Power Systems Analysis
ET-321

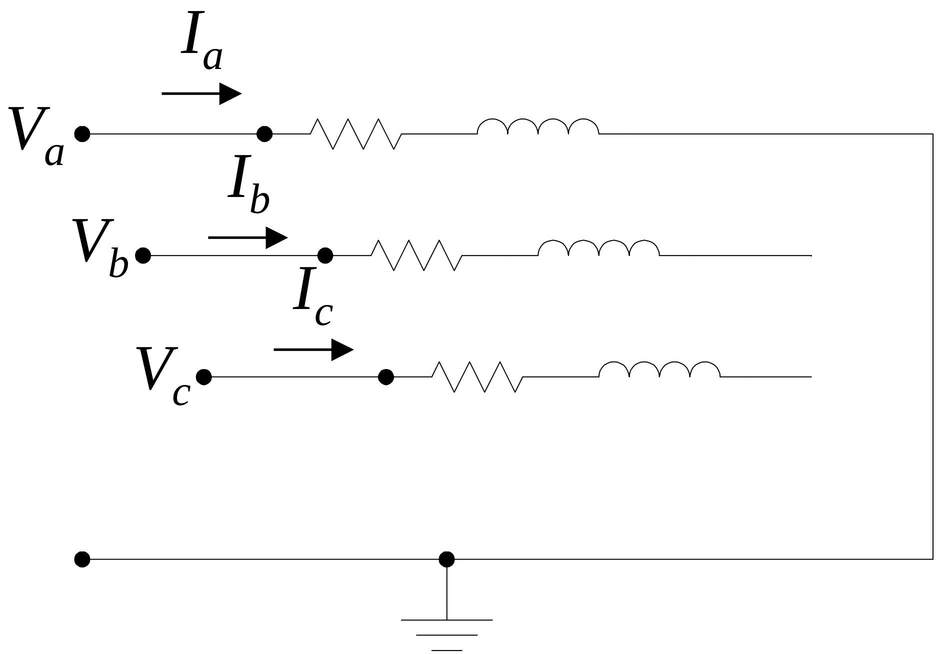
References

(Hadi Sadaat)

Common Unbalanced Network Faults

- **Single-line-to-ground faults**
- **Double-line-to-ground faults**
- **Line-to-line faults**

Single Line to Ground Fault



$$V_a = 0$$

$$I_f = I_a$$

$$I_b = I_c = 0$$

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ 0 \\ 0 \end{bmatrix}$$

$$I_{a0} = I_{a1} = I_{a2} = \tfrac{1}{3} I_a$$

$$V_a = V_{a0} + V_{a1} + V_{a2} = 0$$

Single Line to Ground Fault

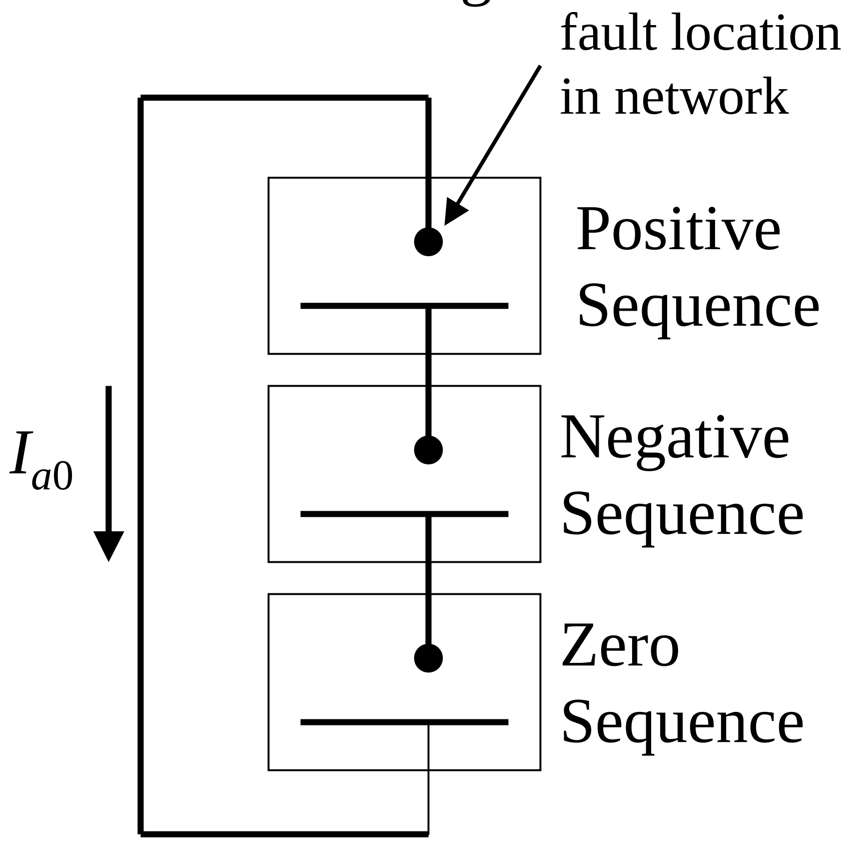
$$V_a = E_a - Z_a I_a = 0$$
$$= E_a - A\left(Z_{012} I_{012}\right)$$
$$= E_a - \left(Z_{a0} I_{a0} + Z_{a1} I_{a1} + Z_{a2} I_{a2}\right)$$
$$= E_a - \left(Z_{a0} + Z_{a1} + Z_{a2}\right) I_{a0}$$

