Power Systems Analysis
ET-321

References

- **Elements of power system analysis**
  (William Stevenson)

- **Power system analysis**
  (Hadi Sadaat)

Symmetrical Fault Current Calculations

ET 321

Introduction

- A **fault** in a circuit is any failure that interferes with the normal system operation.
- Lighting strokes cause most faults on high-voltage transmission lines producing a very high transient that greatly exceeds the rated voltage of the line.
- This high voltage usually causes flashover between the phases and/or the ground creating an arc.
- Since the impedance of this new path is usually low, an excessive current may flow.
- Faults involving ionized current paths are also called transient faults. They usually clear if power is removed from the line for a short time and then restored.

Introduction

- If one, or two, or all three phases break or if insulators break due to fatigue or inclement weather, this fault is called a permanent fault.
- Approximately 75% of all faults in power systems are transient in nature.
- Knowing the magnitude of the fault current is important when selecting protection equipment (type, size, etc..)

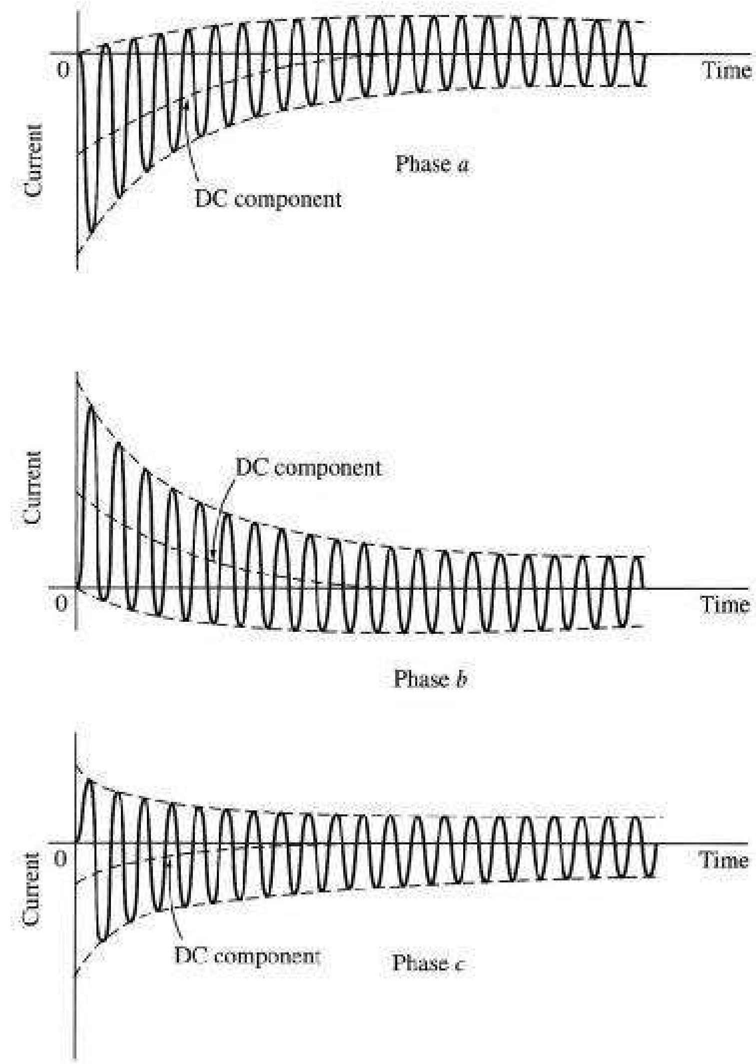
Type of fault	Abbreviation	Type
Single line-to-ground	SLG	Unsymmetrical
Line-to-line	LL	Unsymmetrical
Double line-to-ground	LLG	Unsymmetrical
Symmetrical three-phase	3P	Symmetrical

3-Phase fault current transients in synchronous generators

When a symmetrical 3-phase fault occurs at the terminals of a synchronous generator, the resulting current flow in the phases of the generator appear as shown.

The current can be represented as a transient DC component added on top of a symmetrical AC component.

