Power Systems Analysis
ET-321

References

- **Elements of power system analysis**
  (William Stevenson)

- **Power system analysis**
  (Hadi Sadaat)

General Network Constants

Introduction

- A network having two input and two output terminals is known as a two-port network . It may also be called a two-terminal-pair network or quadriple network . In fig.(1 . a,b) represent the input pair terminals and (c,d) the output pair terminals . The two pairs of terminals are usually shown to be enclosed in a box .

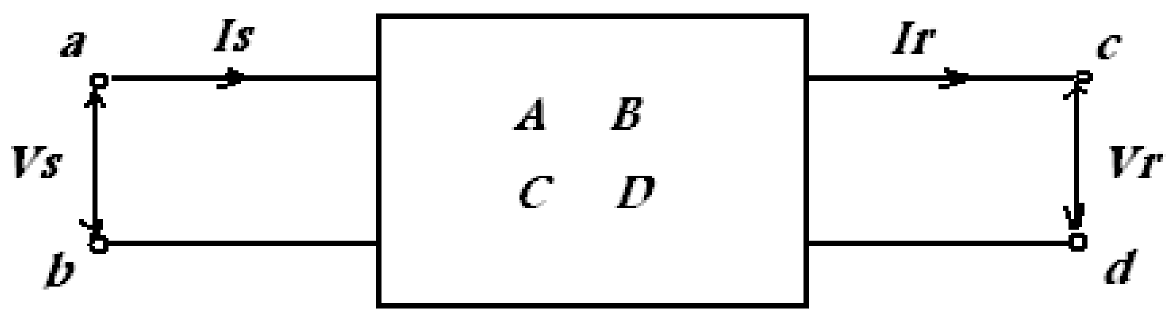

Fig.(1) :Two-port network .

- A circuit consisting of any arrangement of its components is connected to these terminals .

$$\left.\begin{aligned} V_s &= AV_r + BI_r \\ I_s &= CV_r + DI_r \end{aligned}\right\} \qquad (1.1)$$

- Where A , B , C , D are called the general network constants of the system . These constants are known by other names like transmission parameters , chain parameters and auxiliary network constants .

- Equation (1) can be put in the matrix form as :

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} V_r \\ I_r \end{bmatrix} \qquad \textbf{(1.2)}$$

- The matrix $\begin{bmatrix} A & B \\ C & D \end{bmatrix}$ is called the transfer matrix or transmission matrix of the network

Cascaded network

- The overall **A , B , C , D** constants for several 2-port networks connected in cascade (or chain arrangement) can be found out easily . Fig.(2) shows two cascaded networks , and one that is the equivalent of both . The constants of the two component networks are $\mathbf{A_1}$, $\mathbf{B_1}$, $\mathbf{C_1}$, $\mathbf{D_1}$ and $\mathbf{A_2}$, $\mathbf{B_2}$, $\mathbf{C_2}$, $\mathbf{D_2}$. Let the constants for the equivalent network be $\mathbf{A_0}$, $\mathbf{B_0}$, $\mathbf{C_0}$, $\mathbf{D_0}$.