$$I_{a0} = \frac{E_a}{\left(Z_{a0} + Z_{a1} + Z_{a2}\right)}$$

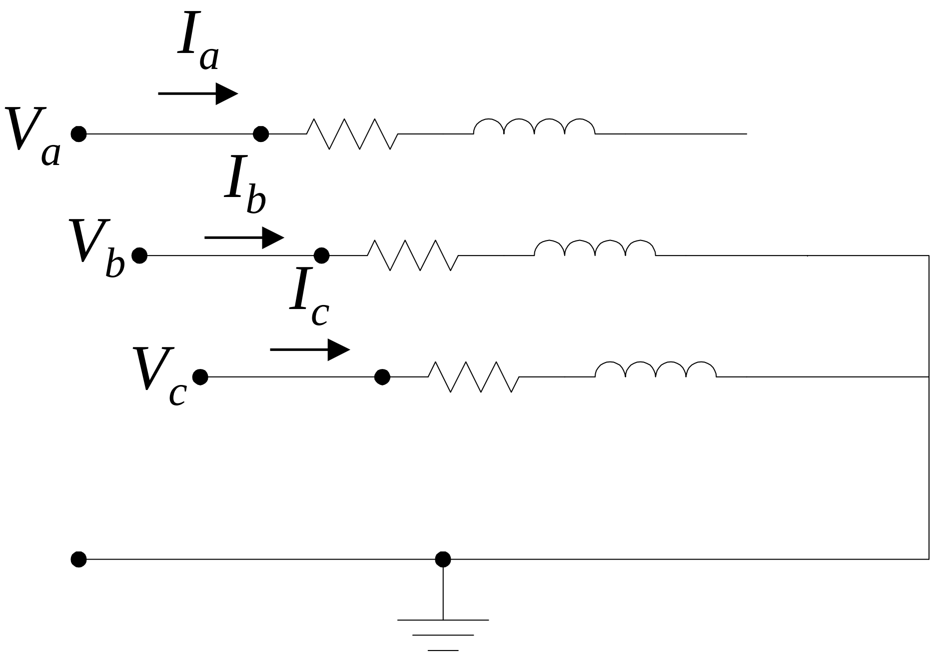
$$I_{a0} = I_{a1} = I_{a2}$$

$$I_f = 3\, I_{a0}$$

Network Diagram



Double Line to Ground Fault



$$V_b = 0$$

$$V_c = 0$$

$$I_a = I_{a0} + I_{a1} + I_{a2} = 0$$

$$I_f = I_b + I_c$$

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_a \\ 0 \\ 0 \end{bmatrix}$$

$$V_{a0} = V_{a1} = V_{a2}$$

Double Line to Ground Fault

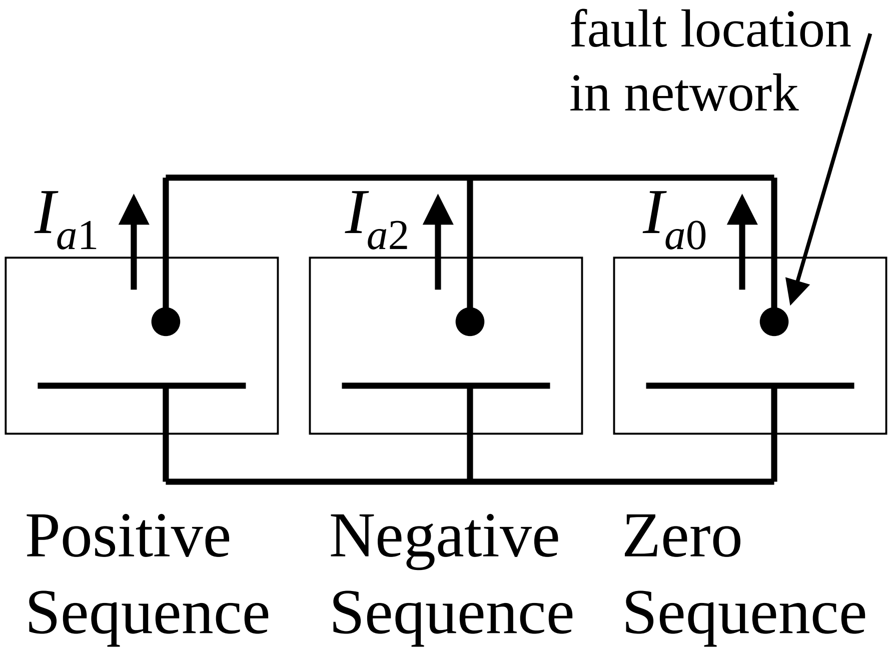
$$I_{a1} = \left(1 + \frac{Z_1}{Z_0} + \frac{Z_1}{Z_2}\right) = V_{a1}\frac{Z_2 + Z_0}{Z_2 Z_0}$$