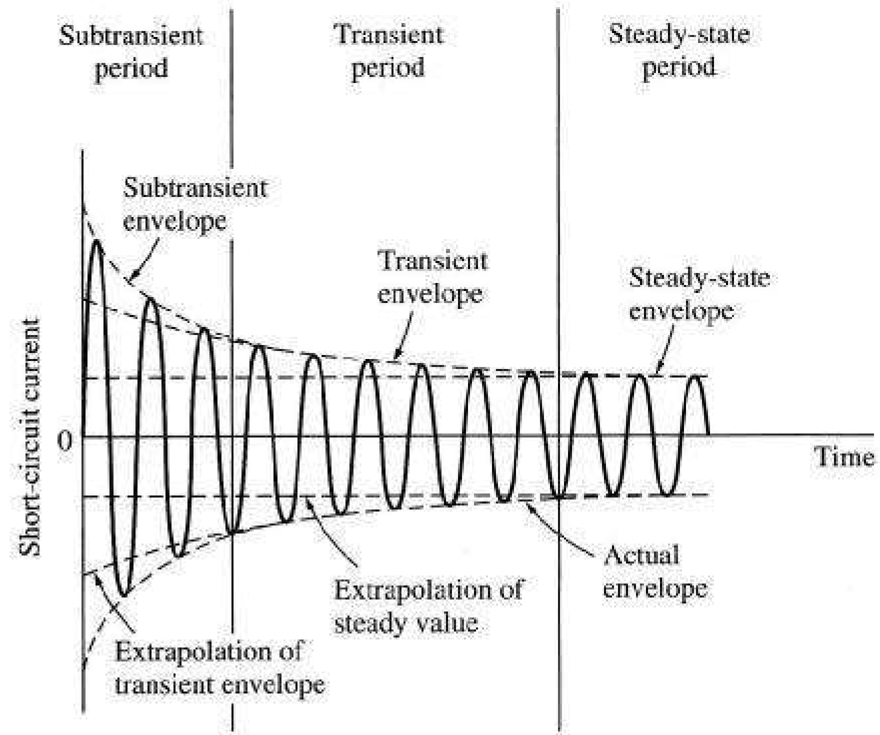
Before the fault, only AC voltages and currents are present, but immediately after the fault, both AC and DC currents are present.



Fault current transients in machines

- When the fault occurs, the AC component of current jumps to a very large value, but the total current cannot change instantly since the series inductance of the machine will prevent this from happening.
- The transient DC component of current is just large enough such that the sum of the AC and DC components just ***after*** the fault equals the AC current just ***before*** the fault.
- Since the instantaneous values of current at the moment of the fault are different in each phase, the magnitude of DC components will be different in different phases.
- These DC components decay fairly quickly, but they initially average about 50 - 60% of the AC current flow the instant after the fault occurs. The total initial current is therefore typically 1.5 or 1.6 times the AC component alone.

Symmetrical AC component of the fault current:



- There are three periods of time:
  - Sub-transient period: first couple of cycles after the fault – AC current is very large and falls rapidly;
  - Transient period: current falls at a slower rate;
  - Steady-state period: current reaches its steady value.
- It is possible to determine the time constants for the sub-transient and transient periods .

Fault current transients in machines

- The AC current flowing in the generator during the sub-transient period is called the sub-transient current and is denoted by *I”.* The time constant of the sub-transient current is denoted by *T”* and it can be determined from the slope. This current can be as much as 10 times the steady-state fault current.
- The AC current flowing in the generator during the transient period is called the transient current and is denoted by *I’*. The time constant of the transient current is denoted by *T’*. This current is often as much as 5 times the steady-state fault current.
- After the transient period, the fault current reaches a steady-state condition I_{ss}. This current is obtained by dividing the induced voltage by the synchronous reactance:

$$I_{ss} = \frac{E_A}{X_s}$$

Fault current transients in machines

- The rms value of the AC fault current in a synchronous generator varies over time as

$$I(t) = (I'' - I')e^{-t/T''} + (I' - I_{ss})e^{-t/T'} + I_{ss}$$

- The sub-transient and transient reactances are defined as the ratio of the internal generated voltage to the sub-transient and transient current components:

$$X'' = \frac{E_A}{I''} \qquad X' = \frac{E_A}{I'}$$

Fault current calculations

Example 1: A 100 MVA, 13.8 kV, Y-connected, 3 phase 60 Hz synchronous generator is operating at the rated voltage and no load when a 3 phase fault occurs at its terminals. Its reactances per unit to the machine's own base are

$$X_s = 1.00 \qquad X' = 0.25 \qquad X'' = 0.12$$

and the time constants are

$$T' = 1.10\,s \qquad T'' = 0.04\,s$$

The initial DC component averages 50% of the initial AC component.

a) What is the AC component of current in this generator the instant after the fault?

b) What is the total current (AC + DC) in the generator right after the fault occurs?

c) What will the AC component of the current be after 2 cycles? After 5 s?

Fault current calculations

The base current of the generator can be computed as

$$I_{L,base} = \frac{S_{base}}{\sqrt{3}V_{L,base}} = \frac{100,000,000}{\sqrt{3} \cdot 13,800} = 4,184\ A$$

The subtransient, transient, and steady-state currents are (per-unit and Amps)

$$I'' = \frac{E_A}{X''} = \frac{1.0}{0.12} = 8.333\ pu = 34,900\ A$$

$$I' = \frac{E_A}{X'} = \frac{1.0}{0.25} = 4\ pu = 16,700\ A$$

$$I_{ss} = \frac{E_A}{X_s} = \frac{1.0}{1.0} = 1\ pu = 4,184\ A$$

Fault current calculations

a) The initial AC component of current is I'' = 34,900 A.

b) The total current (AC and DC) at the beginning of the fault is

$$I_{tot} = 1.5 I'' = 52,350\ A$$

c) The AC component of current as a function of time is

$$I(t) = (I'' - I')e^{-\frac{t}{T''}} + (I' - I_{ss})e^{-\frac{t}{T'}} + I_{ss} = 18,200 \cdot e^{-\frac{t}{0.04}} + 12,516 \cdot e^{-\frac{t}{1.1}} + 4,184\ A$$

After 2 cycles t = 1/30 s and the total AC current is

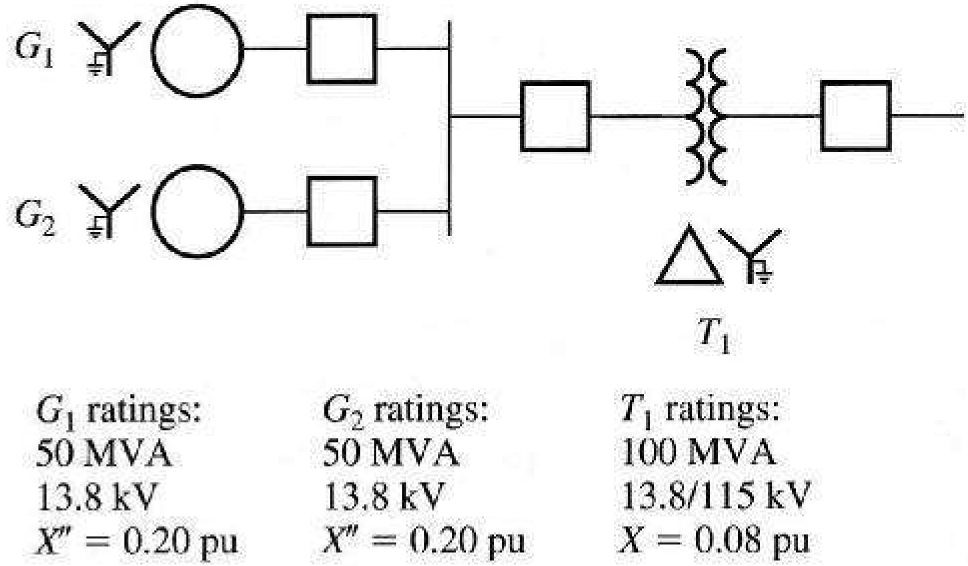
$$I\left(\frac{1}{30}\right) = 7,910 + 12,142 + 4,184 = 24,236\ A$$

At 5 s, the current reduces to

$$I(5) = 0 + 133 + 4,184 = 4,317\ A$$

Fault current transients

<u>Example 2:</u> Two generators are connected in parallel to the low-voltage side of a transformer. Generators G_1 and G_2 are each rated at 50 MVA, 13.8 kV, with a subtransient resistance of 0.2 pu. Transformer T_1 is rated at 100 MVA, 13.8/115 kV with a series reactance of 0.08 pu and negligible resistance.



