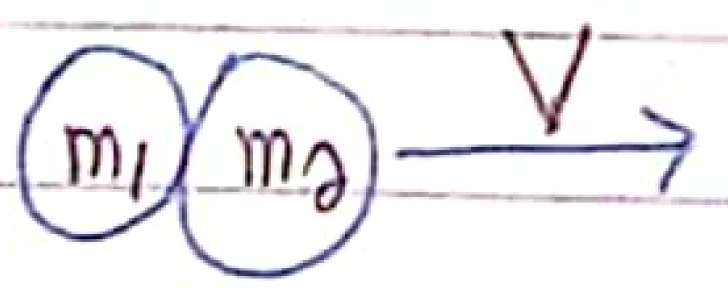
Collisions:

"when particles come close to each other with or without physical contact but must have interaction (attraction or repulsion) then collision occur."

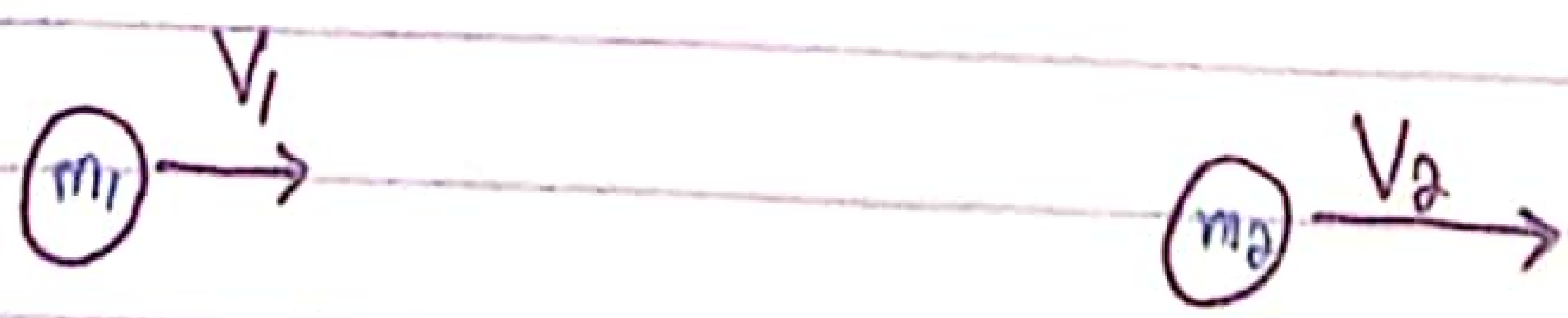
Head ON Collision:

⇒ Before Collision:

⇒ During Collision:



⇒ After Collision:



Deformation Period:

"Time b/w start of interaction to maximum deformation is called deformation period."

Restitution Period:

"Time b/w maximum deformation to the point of seperation is called restitution period."

Impulse:

"Rate of change of momentem is called force and cross product of force and time is called impulse."

$\Rightarrow$

$$\vec{F} = \frac{d\vec{p}}{dt}$$

$\Rightarrow$ $\therefore \vec{I} = \vec{F}dt$

$$d\vec{p} = \vec{F}dt$$

So,

$\Rightarrow$ $\vec{I} = d\vec{p}$

Now, impulse on body 1 during deformation period is,

$\Rightarrow$

$$\vec{I}_{d1} = m_1\vec{V} - m_1\vec{u}_1$$

$\Rightarrow$

$$\vec{u}_1 = -\frac{\vec{I}_{d1}}{m_1} + \vec{V} \longrightarrow ①$$

Similarly,

$\Rightarrow$

$$\vec{I}_{d_2} = m_2\vec{V} - m_2\vec{u}_2$$

$\Rightarrow$ By Newton's law,

$$\vec{F}_1 = -\vec{F}_2$$

So,

$$\vec{I}_{d_1} = -\vec{I}_{d_2}$$

$\Rightarrow$

$$-\vec{I}_{d_1} = m_2\vec{V} - m_2\vec{u}_2$$

$\Rightarrow$

$$\vec{u}_2 = \frac{\vec{I}_{d_1}}{m_2} + \vec{V} \longrightarrow (2)$$

(2) − (1) $\Rightarrow$

$$\vec{u}_2 - \vec{u}_1 = \left(\frac{\vec{I}_{d_1}}{m_2} + \vec{V}\right) - \left(-\frac{\vec{I}_{d_1}}{m_1} + V\right)$$