$$I_{a1} = \frac{V_{a1}(Z_0 + Z_2)}{(Z_1 Z_0 + Z_2 Z_0 + Z_1 Z_2)}$$

$$I_{a0} = \frac{V_{a1}}{\left(Z_1 + \frac{Z_0 Z_2}{Z_0 + Z_2}\right)}$$

$$I_{a0} = -\frac{V_{a0}}{Z_0}, \quad I_{a2} = -\frac{V_{a2}}{Z_2}$$

Network Diagram



Double Line to Ground Fault

$$I_{a0} + I_{a1} + I_{a2} = 0$$

$$\mathbf{I}_{abc} = \mathbf{A}\,\mathbf{I}_{012}$$

$$I_f = I_b + I_c$$

$$= I_{a0} + a^2 I_{a1} + a\, I_{a2} + I_{a0} + a\, I_{a1} + a^2 I_{a2}$$

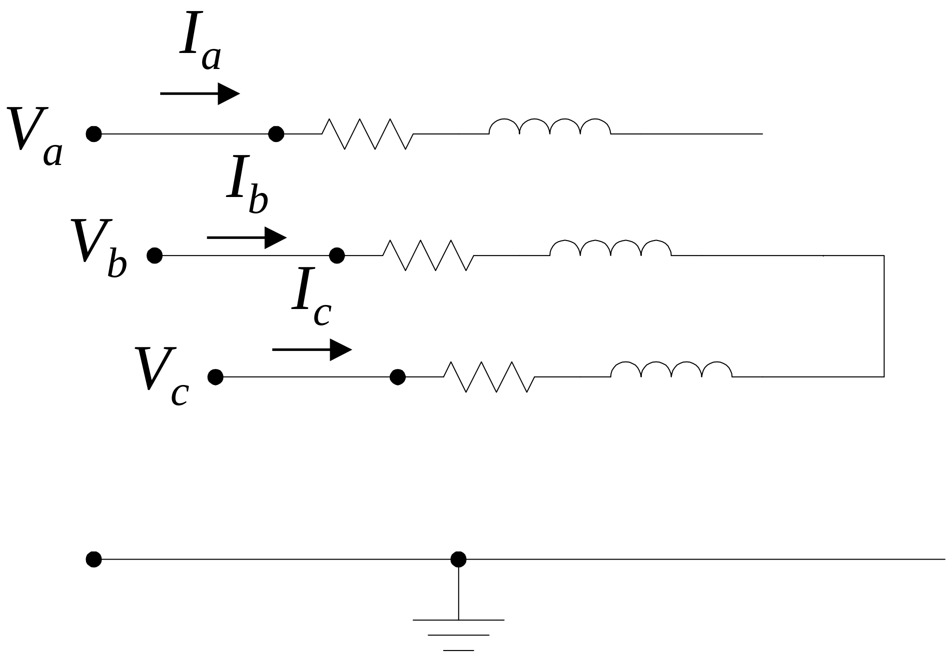
$$= 2\, I_{a0} + a^2 \left(I_{a1} + I_{a2}\right) + a\left(I_{a1} + I_{a2}\right)$$

$$= 2\, I_{a0} + \left(a^2 + a\right)\left(I_{a1} + I_{a2}\right)$$

$$\left(a^2 + a\right) = -1$$

$$I_f = 2\, I_{a0} - \left(I_{a1} + I_{a2}\right)$$

Line-to-Line Fault



$$V_b - V_c = 0$$

$$I_b + I_c = 0$$

$$I_a = I_{a0} + I_{a1} + I_{a2} = 0$$

$$I_f = I_b - I_c$$

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} 0 \\ I_b \\ -I_b \end{bmatrix}$$

Line-to-Line Fault

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3}\begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix}\begin{bmatrix} 0 \\ I_b \\ -I_b \end{bmatrix}$$

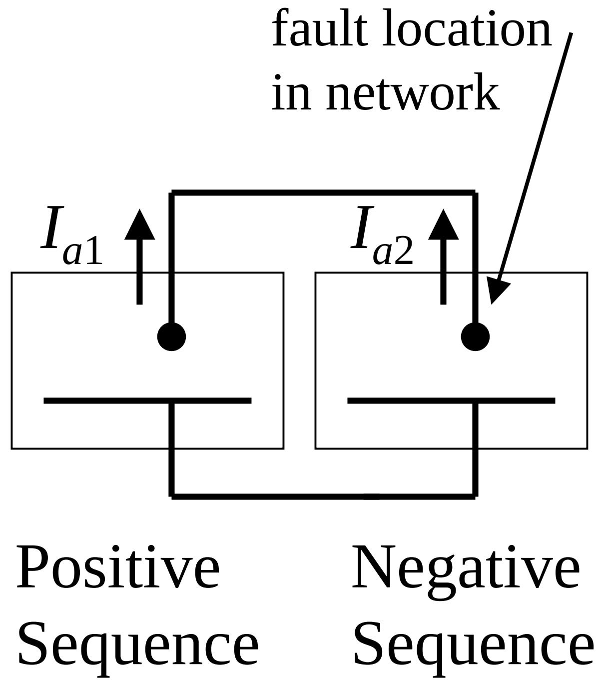
$$I_{a0} = 0$$

$$I_{a1} = \tfrac{1}{3}\left(a - a^2\right)\ I_b$$

$$I_{a2} = \tfrac{1}{3}\left(a^2 - a\right)\ I_b$$

$$I_{a1} = -I_{a2}$$

Network Diagram



Line-to-Line Fault

$$V_b - V_c = \left(a^2 - a\right)\left(V_k^{a1} - V_k^{a2}\right) = 0$$

$$V_k^{a1} = V_a - Z_k^1 I_{a1}$$

$$V_k^{a2} = -Z_k^2 I_k^{a2} = Z_k^2 I_k^{a1}$$

$$\left(a^2 - a\right)\left[V_a - \left(Z_k^1 + Z_k^2\right) I_k^{a1}\right] = 0$$

$$V_k^a - \left(Z_k^1 + Z_k^2\right) I_k^{a1} = 0$$

$$I_k^{a1} = \frac{V_{k,pre-f}^a}{Z_k^1 + Z_k^2}$$

Line-to-Line Fault

$$I_k^{a1} = -I_k^{a2}$$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} 0 \\ I_k^{a1} \\ -I_k^{a1} \end{bmatrix}$$

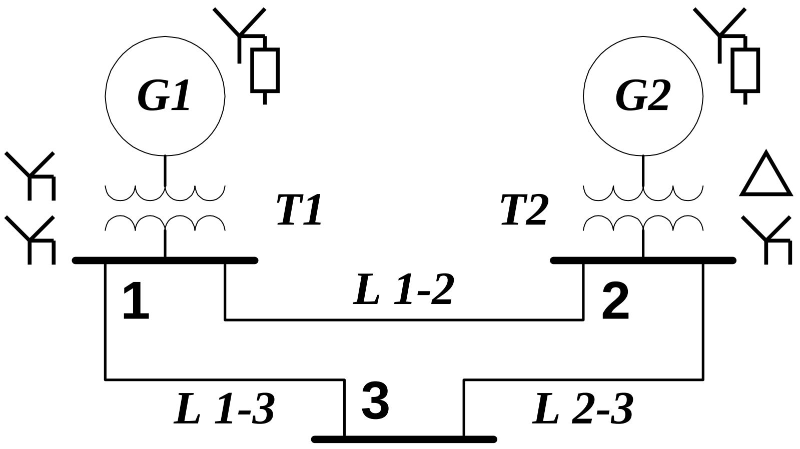
$$I_b = \left(a^2 - a\right) I_k^{a1} = -j\sqrt{3}\, I_k^{a1}$$

