	$SST = \sum\sum X_{ij}^2 - \frac{T_{..}^2}{rc}$		

$F_1 \Rightarrow v_1 = r-1 \qquad v_2 = (r-1)(c-1)$

$F_2 \Rightarrow v_2 = c-1 \qquad v_2 = (r-1)(c-1)$

⇒ when ANOVA reject we move toward
LSD (Least significant difference test)
Duncan's test.