Power Systems Analysis
ET-321

References

- **Elements of power system analysis**

  (William Stevenson)

- **Power system analysis**

  (Hadi Sadaat)

Current and Voltage Relations on a Transmission line

Short Transmission line

- In the case of a short transmission line the capacitance and conductance to earth may be neglected.
- Leaving only the series resistance and inductance to be taken into consideration.
- The current entering the line at the sending-end termination is equal to the current leaving at the receiving-end, and this same current flows through all the line sections.
- The ***R*** and ***L*** parameters may therefore be regarded as ' lumped ' .

- The equivalent circuit diagram and the vector diagram for a short line are shown in fig.(6.1) in which:

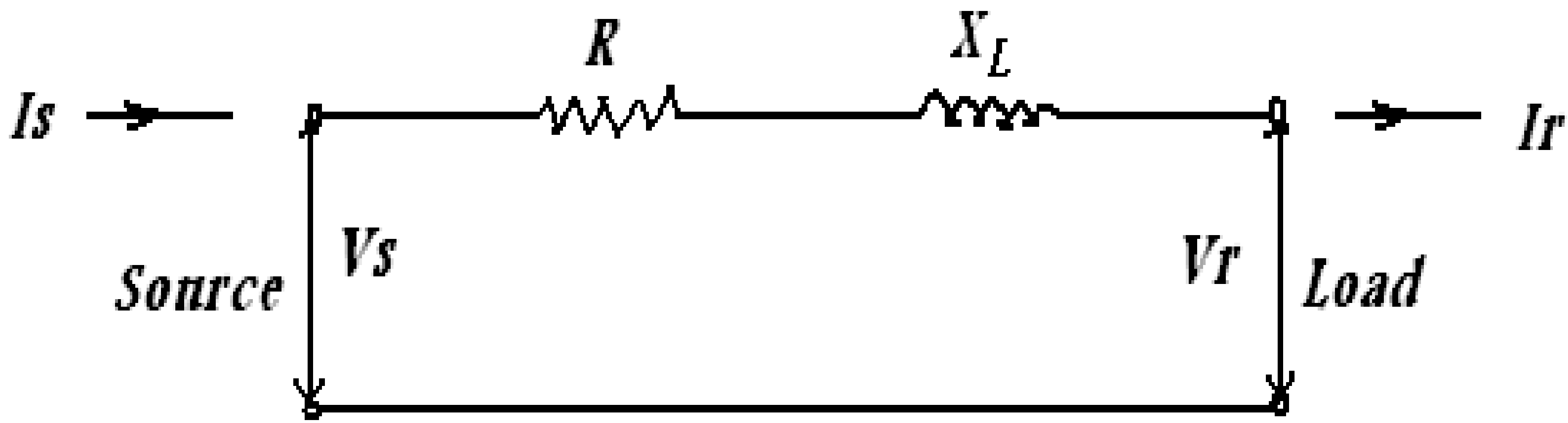


Fig.(6.1 a): Equivalent circuit for a short transmission line

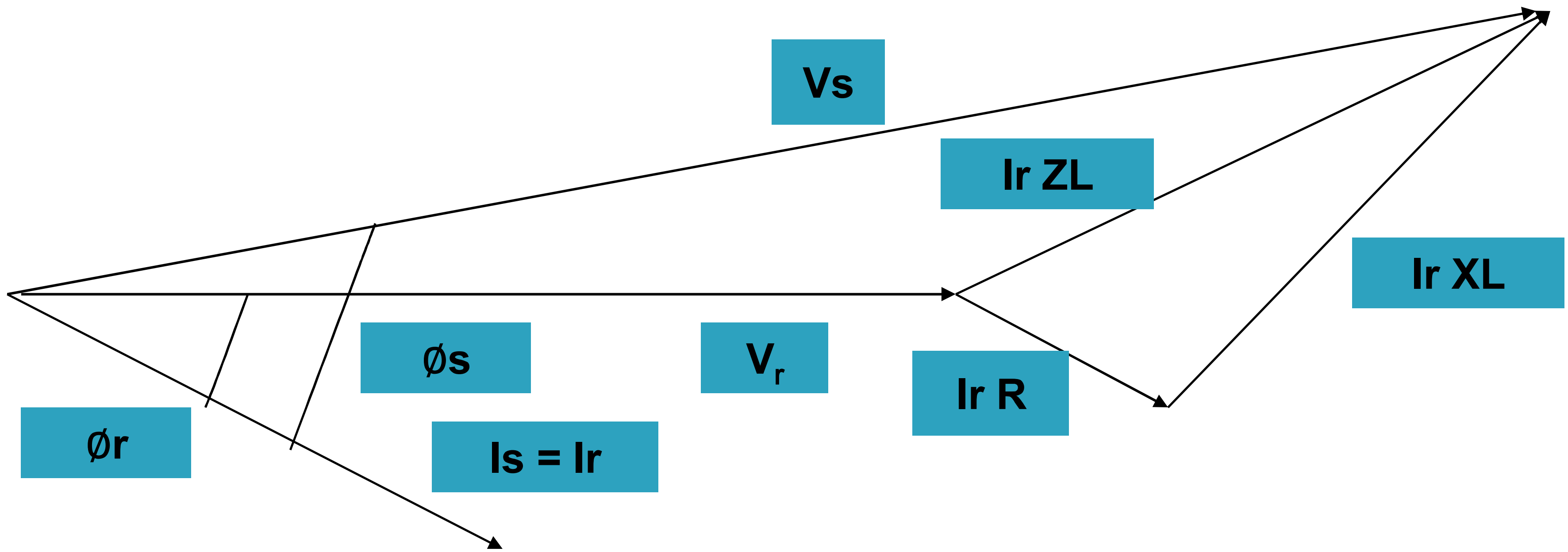


Fig.(6.1 b): Vector diagram for a short transmission line .

- V_S : phase voltage at sending-end .
- V_R : phase voltage at receiving-end
- I_S : phase current at sending-end .
- I_R : phase current at receiving-end
- $\cos\phi_S$: power factor at sending-end .

- $\cos\phi_R$: power factor at receiving-end .
- R : resistance per phase of circuit .
- X_L : inductive reactance per phase of circuit .

- The currents I_S and I_R will be equal in magnitude but not in phase.
- R is obtained from a knowledge of the line length ,the size of conductor and the specifics resistance of the conductor material ,