$$I_b = \frac{-j\sqrt{3}\, V_k^a}{Z_k^1 + Z_k^2}$$

$$I_c = -I_b$$

Example

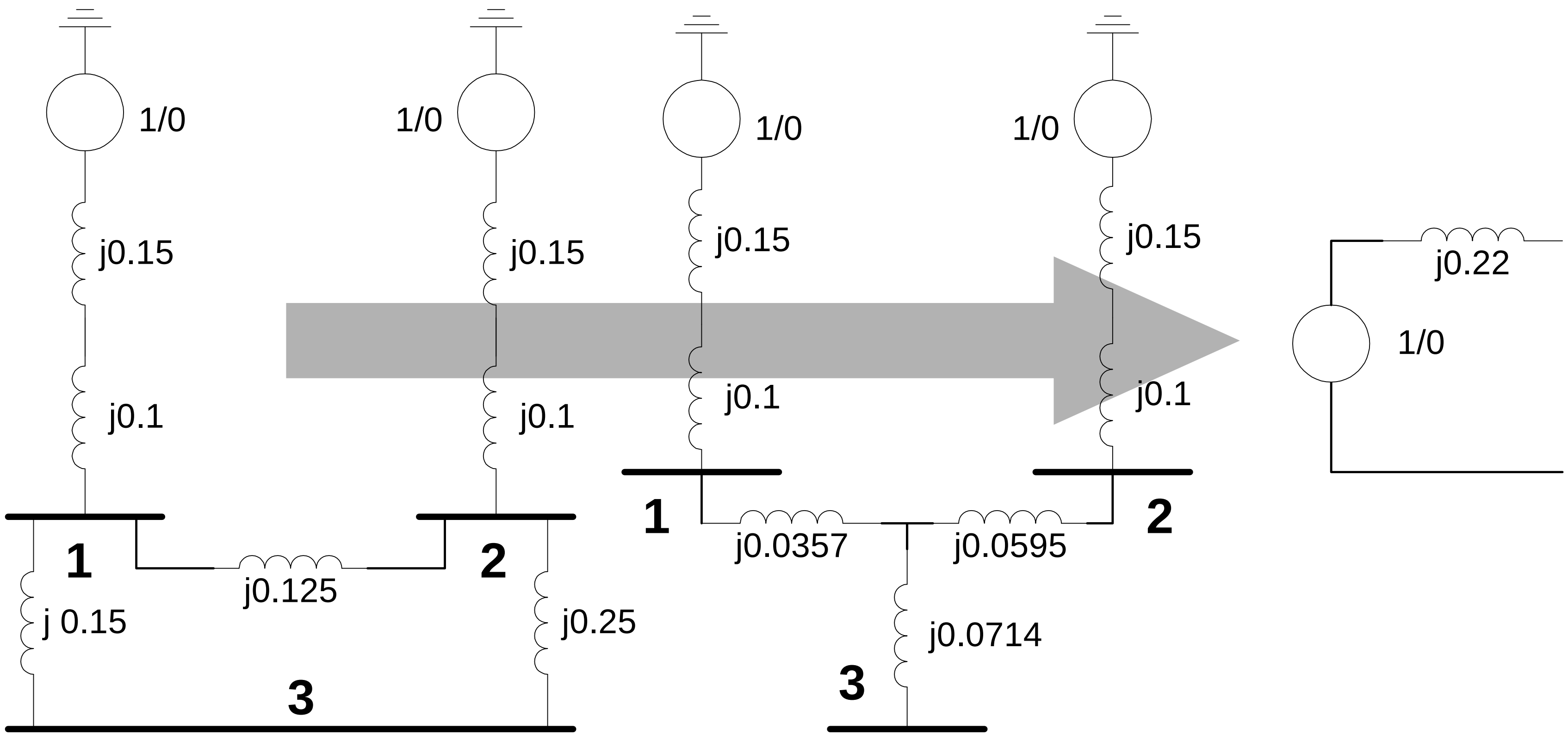
- The neutral of each generator is grounded through a current limiting resistor of 8.333 % on a 100 MVA base
- generators are running at no-load at rated voltage and in phase
- all network data is expressed on a 100 MVA base
- Find the fault current for 3-phs, 1-phs, L-L, L-L-G bolted faults



Item	V Rating	X1	X2	X0
G1	20 kV	15%	15%	5%
G2	20 kV	15%	15%	5%
T1	20/200 kV	10%	10%	10%
T2	20/200 kV	10%	10%	10%
L 1-2	200 kV	12.5%	12.5%	30%
L 1-3	200 kV	15%	15%	35%
L 2-3	200 kV	25%	25%	71.25%