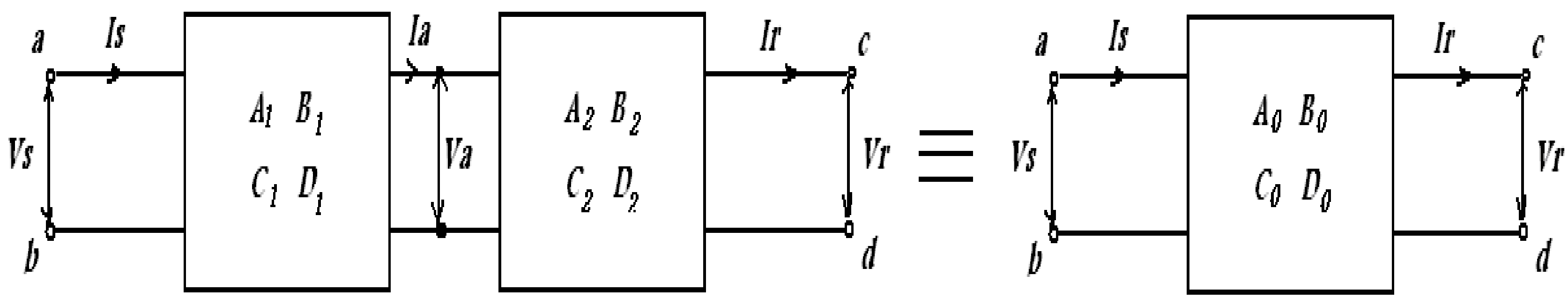

Fig.(2) : Two cascaded networks and their equivalents.

- Let V_a and I_a be the voltage and current respectively at the junction (a) of the two networks.

$$\left.\begin{aligned} V_a &= A_2 V_r + B_2 I_r \\ I_a &= C_2 V_r + D_2 I_r \end{aligned}\right\} \qquad (2.1)$$

- For the network (1) ,

$$\left.\begin{aligned} V_s &= A_1 V_a + B_1 I_a \\ I_s &= C_1 V_a + D_1 I_a \end{aligned}\right\} \qquad (2.2)$$

- Substituting the values of V_a and I_a from the first set of equations in the second set , we have :

$$V_s = A_1 \, (A_2 \, V_r + B_2 \, I_r) + B_1 \, (C_2 \, V_r + D_2 \, I_r)$$
$$= (A_1 A_2 + B_1 C_2) \, V_r + (A_1 B_2 + B_1 D_2) \, I_r \qquad \textbf{(2.3)}$$

$$I_s = C_1 \, (A_2 \, V_r + B_2 \, I_r) + D_1 \, (C_2 \, V_r + D_2 \, I_r)$$
$$= (C_1 A_2 + D_1 C_2) \, V_r + (C_1 B_2 + D_1 D_2) \, I_r \qquad \textbf{(2.4)}$$

- The sending-end voltage and current for the equivalent network with constants **A_0 , B_0 , C_0 , D_0** are given by :

$$\left.\begin{aligned} V_s &= A_0\, V_a + B_0\, I_a \\ I_s &= C_0\, V_a + D_0\, I_a \end{aligned}\right\} \qquad \textbf{(2.5)}$$

- Equating the constants of V_r and I_r , the overall constants for the two networks in cascade are :

$$\left.\begin{aligned} A_0 &= A_1 A_2 + B_1 C_2 \\ B_0 &= A_1 B_2 + B_1 D_2 \\ C_0 &= C_1 A_2 + D_1 C_2 \\ D_0 &= C_1 B_2 + D_1 D_2 \end{aligned}\right\} \qquad \textbf{(2.6)}$$

- Matrix method . For the first network ,

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} V_a \\ I_a \end{bmatrix} \qquad (2.7)$$

- But V_a and I_a are the input voltage and current respectively of the second network , so that :

$$\begin{bmatrix} V_a \\ I_a \end{bmatrix} = \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_r \\ I_r \end{bmatrix} \qquad (2.8)$$

- Combining these equations ,

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \begin{bmatrix} V_r \\ I_r \end{bmatrix} \qquad \textbf{(2.9)}$$

- For the equivalent network ,

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} A_0 & B_0 \\ C_0 & D_0 \end{bmatrix} \begin{bmatrix} V_r \\ I_r \end{bmatrix} \qquad \textbf{(2.10)}$$

- Comparing equations (2.9) and (2.10) we get ,

$$\begin{bmatrix} A_0 & B_0 \\ C_0 & D_0 \end{bmatrix} = \begin{bmatrix} A_1 & B_1 \\ C_1 & D_1 \end{bmatrix} \begin{bmatrix} A_2 & B_2 \\ C_2 & D_2 \end{bmatrix} \qquad \textbf{(2.11)}$$