Assume that initially the voltage on the high side of the transformer is 120 kV, that the transformer is unloaded, and that there are no circulating currents between the generators.

Calculate the subtransient fault current that will flow if a 3 phase fault occurs at the high-voltage side of transformer.

Fault current calculations

Let choose the per-unit base values for this power system to be 100 MVA and 115 kV at the high-voltage side and 13.8 kV at the low-voltage side of the transformer.

The subtransient reactance of the two generators to the system base is

$$Z_{new} = Z_{given}\left(\frac{V_{given}}{V_{new}}\right)^2\left(\frac{S_{new}}{S_{given}}\right)$$

Therefore:

$$X_1'' = X_2'' = 0.2\cdot\left(\frac{13,800}{13,800}\right)^2\left(\frac{100,000}{50,000}\right) = 0.4\ pu$$

The reactance of the transformer is already given on the system base, it will not change

$$X_T = 0.08\ pu$$

Symmetrical fault current calculations

The per-unit voltage on the high-voltage side of the transformer is

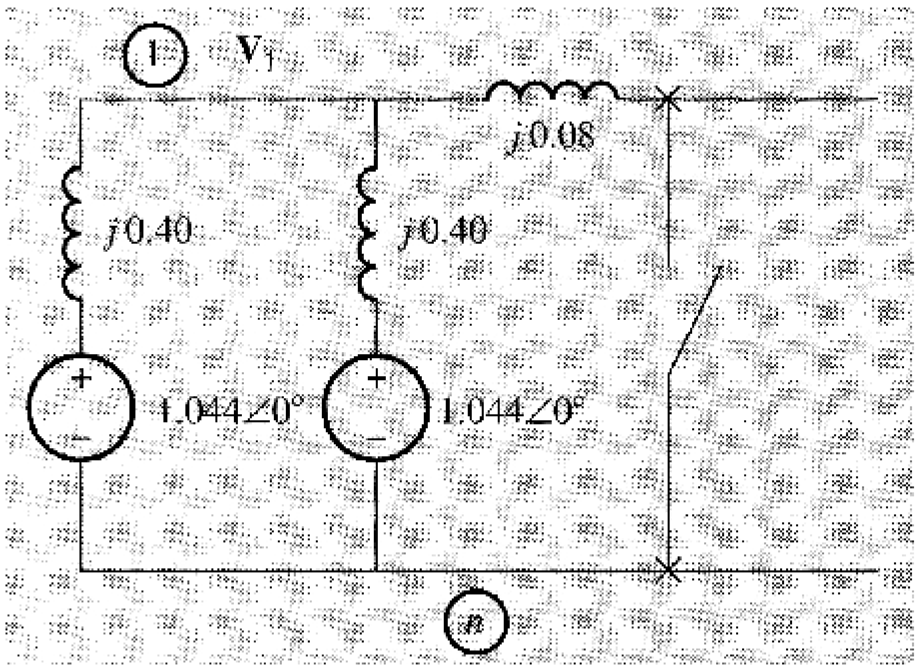
$$V_{pu} = \frac{actual\ \ value}{base\ \ value} = \frac{120,000}{115,000} = 1.044\ pu$$