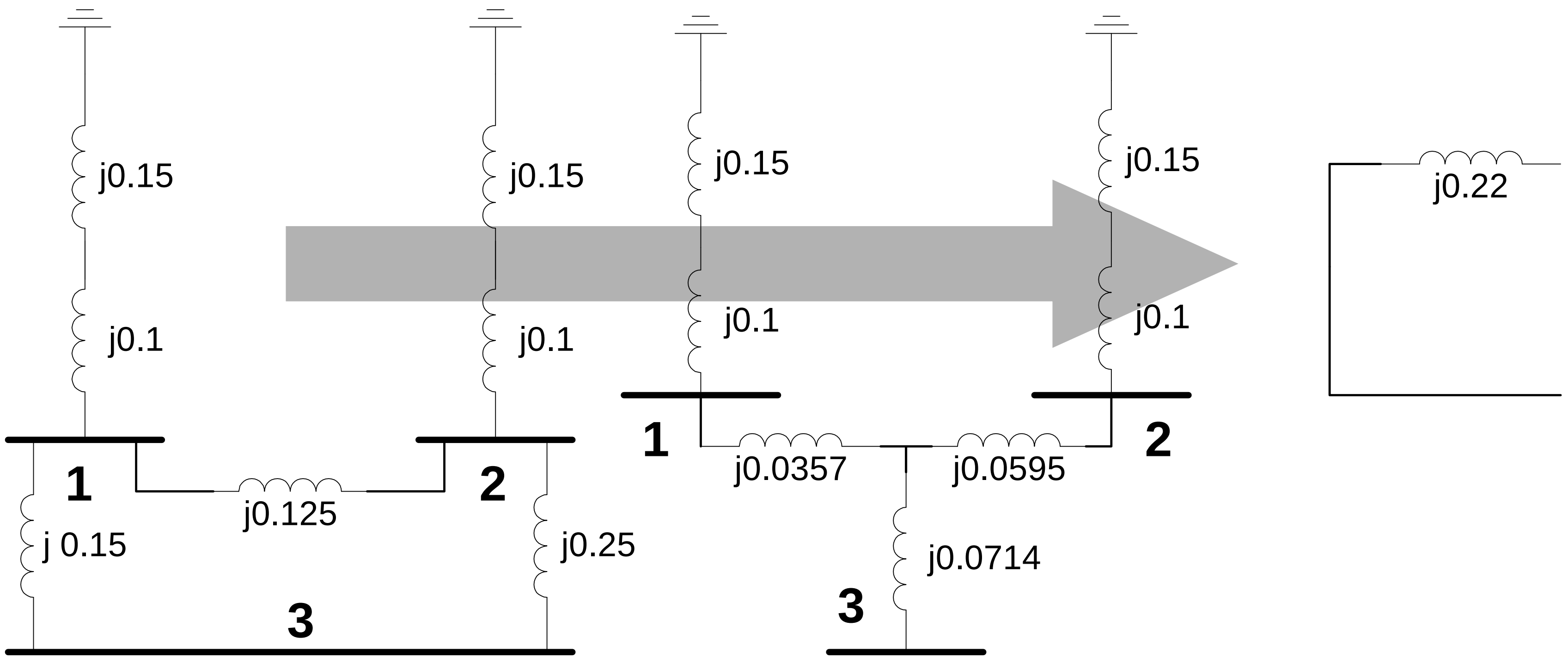
Example

- **Positive Sequence Network**



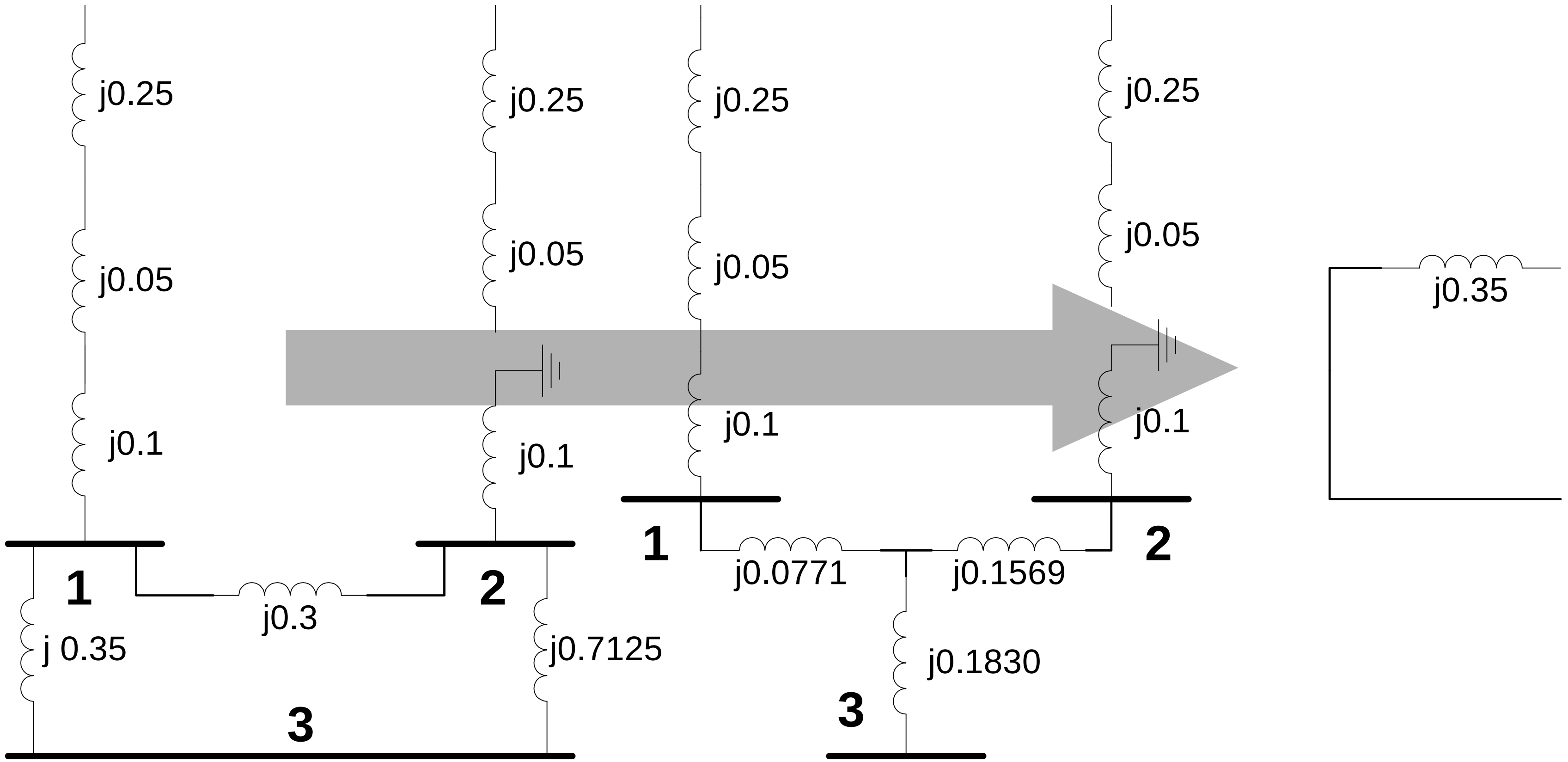
Example

- **Negative Sequence Network**



Example

- **Zero Sequence Network**



Example

- **3-phase fault**

$$I_3^{a1}(F) = \frac{V_3^{a1}}{Z_{33}^1} = \frac{1.0\angle 0^\circ}{j0.22} = -j4.54 \text{ pu}$$