- while X_L is calculated from the conductor spacing and radius .

- Referring to the equivalent circuit :

$$I_S = I_R \qquad (6.1a)$$

$$V_S = V_R + (R + jX_L)I_R \qquad (6.1b)$$

$$= V_R + Z \; I_R$$

- Hence, if the receiving-end conditions are known the necessary sending-end voltage may be calculated .

- It will be noted that (6.1a) and (6.1b) are phasor equations , a more approximate method involving scalar quantities is as follows: Referring to the vector diagram,

$$V_{SX} = V_R + I_R R \cos\phi_R + I_R X_L \sin\phi_R$$

$$V_{SY} = I_R X_L \cos\phi_R - I_R R \sin\phi_R$$

$$\therefore V_S = [(V_R + I_R R \cos\phi_R + I_R X_L \sin\phi_R)^2 + (I_R X_L \cos\phi_R - I_R R \sin\phi_R)^2]^{1/2}$$

- However ($I_R X_L$) and ($I_R R$) are very much less than V_R and the small voltage is in quadrate with the much larger V_{SX} ,

$$\therefore\ V_S \cong V_{SX} \cong V_R + I_R R \cos\phi_R + I_R X_L \sin\phi_R .$$

- The voltage regulation of the line is given by the rise in voltage when full loads is removed , or :

$$\%age\ voltage\ regulation = \frac{V_S - V_R}{V_R} \cong I_R \frac{(R \cos\phi_R + X_L \sin\phi_R)}{V_R}$$

Example

A three-phase line delivers 3 MW at 11 KV for a distance of 15 Km . Line loss is 10 % of power delivered , load power factor is 0.8 lagging . frequency is 50 Hz , 1.7 m equilateral spacing of conductors . Calculate the sending-end voltage and regulation .