Thevenin equivalent circuit:
Vth = 1.044 pu
Zth = j0.28 pu

Short circuit current (pu)
Isc = Vth/Zth = 3.73 pu

Base current on the high voltage side:
Ibase = 502 A

Short circuit current (A):
Isc = 1,872 A



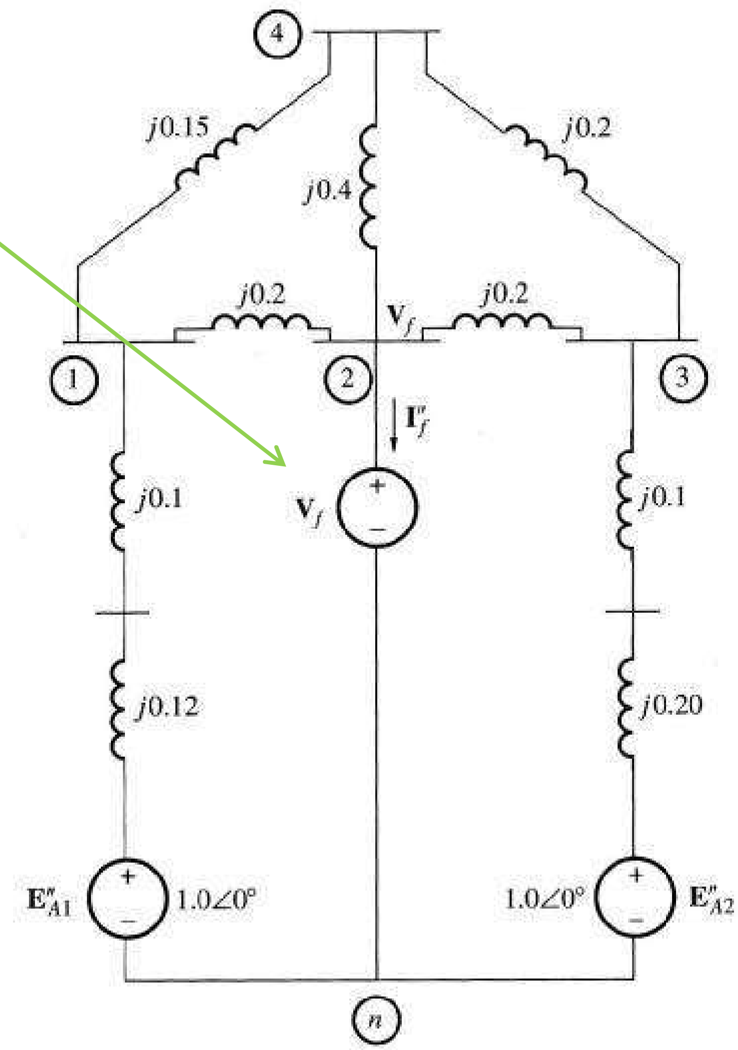
Symmetrical fault current calculations

- To determine the fault current in a large power system:
  - Create a per-phase per-unit equivalent circuit of the power system using either sub-transient reactances (if subtransient currents are needed) or transient reactances (if transient currents are needed).
  - Find the Thevenin equivalent circuit looking from the fault point, then divide the Thevenin voltage by the Thevenin impedance.
- You may use the Z-bus elements to determine the voltages and current flows elsewhere in the system (further away from the faulted bus)
- Today, software tools to do the calculations for us are readily available.

Fault current calculations using the impedance matrix

Before the fault, the voltage on bus 2 was V_f. If we introduce a voltage source of value V_f between bus 2 and the neutral, nothing will change in the system.

Since the system operates normally before the fault, there will be no current I_f'' through that source.