$$I_{af} = I_3^{a1}(F) = -j4.54 \text{ pu}$$

- **SLG fault**

$$I_3^{a0} = I_3^{a1} = I_3^{a2} = \frac{V_3^{a1}}{Z_{33}^0 + Z_{33}^1 + Z_{33}^2}$$

$$= \frac{1.0\angle 0^\circ}{j0.35 + j0.22 + j0.22} = -j1.266 \text{ pu}$$

$$I_{af} = 3I_3^{a1} = -j3.80 \text{ pu}$$

Example

- **L-L fault**

$$I_3^{a0} = 0$$

$$I_3^{a1} = -I_3^{a2} = \frac{V_3^{a1}}{Z_{33}^1 + Z_{33}^2} = \frac{1.0\angle 0^\circ}{j0.22 + j0.22} = -j2.27 \text{ pu}$$

$$I_{bf} = -I_{cf} = -j\sqrt{3}\left(-j2.27\right) = -3.936 \text{ pu}$$

Example

- **DLG fault**

$$I_3^{a1} = \frac{V_3^{a1}}{Z_{33}^1 + \dfrac{Z_{33}^0 \cdot Z_{33}^2}{Z_{33}^0 + Z_{33}^2}} = \frac{1.0\angle 0^\circ}{j0.22 + \dfrac{j0.35 \cdot j0.22}{j0.35 + j0.22}} = -j2.816 \text{ pu}$$

$$I_3^{a2} = -\frac{V_3^{a1} - Z_{33}^1 I_3^{a1}}{Z_{33}^2} = -\frac{1.0\angle 0^\circ - (j0.22)(-j2.816)}{j0.22} = j1.729 \text{ pu}$$

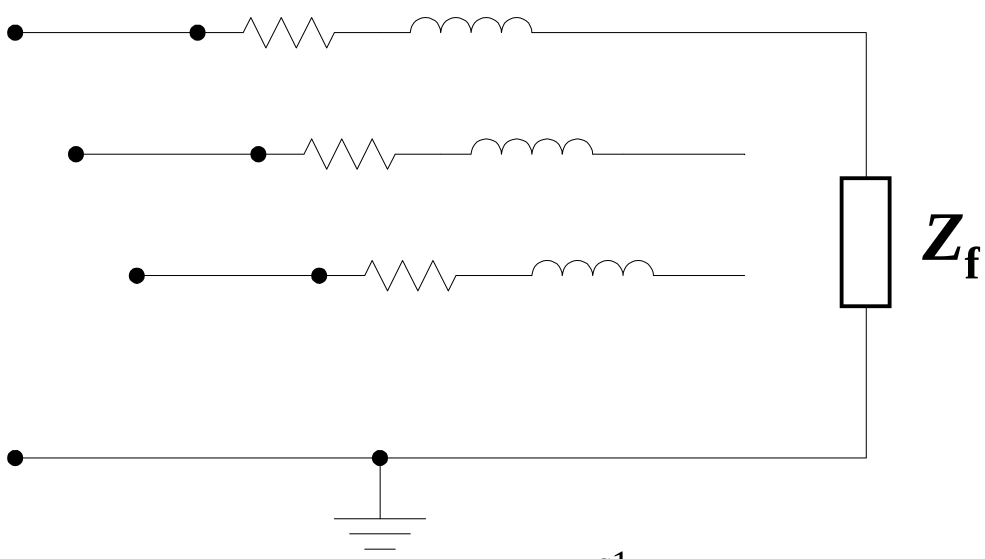
$$I_3^{a0} = -\frac{V_3^{a1} - Z_{33}^1 I_3^{a1}}{Z_{33}^0} = -\frac{1.0\angle 0^\circ - (j0.22)(-j2.816)}{j0.35} = j1.087 \text{ pu}$$

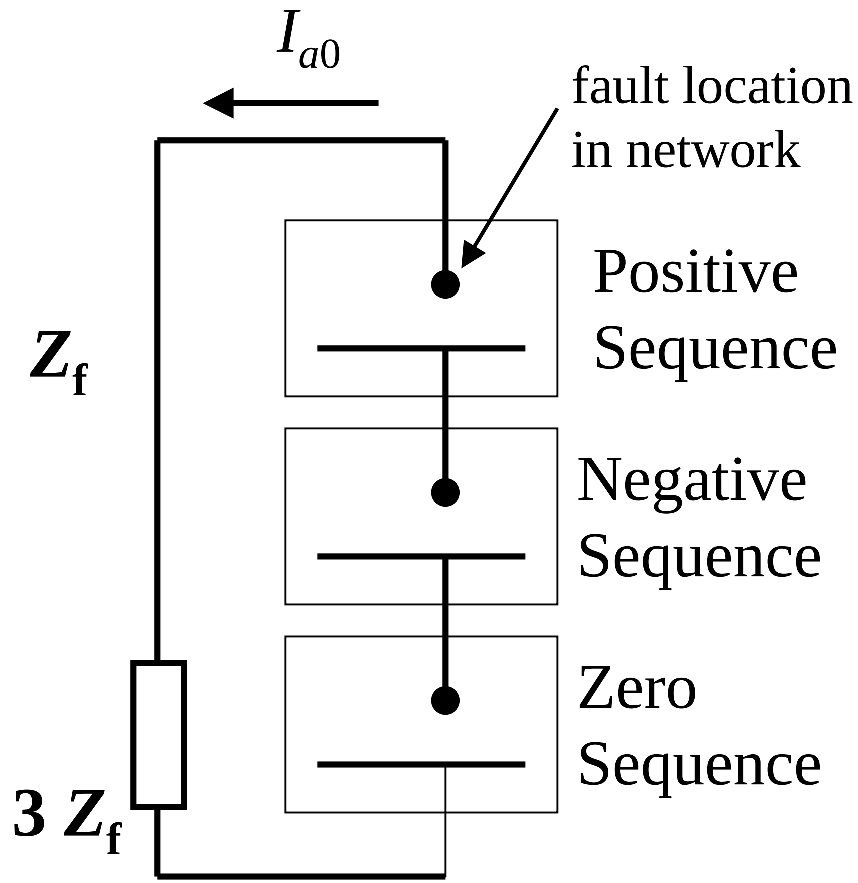
$$I_f = I_3^b + I_3^c = 2I_3^{a0} + (a + a^2)(I_3^{a1} + I_3^{a2})$$