Relations between A,B ,C ,D constants

- The relations between ***A*, *B* , *C* , *D*** constants of a passive , linear and bilateral network can be found with the help of reciprocity theorem . First a voltage ***V*** is applied to the input terminals keeping the output terminals short circuited fig.(3 ,a) . Since under short circuit $V_r = 0$, equations (1.1) give :

$$V = B\, I_{rs} \qquad (3.1)$$

$$I_{ss} = D\, I_{rs} \qquad (3.2)$$

- Now , the voltage V is applied to the output terminals and the input terminals are short circuited fig.(3 ,b) . The directions of flow of currents at the input and output terminals are reversed and the sending-end voltage V_s becomes zero. Equation (1.1) become :

$$0 = AV - BI_r'$$

$$I_r' = \frac{AV}{B} \qquad (3.3)$$

$$-I_s' = CV - DI_r' \qquad (3.4)$$

- Since the network is passive , by the reciprocity theorem ,

$$I_s' = I_{rs} \qquad (3.5)$$

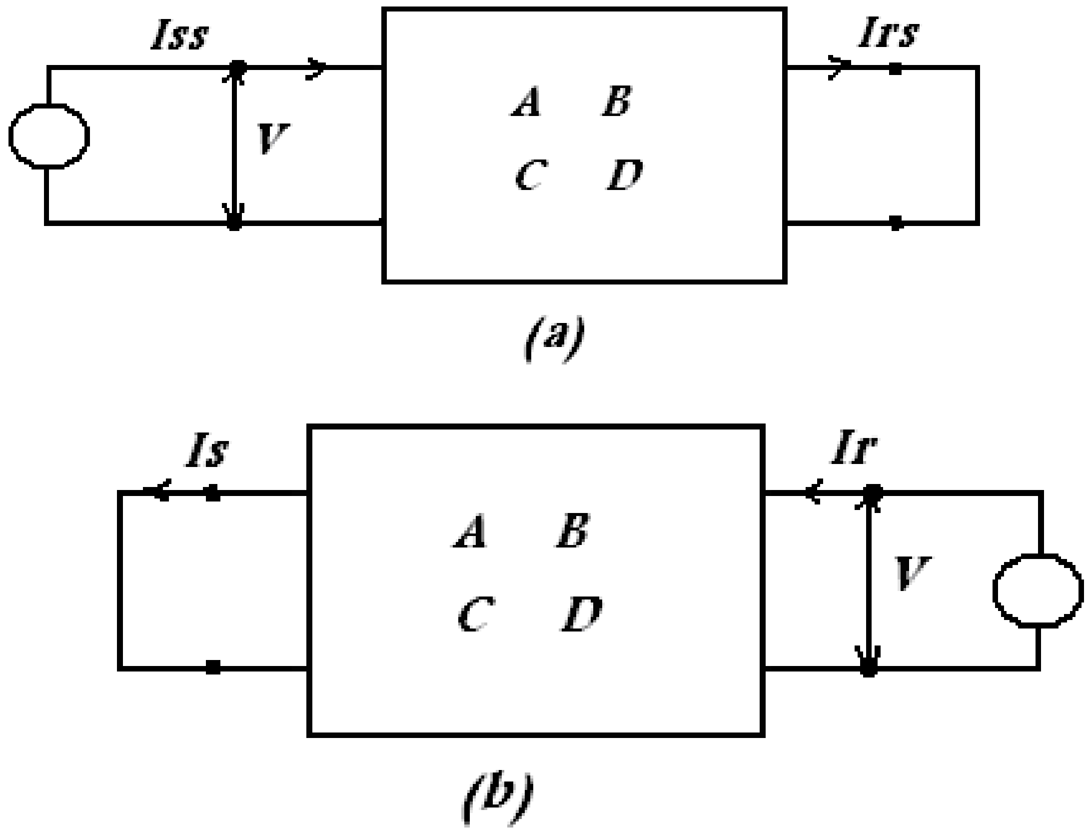

Fig.(3)

- Combining equations (3.1), (3.3) , (3.4) and (3.5) we get ,

$$-I_{rs} = CV - \frac{DAV}{B}$$

$$-\frac{V}{B} = CV - \frac{DAV}{B}$$

- Dividing both the sides of the above equation by ***-V/B*** we get ,

$$AD - BC = 1 \qquad \textbf{(3.6)}$$

- Equation (3.6) is of one of the required relations between the network constants. This relation may also be put in the determinant form as :

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = 1$$

Series impedance circuit

- A circuit having a series impedance Z is shown in fig.(4) . Such a case is found in a short transmission line where the line capacitance is negligible and the shunt admittance Y is zero . A transformer with magnetizing current neglected can also be represented by such a circuit .