Solution

$$\text{Receiving-end phase voltage} = \frac{11{,}000}{\sqrt{3}} = 6.360 = V_R$$

Line current = phase current (assuming a star connection)

$$= \frac{3{,}000 \times 10^3}{\sqrt{3} \times 11 \times 10^3 \times 0.8} = 197 \quad \text{A}$$

Total line loss $=3I^2R$ **(in three conductors)**

$$=\frac{10}{100}\times 3{,}000\times 10^3$$

$$\therefore\ R=\frac{300\times 10^3}{3\times 197^2}$$

$$=2.58\ \text{ohms}$$

- Assuming that the conductors are manufactured from copper having a resistance of 0.0137 ohms per meter for a cross-sectional area of 1 mm^2 , the conductor cross-section is 80 mm^2 corresponding to a radius of 5 mm .

$$\text{Inductance} = L = \frac{1}{2}(1 + 4\ \log_c \frac{d}{r}) \times 10^{-7} \quad H\ /\ metre$$

$$\therefore\ X_L = \omega L \times \text{length}$$

$$= 314 \times \frac{1}{2}(1 + 4\ \log_c \frac{1.7 \times 10^{-3}}{5}) \times 10^{-7} \times 15 \times 10^{3}$$

$$= 5.75 \quad \text{ohms}$$

$$\therefore\ V_S \cong V_R + I_R R\ \cos\phi_R + I_R X_L\ \sin\phi_R$$

$$= 6{,}350 + (197 \times 2.58 \times 0.8) + (197 \times 5.75 \times 0.6)$$

$$= 6{,}350 + 1057$$

$$= 7{,}407 \quad \text{V per phase}$$

$$= 12{,}780 \quad \text{V line}$$

$$\textbf{Regulation} = I_R \frac{(R\ \cos\phi_R + X_L\ \sin\phi_R)}{V_R} = \frac{V_S - V_R}{V_R}$$

$$= \frac{1{,}057}{6{,}350} = 16.7\ \%$$

Medium Transmission line

- It has been mentioned in section 6.2 that the capacitance of medium length lines is significant.

- When the effect of capacitance is not negligible , it may be assumed to be concentrated at one or more definite points along the line.

- A number of localized capacitance methods have been used to make approximate line performance calculations.

- The following methods are more commonly used :

a) Nominal T method .

b) Nominal π method .

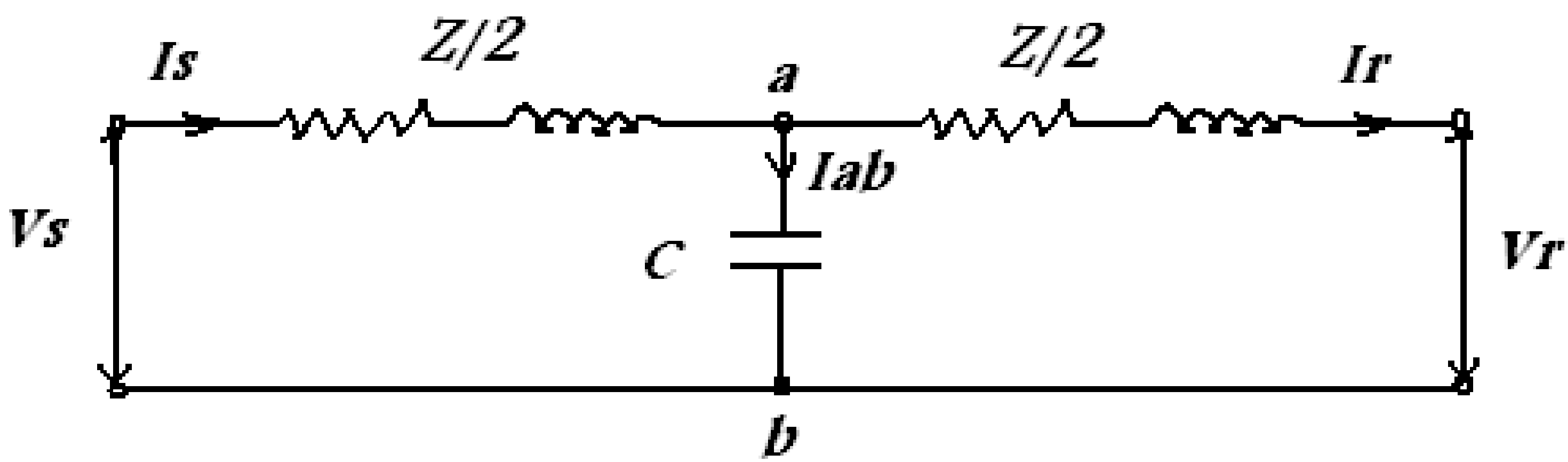
- These methods of calculation give reasonably accurate results for the solution of most transmission-line problems .