$$= 2(j1.087) - (-j2.816 + j1.729) = j3.261 \text{ pu}$$

Fault Analysis with Fault Impedances

- **Single Line to Ground Fault**

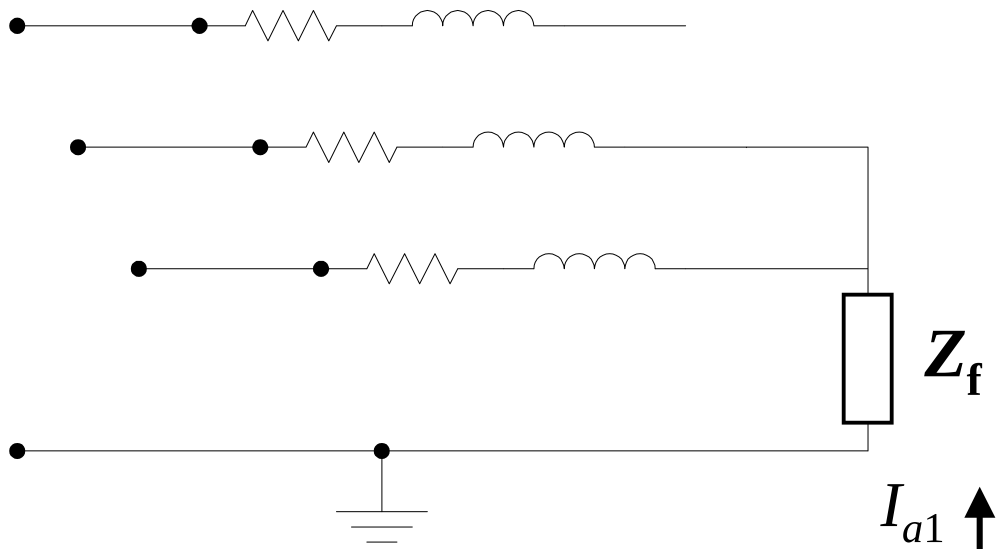


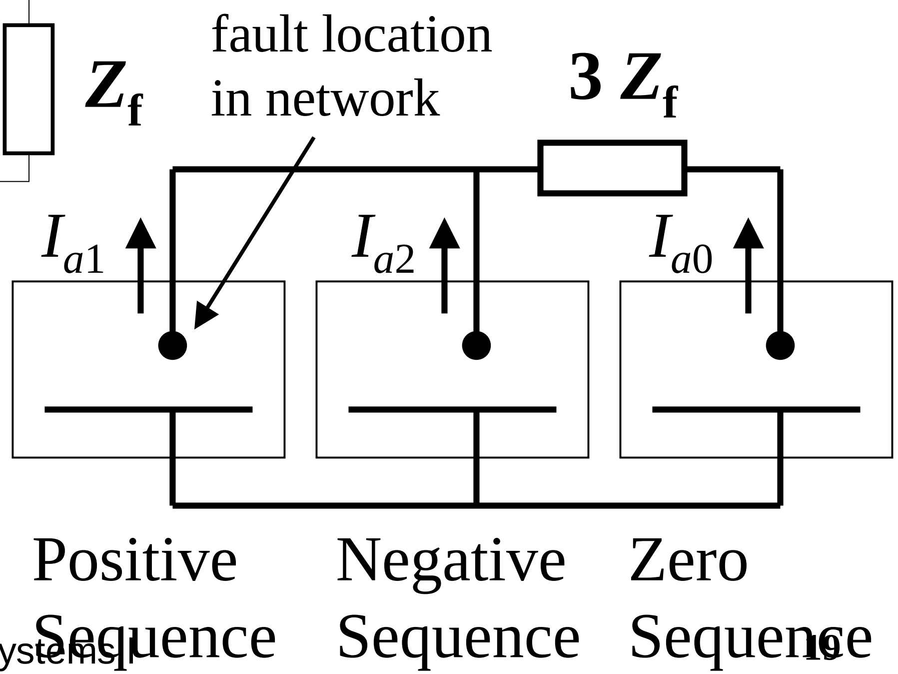


$$I_k^{a0} = I_k^{a1} = I_k^{a2} = \frac{V_{k,pre-f}^{a1}}{Z_k^0 + Z_k^1 + Z_k^2 + 3Z_f}$$