$\Rightarrow$

$$= \frac{\vec{I}_{d_1}}{m_2} + \cancel{\vec{V}} + \frac{\vec{I}_{d_1}}{m_1} - \cancel{\vec{V}}$$

$\Rightarrow$

$$\vec{u}_2 - \vec{u}_1 = \vec{I}_{d_1}\left(\frac{1}{m_1} + \frac{1}{m_2}\right) \rightarrow (3)$$

Now, impulse acting upon body 1, during the restitution period,

$\Rightarrow$

$$\vec{I}_{r_1} = m_1\vec{v}_1 - m_1\vec{V}$$

$\Rightarrow$

$$\vec{v}_1 = \frac{\vec{I}_{r_1}}{m_1} + \vec{V} \rightarrow (4)$$

Similarly,

$$\vec{I}_{r_2} = m_2\vec{v}_2 - m_2\vec{V}$$

$\Rightarrow$ $\quad\because \vec{I}_{r_2} = -\vec{I}_{r_1}$

$$-I_{r_1} = m_2\vec{v}_2 - m_2\vec{V}$$

$\Rightarrow$

$$\vec{v}_2 = -\frac{I_{r_1}}{m_2} + V \longrightarrow (5)$$

(5) − (4) $\Rightarrow$

$$\vec{v}_2 - \vec{v}_1 = \left(-\frac{I_{r_1}}{m_2} + V\right) - \left(\frac{\vec{I}_{r_1}}{m_1} + \vec{V}\right)$$

$\Rightarrow$

$$\vec{V_2} - \vec{V_1} = -I_{r_1}\left(\frac{1}{m_1} + \frac{1}{m_2}\right) \longrightarrow (6)$$

(6) $\div$ (3) $\Rightarrow$

$$\Rightarrow \frac{\vec{V_2} - \vec{V_1}}{\vec{U_2} - \vec{U_1}} = -\frac{I_{r_1}\cancel{\left(\frac{1}{m_1} + \frac{1}{m_2}\right)}}{I_{d_1}\cancel{\left(\frac{1}{m_1} + \frac{1}{m_2}\right)}}$$

$$\Rightarrow \frac{\vec{V_2} - \vec{V_1}}{\vec{U_2} - \vec{U_1}} = -\frac{\vec{I_{r_1}}}{\vec{I_{d_1}}}$$

Let,

$$e = -\frac{\vec{I_{r_1}}}{\vec{I_{d_1}}}$$

$$\Rightarrow \boxed{\frac{\vec{V_2} - \vec{V_1}}{\vec{U_2} - \vec{U_1}} = e}$$

where,

e = efficient of restitution.

$$0 \leq e \leq 1$$

$\Rightarrow$ For perfectly elastic collision, $e=1$

$\Rightarrow$ For perfectly inelastic collision, $e=0$

"The collision in which total energy and momentum is conserved is called elastic collision."

Q= Why $e=1$ for elastic collision?

Ans=

As we know, In elastic collision, Linear momentum and energy of system remains constant.

As, cofficient of restitution is,

$$e = -\frac{(\vec{V_2}-\vec{V_1})}{(\vec{U_2}-\vec{U_1})}$$

By Using Law of Conservation of Momentum:

$$\Rightarrow m_1 V_1 + m_2 V_2 = m_1 U_1 + m_2 V_2 \rightarrow a)$$

By Using Law of Energy Conservation:

$$\Rightarrow \frac{1}{2} m_1 V_1^2 + \frac{1}{2} m_2 V_2^2 = \frac{1}{2} m_1 U_1^2 + \frac{1}{2} m_2 U_2^2 \rightarrow b)$$

$$\Rightarrow m_1 V_1^2 + m_2 V_2^2 = m_1 U_1^2 + m_2 U_2^2 \rightarrow c)$$

a) ⇒

$$m_1(v_1 - u_1) = m_2(u_2 - v_2) \rightarrow ①$$

c) ⇒

$$m_1(v_1^2 - u_1^2) = m_2(u_2^2 - v_2^2) \rightarrow ②$$

Dividing ② by 1).

⇒

$$\frac{m_1(v_1^2 - u_1^2)}{m_1(v_1 - u_1)} = \frac{m_2(u_2^2 - v_2^2)}{m_2(u_2 - v_2)}$$

⇒

$$v_1 + u_1 = v_2 + u_2$$

⇒

$$-u_1 + u_2 = -v_2 + v_1$$

⇒

$$u_2 - u_1 = -(v_2 - v_1)$$

⇒

$$\frac{u_2 - u_1}{v_2 - v_1} = -1$$

Puting in equations

⇒

$$e = -(-1)$$

$$e = +1$$

⇒ So, it is for perfectly elastic collision.