Nominal T method .

- In a nominal ***T*** method the total line capacitance is assumed to be concentrated at the middle point of the line . The ***T*** representation of a line is shown in fig.(6.12).

Series impedance of the line $\bar{Z} = R + jX$

Shunt admittance $\bar{Y} = j\omega C$



- With the usual meanings of the quantities given in fig.(2) ,
- Voltage at the mid-point of the line .

$$\bar{V}_{ab} = \bar{V}_r + \bar{I}_r \frac{\bar{Z}}{2}$$

- Current in the capacitor ,

$$\bar{I}_{ab} = \bar{V}_{ab} \; \bar{Y}$$

- Sending-end current ,

$$\bar{I}_s = \bar{I}_r + \bar{I}_{ab}$$

$$= \bar{I}_r + \bar{V}_{ab} \; \bar{Y}$$

$$= \bar{I}_r + (\bar{V}_r + \bar{I}_r \frac{\bar{Z}}{2}) \; \bar{Y}$$

$$\bar{I}_s = \bar{I}_r (1 + \frac{\bar{Z} \; \bar{Y}}{2}) + \bar{Y} \; \bar{V}_r \qquad (6.13.1)$$

- Sending-end voltage ,

$$\bar{V}_s = \bar{V}_{ab} + \bar{I}_s \frac{\bar{Z}}{2}$$

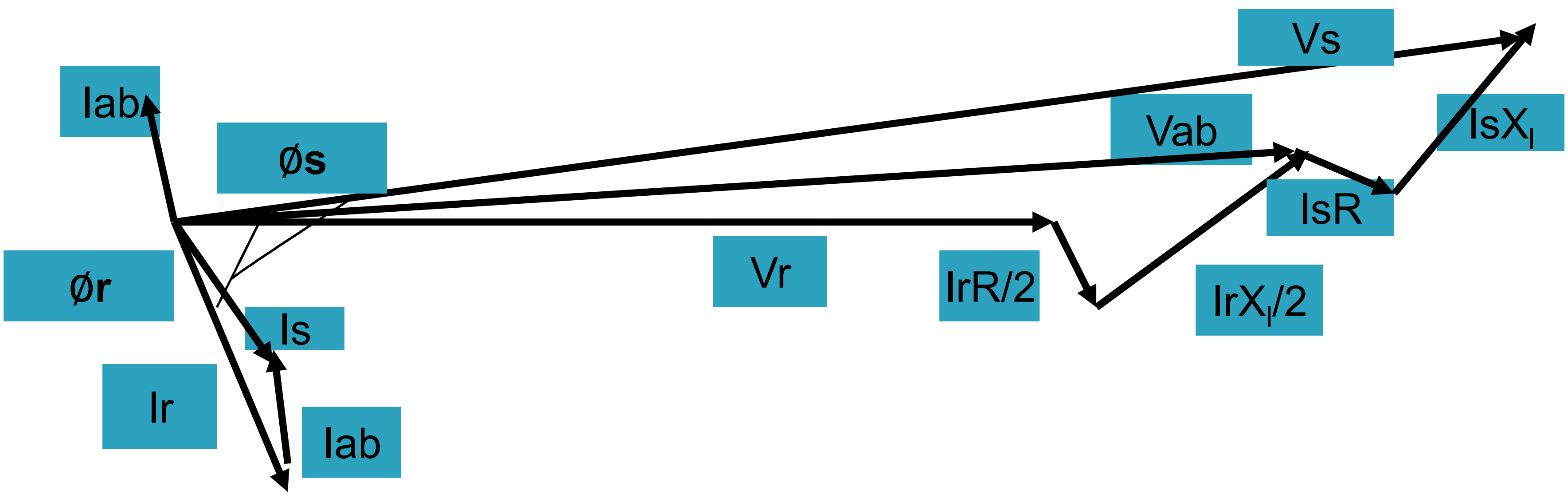
$$= \bar{V}_r + \bar{I}_r \frac{\bar{Z}}{2} + [\bar{I}_r (1 + \frac{\bar{Z}\ \bar{Y}}{2} + \bar{Y}\ \bar{V}_r)]\ \frac{\bar{Z}}{2}$$