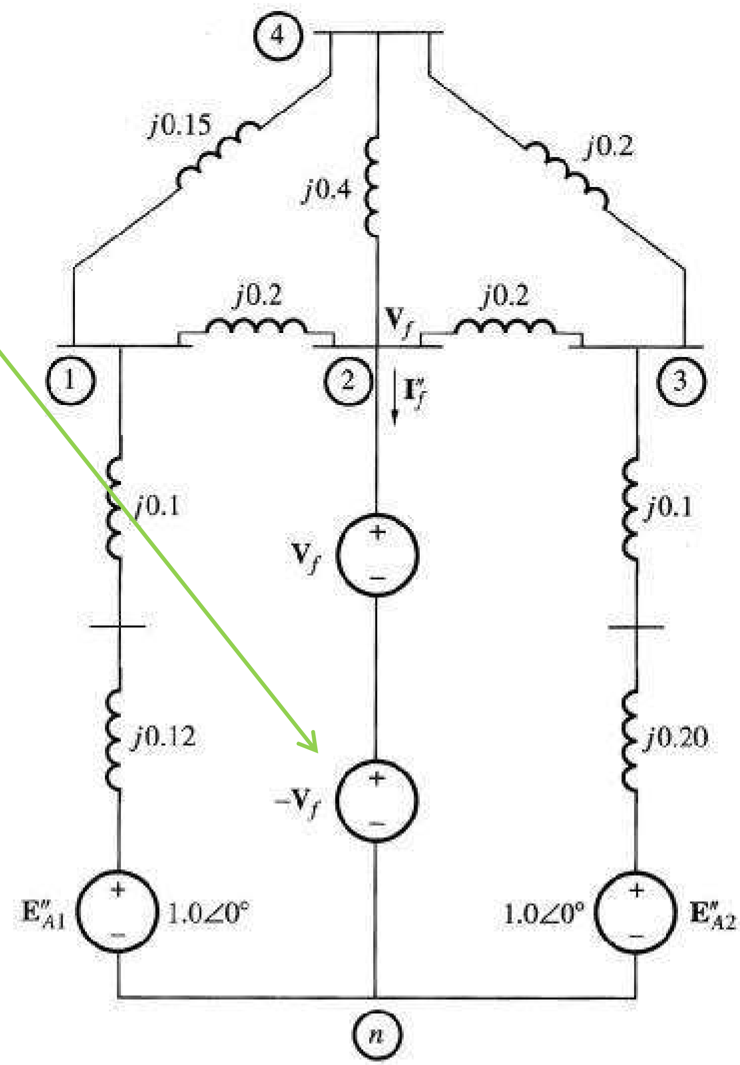


Fault current calculations using the impedance matrix

Assume that we create a short circuit on bus 2, which forces the voltage on bus 2 to 0. This is equivalent to inserting an additional voltage source of value $-V_f$ in series with the existing voltage source. The later will make the total voltage at bus 2 become 0.

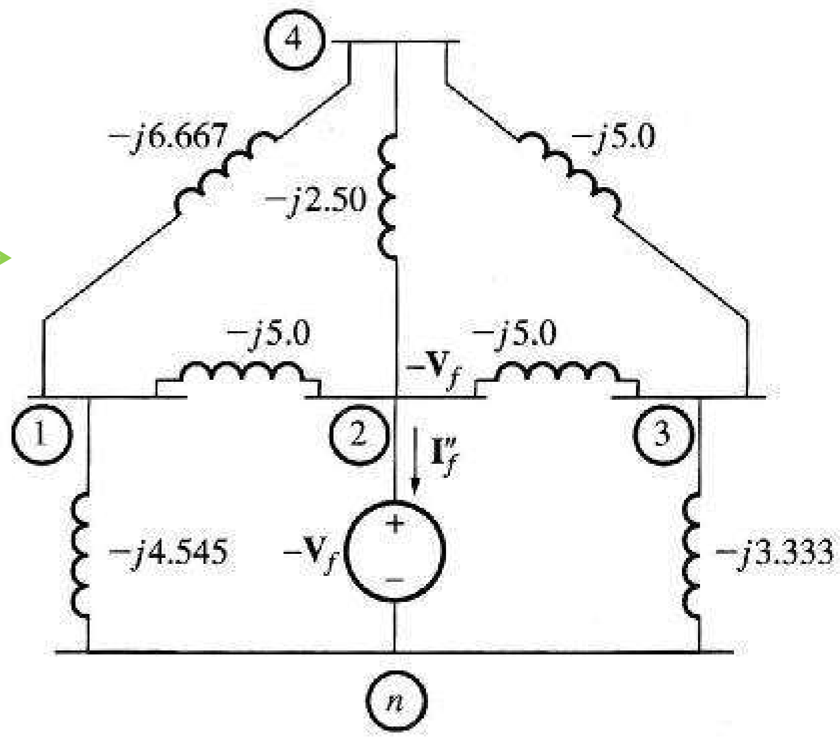
With this additional voltage source, there will be a fault current I_f”, which is entirely due to the insertion of the new voltage source to the system. Therefore, we can use superposition to analyze the effects of the new voltage source on the system.