LAB and COM System:

LAB Co-ordinate System:

"when an observer is at rest and COM is moving with respect to observer is called LAB-Coordinate system."

COM Co-ordinate System:

"when COM is at rest w.r.t observer then system is called COM Coordinate system."

Elastic Collision in LAB and COM:

Let,

m_1 = mass of in LAB System.

m_2 = " " " COM System.

⇒

$\vec{U_1}$ = initial velocity of object 1 in LAB Systems before collision.

$\vec{U_2}$ = " " " " 2 " " " " "

$\vec{V_1}$ = final velocity of object 1 in LAB system after collision

$\vec{V_2}$ = " " " " 2 " " " " "

Unprimed Variable ⇒ LAB System

Primed Variable ⇒ COM system

$\vec{U_1}'$ = initial velocity of object 1 in COM before collision.

$\vec{U_2}'$ = " " " " 2 " " " "

$\vec{V_1}'$ = final velocity of object 1 in COM after collision.

$\vec{V_2}'$ = " " " " 2 " " " "

$\vec{V}$ = velocity of COM in LAB System
$\Rightarrow$ θ = Scattering angle of m_1 in COM system

ψ = Scattering angle of m_1 after collision in LAB System.

ϕ = " " " m_2 " " " " ".

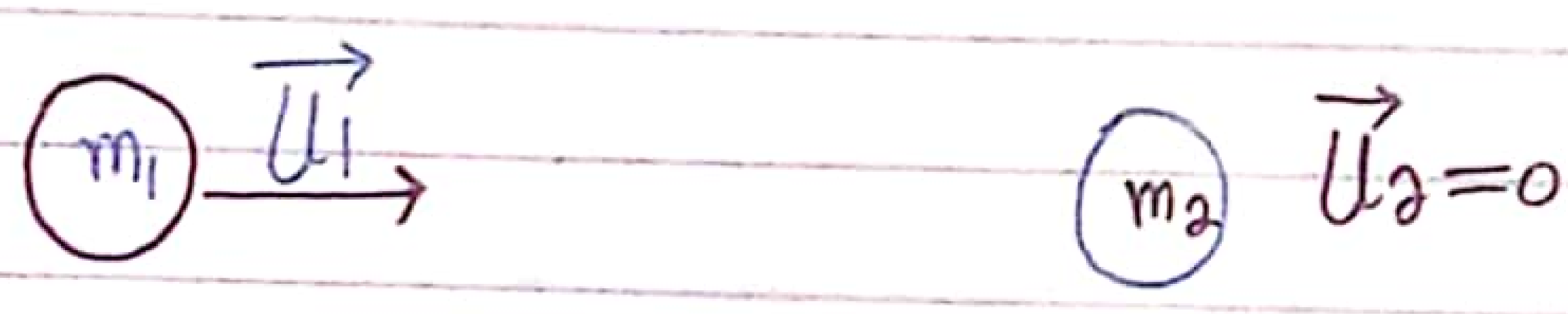
COM is always on the straight line joining two objects.

In LAB System :