$$\bar{V}_s = \bar{V}_r (1 + \frac{\bar{Z}\ \bar{Y}}{2}) + \bar{I}_r (\bar{Z} + \frac{\bar{Z}\ \bar{Y}}{4}) \qquad (6.13.2)$$

- Equations (6.13.1) and (6.13.2) give the sending-end current and sending-end voltage respectively.
- Other quantities , such as phase shift, power input, efficiency, regulation, etc, can be determined in the usual manner .

Phasor diagram

- The phasor diagram of the nominal ***T*** circuit can be drawn for a lagging power factor as :



Phasor diagram of a nominal *T* network

- In the phasor diagram :

$OA = \overline{V}_r$: receiving-end voltage to neutral . It is taken as a reference phasor .

$OB = \overline{I}_r$: load current lagging behind $\overline{V}_r$ by an angle ϕ_r , $\cos\phi_r$ is the power factor of the load .

$AC = I_r \frac{R}{2}$: voltage drop in the resistance of the right-hand half of the line . It is parallel to I_r .

$CD_1 = I_r \frac{X}{2}$: voltage drop in the reactance of the right-hand half of the line . It is perpendicular to OB , i.e. , $\overline{I}_r$.

$OD_1 = \overline{V}_{ab}$: voltage at the mid-point of the line across the capacitance C .

$BE = \overline{\mathbf{I}}_{ab}$: current in the capacitor . It leads the voltage $\overline{\mathbf{V}}_{ab}$ by 90° .

$OE = \overline{\mathbf{I}}_{s}$: sending-end current , the phasor sum of load current and capacitor current .

$D_1C_1 = \mathrm{I}_s \dfrac{R}{2}$: voltage drop in the resistance in the left-hand side of the line . It is drawn parallel to $\overline{\mathbf{I}}_{s}$.