Fault Analysis with Fault Impedances

- **Double Line to Ground Fault**





Fault Analysis with Fault Impedances

- **Double Line to Ground Fault**

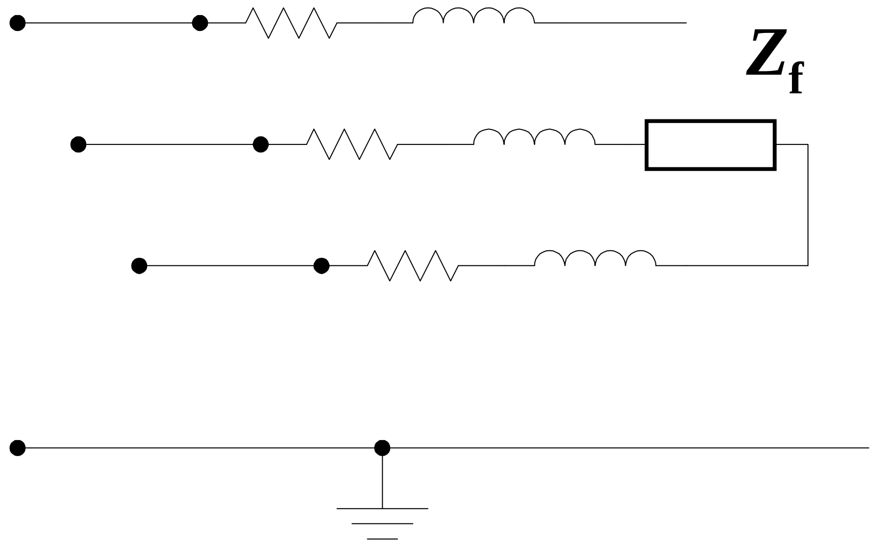
$$I_k^{a1} = \frac{V_k^{a1}}{Z_k^1 + \left(Z_k^0 + 3Z_f\right) \cdot Z_k^2 \Big/ \left(Z_k^0 + 3Z_f + Z_k^2\right)}$$

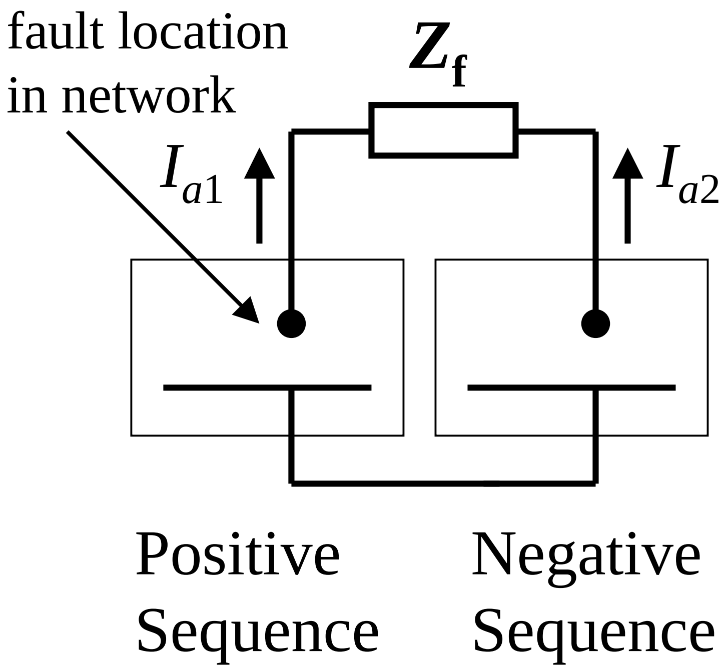
$$I_k^{a2} = -\frac{V_k^{a1} - Z_k^1 I_k^{a1}}{Z_k^2}$$

$$I_k^{a0} = -\frac{V_k^{a1} - Z_k^1 I_k^{a1}}{Z_k^0 + 3Z_f}$$

Fault Analysis with Fault Impedances

- **Line to Line Fault**





$$I_k^{a1} = -I_k^{a2} = \frac{V_k^a}{Z_k^1 + Z_k^2 + Z_f}$$

Fault Analysis using Z-Bus Matrix

- **Form the Positive Sequence and the Zero Sequence bus impedance matrices**
  - The Negative Sequence matrix is the same as the Positive Sequence matrix
- **Use the diagonal element of the faulted bus with the following modified fault equations**

Fault Analysis using Z-Bus Matrix

- **Single Line to Ground Fault**

$$I_k^{a0} = I_k^{a1} = I_k^{a2} = \frac{V_{k,pre-f}^{a1}}{\mathbf{Z}_{kk}^{0} + 2 \cdot \mathbf{Z}_{kk}^{1} + Z_f}$$

- **Line to Line Fault**

$$I_k^{a1} = -I_k^{a2} = \frac{V_k^{a}}{2 \cdot \mathbf{Z}_{kk}^{1} + Z_f}$$

Fault Analysis using Z-Bus Matrix

- **Double Line to Ground Fault**

$$I_k^{a1} = \frac{V_k^{a1}}{\mathbf{Z}_{kk}^{1} + \left(\mathbf{Z}_{kk}^{0} + 3Z_f\right) \cdot \mathbf{Z}_{kk}^{1} \Big/ \left(\mathbf{Z}_{kk}^{0} + 3Z_f + \mathbf{Z}_{kk}^{1}\right)}$$

$$I_k^{a2} = -\frac{V_k^{a1} - \mathbf{Z}_{kk}^{1} I_k^{a1}}{\mathbf{Z}_{kk}^{1}}$$

$$I_k^{a0} = -\frac{V_k^{a1} - \mathbf{Z}_{kk}^{1} I_k^{a1}}{\mathbf{Z}_{kk}^{0} + 3Z_f}$$

Fault Analysis using Z-Bus Matrix

- **Bus voltages during fault**

$$V_i^{a0} = 0 - Z_{ik}^0 I_k^{a0}$$

$$V_i^{a1} = V_i^a - Z_{ik}^1 I_i^{a1}$$

$$V_i^{a2} = 0 - Z_{ik}^2 I_k^{a2}$$

- **Line currents during fault**

$$I_{ij}^{a0} = \left(V_i^{a0} - V_j^{a0}\right) / z_{ij}^0$$

$$I_{ij}^{a1} = \left(V_i^{a1} - V_j^{a1}\right) / z_{ij}^1$$

$$I_{ij}^{a2} = \left(V_i^{a2} - V_j^{a2}\right) / z_{ij}^2$$