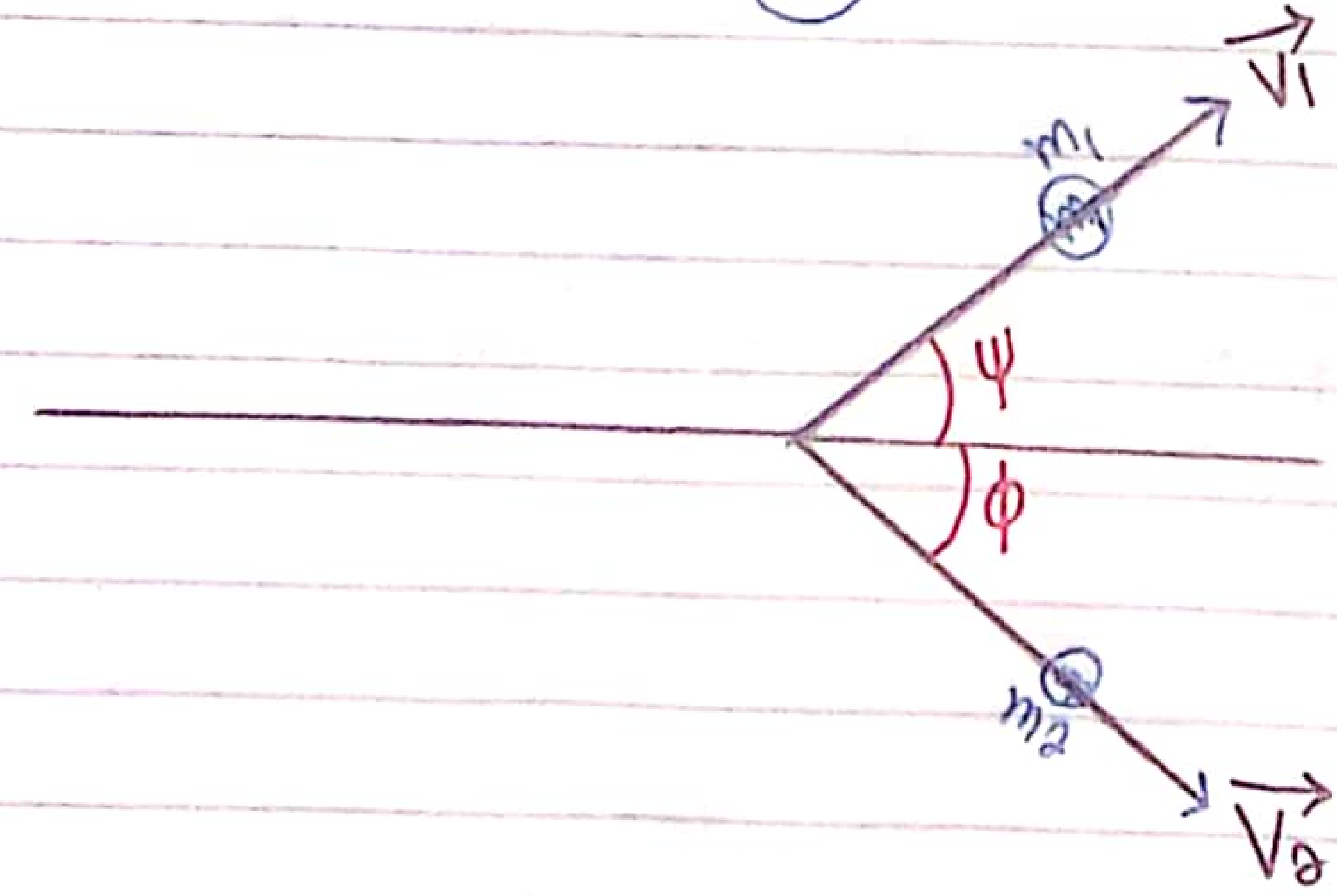
Let, us consider two objects in LAB System having masses m_1 and m_2 respectively with initial velocities $\vec{u_1}$ and $\vec{u_2}$ where m_2 is at rest having,

$$\vec{u_2} = 0$$

Before Collision:

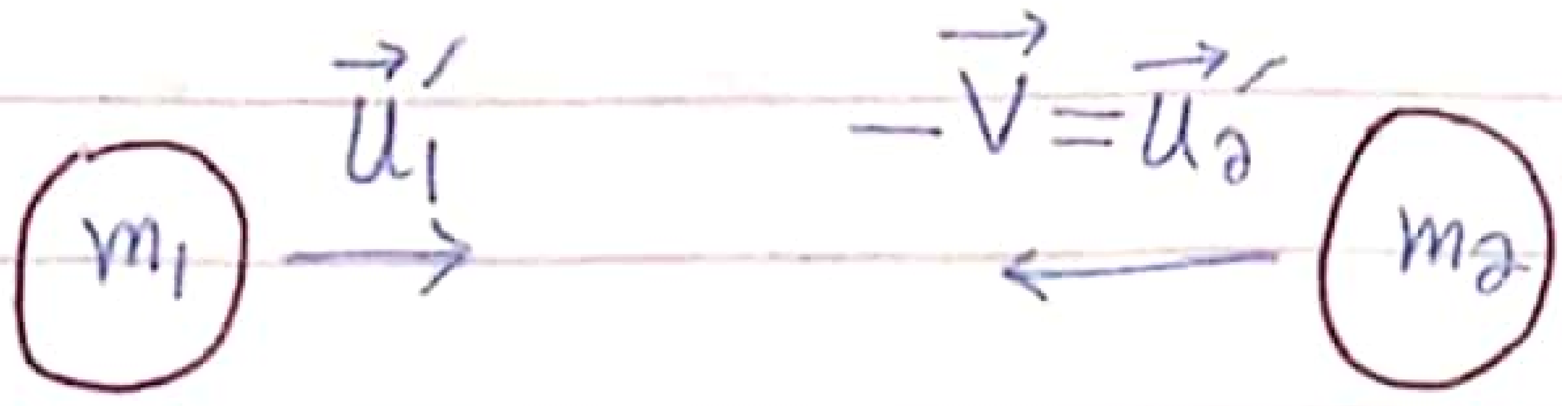


After Collision:



In COM System:

Consider two objects having m_1 and m_2 masses with $\vec{u_1'}$ and $\vec{u_2'}$ velocities. In COM, we assume that COM is at rest.