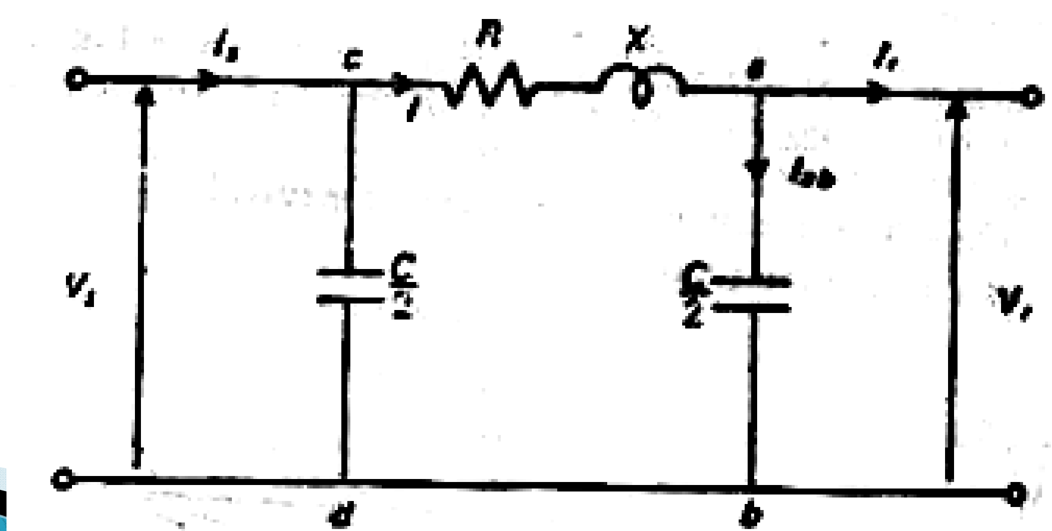
$C_1D = \mathrm{I}_s \dfrac{X}{2}$: voltage drop in the reactance in the left-hand half of the line .It is perpendicular to $\overline{\mathbf{I}}_{s}$.

$OD = \overline{\mathbf{V}}$: sending-end voltage . It is the phasor sum of $\overline{\mathbf{V}}_{ab}$ and the impedance drop in the left-hand half of the line .

ϕ_S : phase angle at the sending-end , $\cos\phi_S$ is the power factor at the sending end .

Nominal π method .

- This method assumed that one-half of the total line capacitance is concentrated at each end of the line and the total resistance and inductive reactance are concentrated at the center .
- Fig.(6.14) shows the nominal **π** representation of the line.



- From fig.(6.14),

$$\bar{I}_{ab} = \bar{V}_r \frac{\bar{Y}}{2}$$

$$\bar{I} = \bar{I}_r + \bar{I}_{ab} = \bar{I}_r + \bar{V}_r \frac{\bar{Y}}{2}$$

- Voltage at the sending-end ,

$$\bar{V}_s = \bar{V}_r + \bar{I}\,\bar{Z}$$

$$= \bar{V}_r + (\bar{I}_r + \bar{V}_r \frac{\bar{Y}}{2})\,\bar{Z}$$

$$= (1 + \frac{\bar{Z}\,\bar{Y}}{2})\bar{V}_r + \bar{Z}\;\bar{I}_r \qquad (6.14.1)$$

$$\bar{I}_{cd} = \bar{V}_s \frac{\bar{Y}}{2}$$

$$= [(1 + \frac{\bar{Z}\ \bar{Y}}{2})\bar{V}_r + \bar{Z}\ \bar{I}_r] \frac{\bar{Y}}{2}$$