The resulting current I_f” will be the current for the entire power system, since the other sources in the system produced a net zero current.



Fault current calculations using the impedance matrix

If all voltage sources except $-V_f''$ are set to zero and the impedances are converted to admittances, the power system appears as shown.



For this system, we can construct the bus admittance matrix as discussed previously:

$$Y_{bus} = \begin{bmatrix} -j16.212 & j5.0 & 0 & j6.667 \\ j5.0 & -j12.5 & j5.0 & j2.5 \\ 0 & j5.0 & -j13.333 & j5.0 \\ j6.667 & j2.5 & j5.0 & -j14.167 \end{bmatrix}$$

The nodal equation describing this power system is

$$\mathbf{Y}_{\mathbf{bus}}\mathbf{V} = \mathbf{I}$$

Fault current calculations using the impedance matrix

With all other voltage sources set to zero, the voltage at bus 2 is $-V_f$, and the current entering the bus 2 is $-I_f''$. Therefore, the nodal equation becomes

$$\begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \begin{bmatrix} \Delta V_1 \\ -V_f \\ \Delta V_3 \\ \Delta V_4 \end{bmatrix} = \begin{bmatrix} 0 \\ -I_f'' \\ 0 \\ 0 \end{bmatrix}$$

where ΔV_1, ΔV_3, and ΔV_4 are the changes in the voltages at those busses due to the current $-I_f''$ injected at bus 2 by the fault.

The solution is found as

$$\mathbf{V} = \mathbf{Y}_{\mathbf{bus}}^{-1}\mathbf{I} = \mathbf{Z}_{\mathbf{bus}}\mathbf{I}$$

Fault current calculations using the impedance matrix

Which, in the case considered, is

$$\begin{bmatrix} \Delta V_1 \\ -V_f \\ \Delta V_3 \\ \Delta V_4 \end{bmatrix} = \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{14} \\ Z_{21} & Z_{22} & Z_{23} & Z_{24} \\ Z_{31} & Z_{32} & Z_{33} & Z_{34} \\ Z_{41} & Z_{42} & Z_{43} & Z_{44} \end{bmatrix} \begin{bmatrix} 0 \\ -I_f'' \\ 0 \\ 0 \end{bmatrix}$$

where $\mathbf{Z}_{bus} = \mathbf{Y}_{bus}^{-1}$. Since only bus 2 has current injected at it, the system (12.38.1) reduces to

$$\Delta V_1 = -Z_{12} I_f''$$

$$-V_f = -Z_{22} I_f''$$

$$\Delta V_3 = -Z_{32} I_f''$$

$$\Delta V_4 = -Z_{42} I_f''$$

Fault current calculations using the impedance matrix

Therefore, the fault current at bus 2 is just the prefault voltage V_f at bus 2 divided by Z_{22}, the driving point impedance at bus 2.

$$I_f^{''} = \frac{V_f}{Z_{22}}$$

The voltage differences at each of the nodes due to the fault current can be calculated by substitution:

$$\Delta V_1 = -Z_{12} I_f^{''} = -\frac{Z_{12}}{Z_{22}} V_f$$

$$\Delta V_2 = -V_f = -V_f$$

$$\Delta V_3 = -Z_{32} I_f^{''} = -\frac{Z_{32}}{Z_{22}} V_f$$

$$\Delta V_4 = -Z_{42} I_f^{''} = -\frac{Z_{42}}{Z_{22}} V_f$$

Fault current calculations using the impedance matrix

Assuming that the power system was running at no load conditions before the fault, it is easy to calculate the voltages at every bus during the fault. At no load, the voltage will be the same on every bus in the power system, so the voltage on every bus in the system is V_f. The change in voltage on every bus caused by the fault current $-I_f''$ - so the total voltage during the fault is