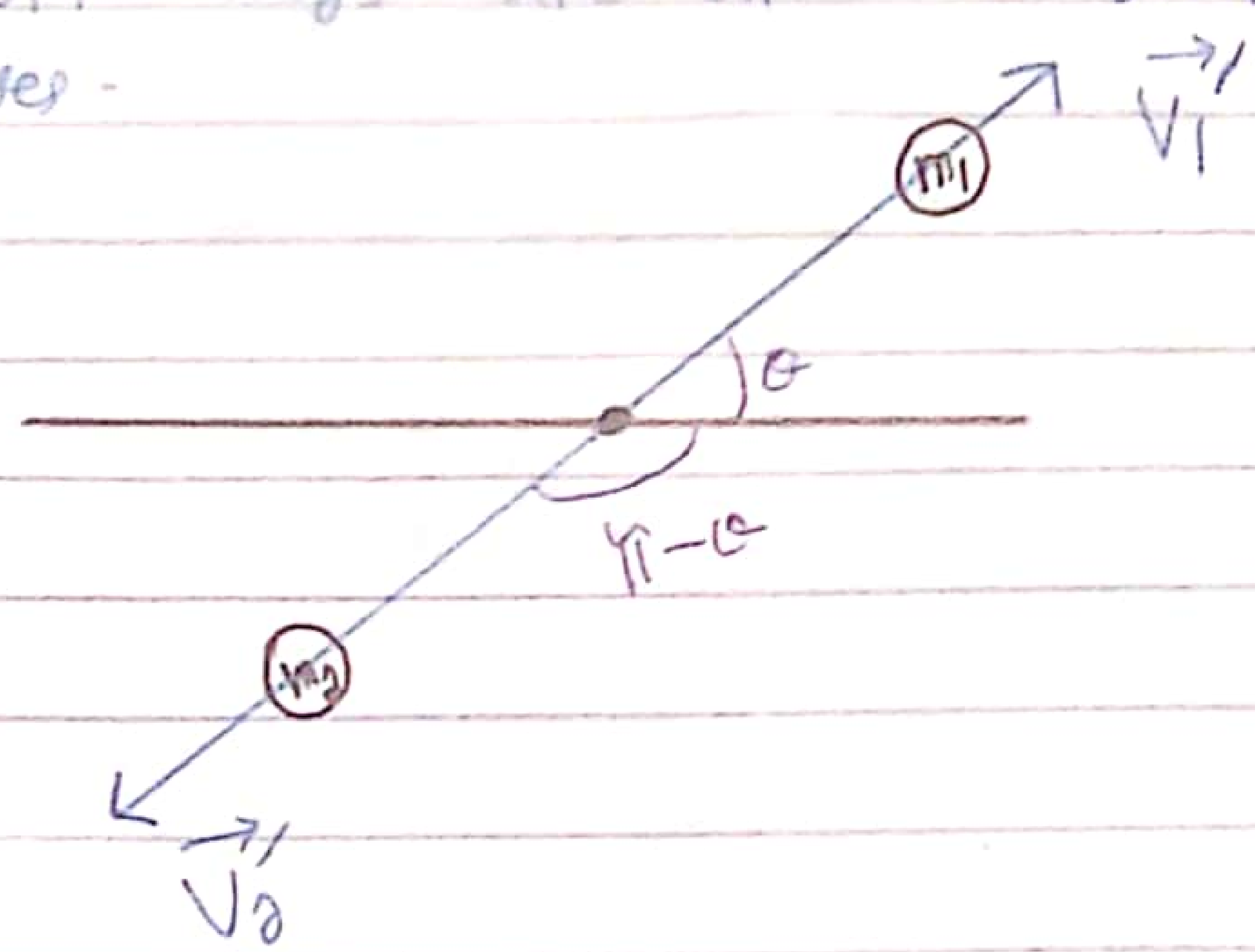
Since we placed COM on m_2 so m_2 has same velocity as COM but in opposite direction.