- Sending-end current,

$$\bar{I}_s = \bar{I} + \bar{I}_{cd}$$

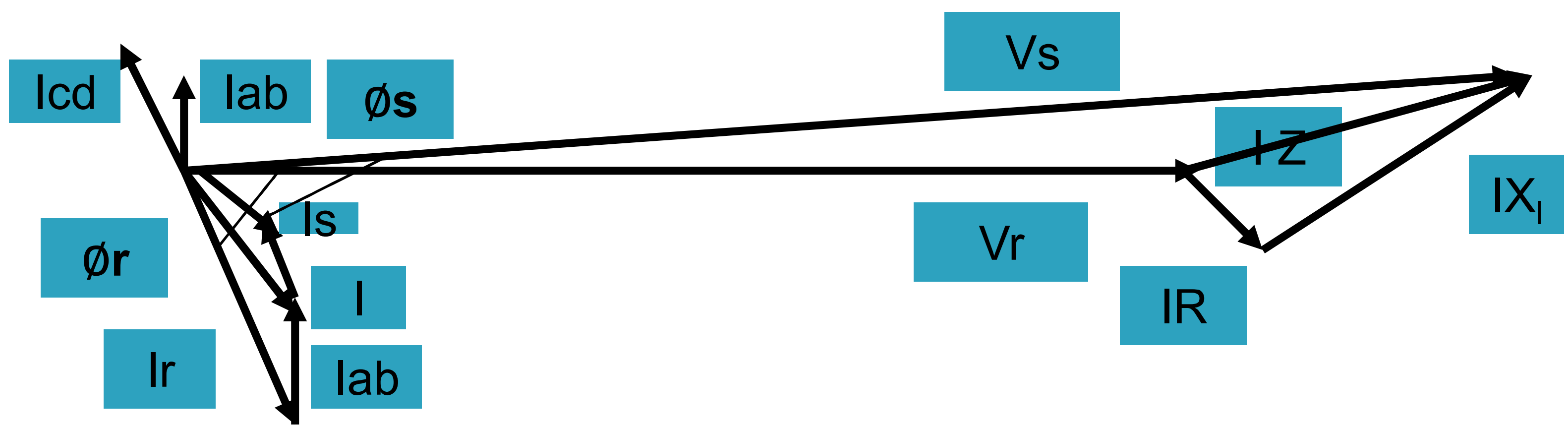
$$\bar{I}_s = \bar{I}_r + \bar{V}_r \frac{\bar{Y}}{2} + [(1 + \frac{\bar{Z}\ \bar{Y}}{2})\bar{V}_r + \bar{Z}\ \bar{I}_r] \frac{\bar{Y}}{2}$$

$$= (\ \bar{Y} + \frac{\bar{Z}\ \bar{Y}^2}{4}\ )\ \bar{V}_r + (\ 1 + \frac{\bar{Z}\ \bar{Y}}{2}\ )\ \bar{I}_r \qquad \textbf{(6.14.2)}$$

- Equations (6.14.1) and (6.14.2) give the sending-end voltage and current respectively . The other calculations can be made in the usual manner.

Phasor diagram

- The phasor diagram of a nominal π circuit is can be drawn for a lagging power factor of the load as :



Phasor diagram of a nominal *II* network

Example

A three-phase, 50 Hz, transmission line, 40 km long delivers 36 Mw at 0.8 power factor lagging at 60 kv (phase). The line constants per conductor are ,

$$R = 2.5 \ \Omega \ , \ L = 0.1 \ H \ , \ C = 0.25 \ \mu F$$

Shunt leakage may be neglected. Find the sending-end voltage , current , phase angle, and the efficiency . Use (a) nominal **T** method, (b) nominal **π** method.

Solution

Phase voltage at the receiving-end

$$V_r = 60 \ \text{kv} = 60 \times 10^3 \quad \text{v}$$

Power per phase $= \frac{1}{3} \times 36 \ \text{Mw} = 12 \times 10^6 \quad \text{w}$

- Therefore, the receiving-end current ,

$$I_r = \frac{12\times10^6}{60\times10^3\times0.8} = 250 \qquad A$$

- Taking $\overline{V}_r$ as the reference phasor,

$$\overline{V}_r = V_r + jQ$$

$$\cos\phi_r = 0.8 \quad , \quad \sin\phi_r = 0.6$$

$$\overline{I}_r = I_r \cos\phi_r - j\ I_r \sin\phi_r$$

$$= 250\times0.8 - j\,250\times0.6 = 200 - j\,150$$

- Reference per phase,

$$R = 2.5 \quad \Omega$$

- Inductive reactance per phase ,

$$X = 2\ \pi f L = 2 \times 3.14 \times 50 \times 0.1 = 31.4 \quad \Omega$$

- Series impedance per phase,

$$\bar{Z} = R + jX = 2.5 + j\,31.4 = 31.4 \angle \tan^{-1} 12.56$$

$$= 31.4 \angle 85.4° \quad \Omega$$

- Shunt admittance per phase

$$Y = 2\ \pi f C = 2 \times 3.14 \times 50 \times 0.25 \times 10^{-6}$$

$$= 78.5 \times 10^{-6} \quad \text{siemens}$$

$$Y = 0 + j\,78.5 \times 10^{-6} = 78.6 \times 10^{-6} \angle 90°$$

Calculation by nominal *T* method

- The nominal T circuit for the line is shown in Fig.(6.1).