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} = \begin{bmatrix} V_f \\ V_f \\ V_f \\ V_f \end{bmatrix} + \begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \Delta V_3 \\ \Delta V_4 \end{bmatrix} = \begin{bmatrix} V_f \\ V_f \\ V_f \\ V_f \end{bmatrix} + \begin{bmatrix} -\frac{Z_{12}}{Z_{22}} V_f \\ -V_f \\ -\frac{Z_{32}}{Z_{22}} V_f \\ -\frac{Z_{42}}{Z_{22}} V_f \end{bmatrix} = \begin{bmatrix} 1-\frac{Z_{12}}{Z_{22}} \\ 0 \\ 1-\frac{Z_{32}}{Z_{22}} \\ 1-\frac{Z_{42}}{Z_{22}} \end{bmatrix} V_f$$

The current through a line between bus *i* and bus *j* is found by

$$I_{ij} = -Y_{ij}\left(V_i - V_j\right)$$

Fault current calculations using the impedance matrix

Example: The power system depicted earlier is working at no load when a symmetrical 3 phase fault is developed on bus 2.

1. Calculate the per-unit subtransient fault current I_f" at bus 2.
2. Calculate the per-unit voltage at every bus in the system during the subtransient period.
3. Calculate the per-unit current I_1 flowing in line 1 during the subtransient period of the fault.

Solution:

1. The per-phase per-unit equivalent circuit and the bus admittance matrix was previously calculated as

$$Y_{bus} = \begin{bmatrix} -j16.212 & j5.0 & 0 & j6.667 \\ j5.0 & -j12.5 & j5.0 & j2.5 \\ 0 & j5.0 & -j13.333 & j5.0 \\ j6.667 & j2.5 & j5.0 & -j14.167 \end{bmatrix}$$

Fault current calculations using the impedance matrix

The bus impedance matrix calculated using Matlab as the inverse of Y_{bus} is

$$Z_{bus} = \begin{bmatrix} 0+j0.1515 & 0+j0.1232 & 0+j0.0934 & 0+j0.1260 \\ 0+j0.1232 & 0+j0.2104 & 0+j0.1321 & 0+j0.1417 \\ 0+j0.0934 & 0+j0.1321 & 0+j0.1726 & 0+j0.1282 \\ 0+j0.1260 & 0+j0.1417 & 0+j0.1282 & 0+j0.2001 \end{bmatrix}$$

For the given power system, the no-load voltage at every bus is equal to the pre-fault voltage at the bus that is

$$V_f = 1.00\angle 0^\circ \quad pu$$

The current at the faulted bus is computed as

$$I''_{f,2} = \frac{V_f}{Z_{22}} = \frac{1.00\angle 0^\circ}{j0.2104} = 4.753\angle -90^\circ \quad pu$$

Fault current calculations using the impedance matrix

The voltage at bus *j* during a symmetrical 3 phase fault at bus *l* can be found as

$$V_1 = \left(1 - \frac{Z_{12}}{Z_{22}}\right) V_f = \left(1 - \frac{j0.1232}{j0.2104}\right) \cdot 1.0\angle 0^\circ = 0.414\angle 0^\circ \quad pu$$

$$V_2 = 0.0\angle 0^\circ \quad pu$$

$$V_3 = \left(1 - \frac{Z_{32}}{Z_{22}}\right) V_f = \left(1 - \frac{j0.1321}{j0.2104}\right) \cdot 1.0\angle 0^\circ = 0.372\angle 0^\circ \quad pu$$

$$V_4 = \left(1 - \frac{Z_{42}}{Z_{22}}\right) V_f = \left(1 - \frac{j0.1417}{j0.2104}\right) \cdot 1.0\angle 0^\circ = 0.327\angle 0^\circ \quad pu$$

The current through the transmission line 1 is computed as

$$I_{12} = -Y_{12}\left(V_1 - V_2\right) = \left(0.414\angle 0^\circ - 0.0\angle 0^\circ\right) \cdot j5.0 = 2.07\angle 90^\circ \quad pu$$