$\Rightarrow$

$$\vec{u_2'} = -\vec{V}$$

After collision

Since COM always lie on line joing the two masses.



In COM system, collision is colinear or 1-D. Momentum is zero in COM.

Conservation of linear Momentum In COM:

So,

$$m_1 u_1' + m_2 u_2' = m_1 V_1' + m_2 V_2' = 0$$

$\Rightarrow$

$$m_1 u_1' + m_2 u_2' = 0 \rightarrow (1) \qquad m_1 V_1' + m_2 V_2' = 0 \rightarrow (2)$$

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Conservation of K.E:

$$\Rightarrow \frac{1}{2}m_1 u_1'^2 + \frac{1}{2}m_2 u_2'^2 = \frac{1}{2}m_1 v_1'^2 + \frac{1}{2}m_2 v_2'^2 \rightarrow (3)$$

Putting value of u_2' and v_2' from (1) and (2),

(1) $\Rightarrow$

$$u_2' = -\frac{m_1}{m_2}u_1'$$

(2) $\Rightarrow$

$$v_2' = -\frac{m_1}{m_2}v_1'$$

3) $\Rightarrow$

$$m_1 \vec{u}_1'^2 + m_2\left(-\frac{m_1}{m_2}u_1'\right)^2 = m_1 v_1'^2 + m_2\left(-\frac{m_1}{m_2}v_1'\right)^2$$

$$\Rightarrow m_1 u_1'^2 + \frac{m_1^2}{m_2}u_1'^2 = m_1 v_1'^2 + \frac{m_1^2}{m_2}v_1'^2$$

$$\Rightarrow m u_1'^2 \cancel{\times}\left(1+\frac{m_1}{m_2}\right) = m_1 v_1'^2\left(1+\frac{m_1}{m_2}\right)$$

$$\Rightarrow m_1 u_1'^2 = m_1 v_1'^2$$

$$\Rightarrow u_1'^2 = v_1'^2$$

$$\Rightarrow \vec{u}_1' = \vec{v}_1'$$

Similarly,

$$u_2' = v_2'$$

So, velocties are same before and after collision.

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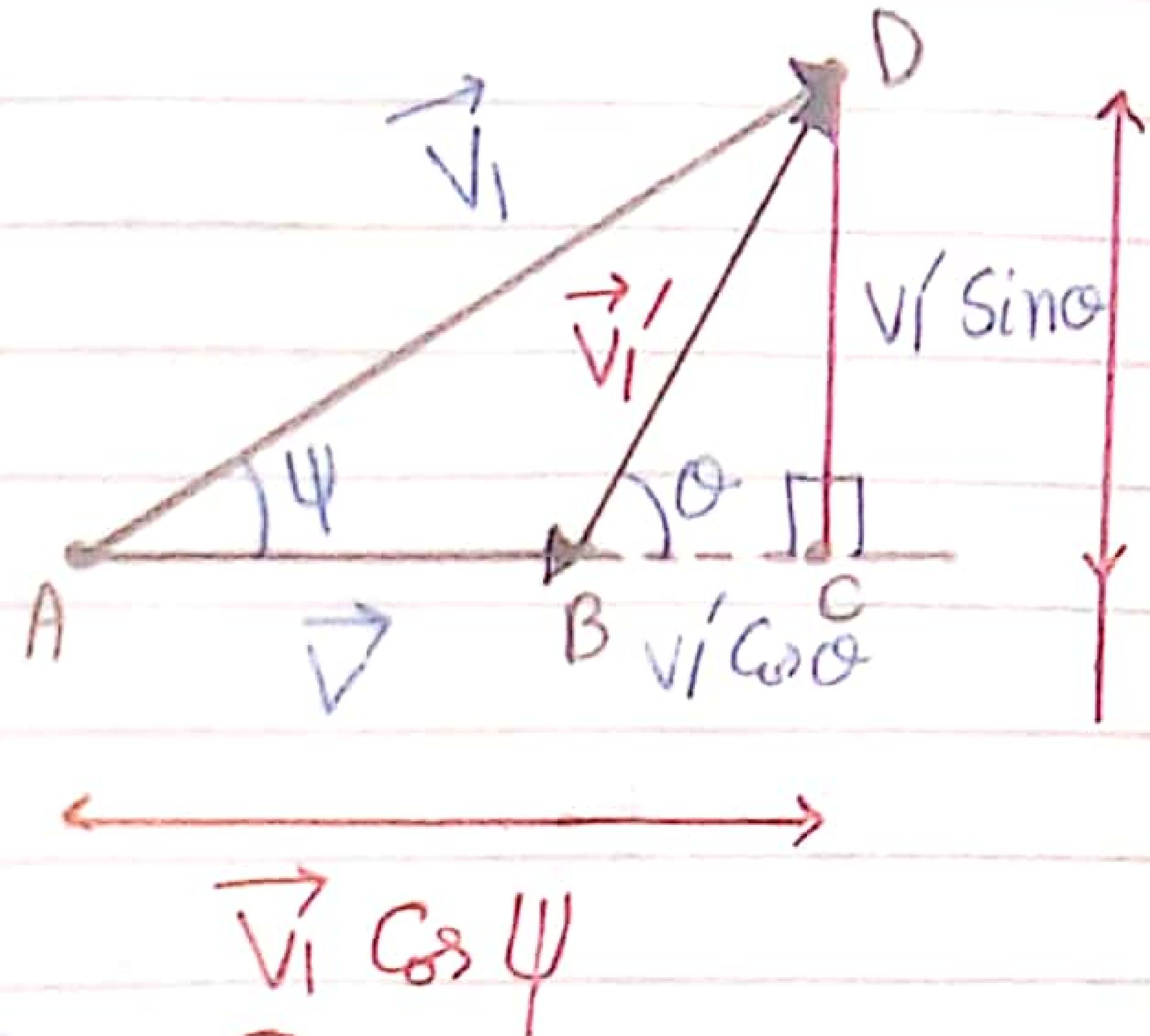
Relation b/w LAB and COM System:

As,

$$V_1 = V_1' + V$$

$\Rightarrow$ (Because we added the velocity of COM.)

$$V_2 = V_2' + V$$



By using Head to tail rule,

$$V_1 \cos\psi = V + V_1' \cos\theta \rightarrow (1)$$

$$V_1 \sin\psi = V_1' \sin\theta \rightarrow (2)$$

$\Rightarrow$ Dividing (2) by (1),

$$\frac{V_1 \sin\psi}{V_1 \cos\psi} = \frac{V_1' \sin\theta}{V + V_1' \cos\theta} \rightarrow A)$$

$\Rightarrow$

$$\tan\psi = \frac{\sin\theta}{\frac{V}{V_1'} + \cos\theta}$$

$$\because \vec{V} = \frac{m_1 u_1}{m_1 + m_2}$$

Putting value of "$\vec{V}$",

$$\Rightarrow \tan\psi = \frac{\sin\theta}{\left(\frac{m_1 u_1}{m_1+m_2}\right) \times \left(\frac{m_2+m_1}{m_2 u_1}\right) + \cos\theta}$$

$$\Rightarrow \tan\psi = \frac{\sin\theta}{\frac{m_1}{m_2} + \cos\theta}$$

It is relation b/w ψ and θ in going LAB system.

Cases:

I- If,

$$m_1 = m_2$$

$$\Rightarrow \tan\psi = \frac{\sin\theta}{1+\cos\theta}$$

$$= \frac{2\sin\frac{\theta}{2}\cos\theta/2}{2\cos^2\frac{\theta}{2}}$$

$$\tan\psi = \tan\theta/2$$

$$\Rightarrow \boxed{\psi = \frac{\theta}{2}}$$

II- If,

$$m_1 \ll m_2, \quad m_1 \approx 0$$

$$\Rightarrow \tan\psi = \frac{\sin\theta}{0+\cos\theta}$$

$$\Rightarrow \psi = \theta$$

III - If,

$m_2 \ll m_1$,

$\Rightarrow$

$$\tan\psi = \frac{\sin\theta}{\infty}$$

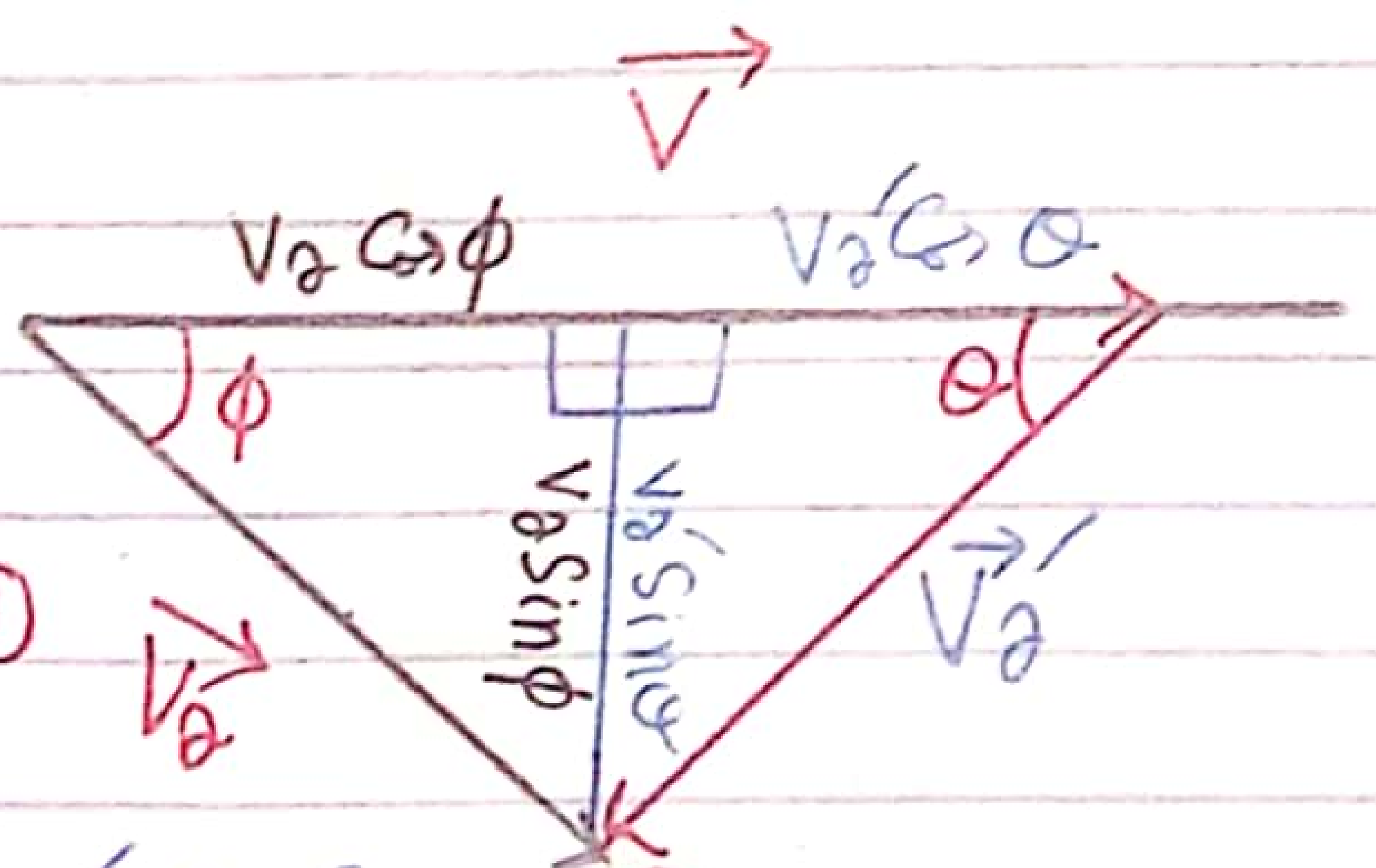
$$= 0$$

$\Rightarrow$

$$\boxed{\psi = 0}$$

Now,

$$V_2 = V_2' + V$$



So from figure

$$V_2\sin\phi = V_2'\sin\theta \rightarrow (3)$$

$$V = V_2\cos\phi + V_2'\cos\theta \rightarrow (4)$$

$\Rightarrow$

$$V_2\cos\phi = V - V_2'\cos\theta$$

$\Rightarrow$

$\frac{3)}{4)} \Rightarrow$

$$\frac{V_2\sin\phi}{V_2\cos\phi} = \frac{V_2'\sin\theta}{V - V_2'\cos\theta}$$

$\Rightarrow$

$$\tan\phi = \frac{\sin\theta}{\frac{V}{V_2'} - \cos\theta}$$

As,

$$\vec{V} = \frac{m_1 u_1'}{m_1 + m_2}$$

or

$$V_2' = V$$

$\Rightarrow$

$$\tan\phi = \frac{\sin\theta}{1 - \cos\theta}$$

$\Rightarrow$

$$\tan\phi = \frac{2\sin\theta/2 \cos\theta/2}{2\sin^2\theta/2}$$

$\Rightarrow$

$$\boxed{\tan\phi = \cot\frac{\theta}{2}}$$

$\Rightarrow$

$$\tan\phi = \tan\left(\frac{\pi}{2} - \frac{\theta}{2}\right)$$

$\Rightarrow$

$$\boxed{\phi = \frac{\pi}{2} - \theta/2}$$