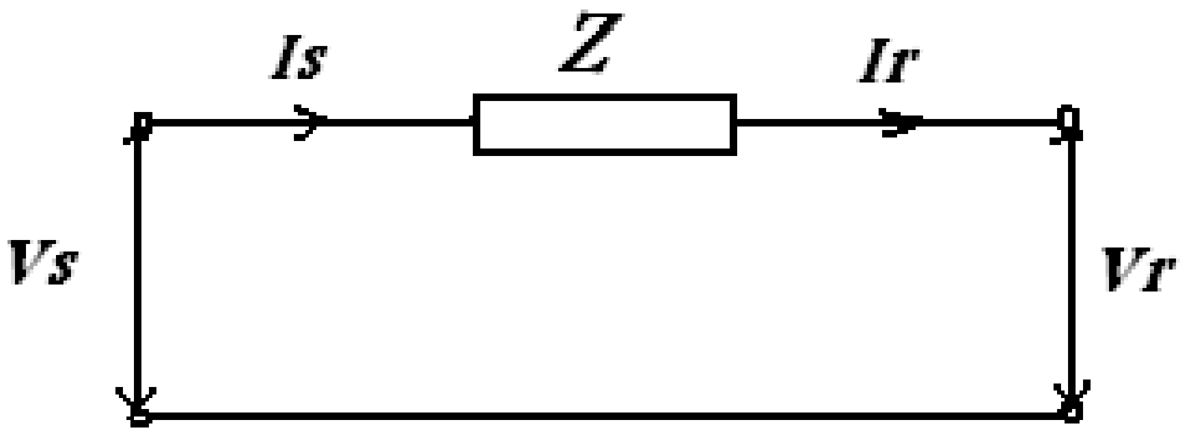

Fig.(4) :Series impedance circuit .

- For the network shown in fig.(4) we may write :

$$\left.\begin{aligned} V_s &= V_r + Z\ I_r \\ I_s &= I_r \end{aligned}\right\} \qquad \textbf{(4.1)}$$

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} V_r \\ I_r \end{bmatrix} \qquad \textbf{(4.2)}$$

- By comparing these equations with the general equations (1.1) and (1.2) the general constants for the series impedance network can be written as :

$$\left.\begin{aligned} A &= 1 \qquad B = Z \\ C &= 0 \qquad D = 1 \end{aligned}\right\} \qquad \textbf{(4.3)}$$

- The transfer matrix for the network is $\begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix}$

Shunt admittance circuit

- Fig,(5) , shows a transmission network with a shunt admittance Y . Such a network may represent the magnetizing current circuit of a transformer or a shunt capacitor .

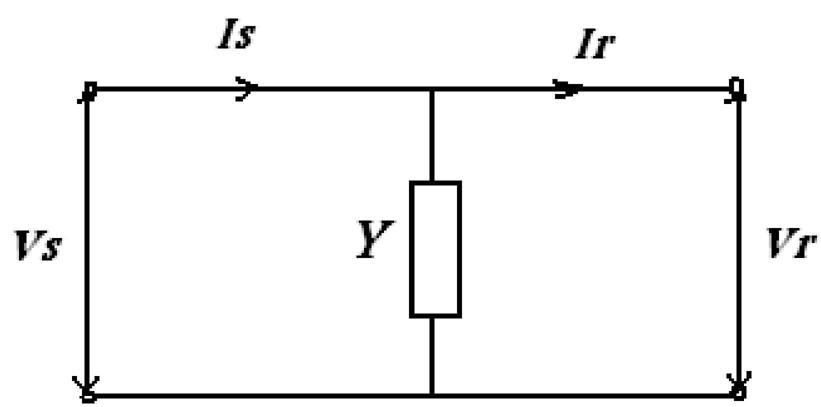

Fig.(5) : Shunt admittance circuit

- For the network shown in fig.(5) we may write :

$$\left.\begin{aligned} V_s &= V_r \\ I_s &= Y\ V_r + I_r \end{aligned}\right\} \qquad \textbf{(5.1)}$$

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} \begin{bmatrix} V_r \\ I_r \end{bmatrix} \qquad \textbf{(5.2)}$$

- Hence,

$$\left.\begin{aligned} A &= 1 \qquad B = 0 \\ C &= Y \qquad \mathrm{D} = 1 \end{aligned}\right\} \qquad \textbf{(5.3)}$$

Half – T network

- A half – *T* network is shown in fig.(6) .

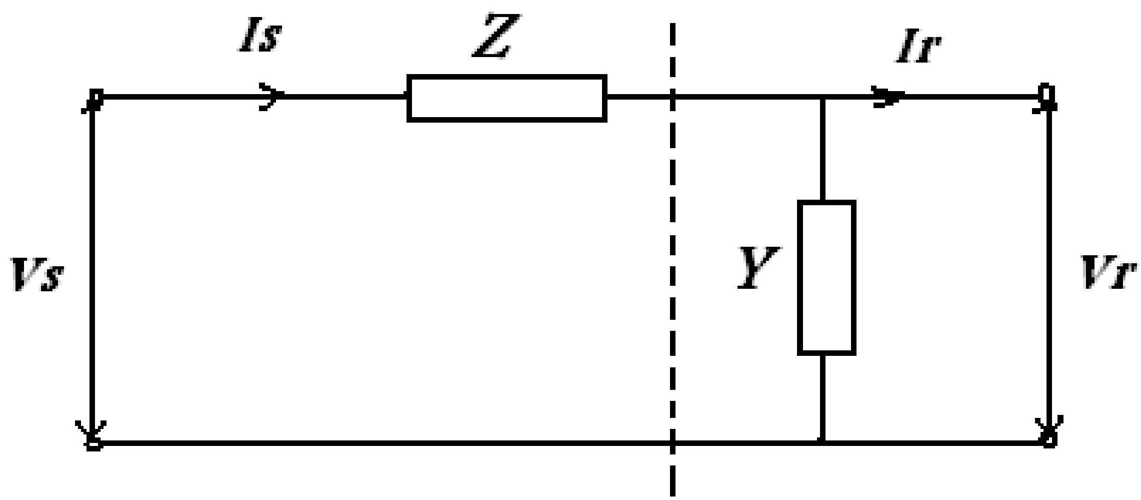

Fig.(6): Half–T network .

$$V_s = V_r + Z\, I_r$$
$$= V_r + (Y\, V_r + I_r)\, Z$$
$$= (1 + Z\, Y)\, V_r + Z\, I_r$$

- Hence, $\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} (1 + Z\, Y) & Z \\ Y & 1 \end{bmatrix} \begin{bmatrix} V_r \\ I_r \end{bmatrix}$ **(6.1)**

$$\left.\begin{array}{ll} A = 1 + Z\, Y & B = Z \\ C = Y & \mathrm{D} = 1 \end{array}\right\} \qquad \textbf{(6.2)}$$

- Matrix method , The half – T network can be considered as the cascade connection of two sections . One section is a series impedance Z and the other a shunt admittance Y . The overall constants are obtained from the matrix product of the transfer matrices of each section in the correct order.

$$\begin{bmatrix} V_s \\ I_s \end{bmatrix} = \begin{bmatrix} 1 & Z \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ Y & 1 \end{bmatrix} \begin{bmatrix} V_r \\ I_r \end{bmatrix}$$

$$= \begin{bmatrix} (1+Z\ Y) & Z \\ Y & 1 \end{bmatrix} \begin{bmatrix} V_r \\ I_r \end{bmatrix}$$