$$\bar{V}_{ab} = \bar{V}_r + \bar{I}_r \frac{\bar{Z}}{2}$$

$$= V_r + j\,0 + (I_r \cos\phi_r - j\; I_r \sin\phi_r)(\frac{R}{2} + j\,\frac{X}{2})$$

$$= 60\times10^3 + (200 - j\,150)(1.25 + j\,15.7)$$

$$= 60\times10^3 + 2.605 + j\,2959$$

$$= (62.605 - j\,2.959)\times10^3 \quad \text{v}$$

- The current in the capacitor ,

$$\bar{I}_{ab} = \bar{Y}\,\bar{V}_{ab}$$

$$= j\,78.6\times10^{-6}\,(62.605 - j\,2.959)\times10^3$$

$$= 0.2315 + j\,4.903$$

- The current at the sending-end ,

$$\bar{I}_s = \bar{I}_r + \bar{I}_{ab}$$

$$= (200 - j\,150) + (-0.2315 + j\,4.903)$$

$$= 199.8 - j\,145$$

$$= [(199.8)^2 + (145)^2]^{1/2} \angle \tan^{-1}(-145/199.8)$$

$$= 247 \angle -\tan^{-1} 0.7257$$

$$= 247 \angle -35^\circ \;\; 57'$$

- Voltage drop in the left-hand half of the line ,

$$= \bar{I}_s \frac{\bar{Z}}{2} = (199.8 - j\,145)(1.25 + j\,15.7)$$

$$= 2527 + j\,2959$$

- Voltage at the sending-end ,

$$\bar{V}_s = \bar{V}_{ab} + \bar{I}_s \frac{\bar{Z}}{2}$$

$$= 62.605 + j\,2959 + 2527 + j\,2959$$

$$= 65132 + j\,5918$$

$$= [(65132)^2 + (5918)^2]^{1/2} \angle \tan^{-1}(5918/65132)$$

$$= 65450 \angle \tan^{-1} 0.09077$$

$$= 65450 \angle 5^\circ\ 11' \qquad \text{v/phase}$$

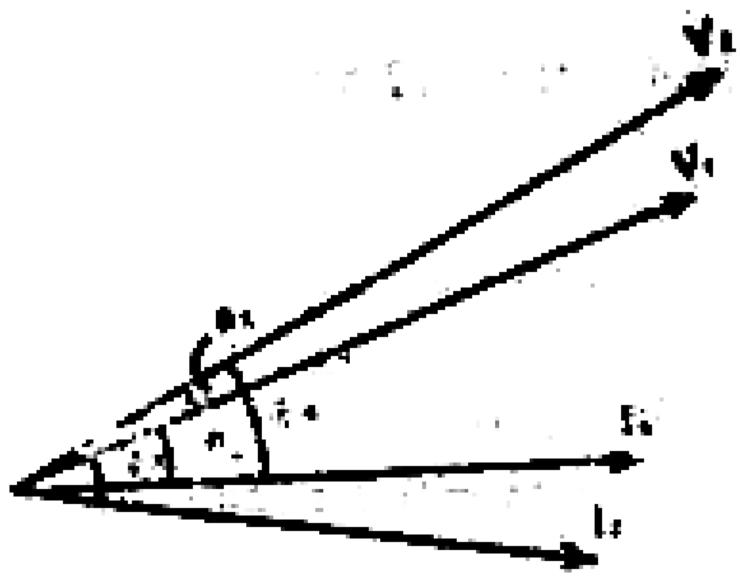


Fig.(6.16):Phasor diagram

- Sending-end line voltage ,

$$=65450\sqrt{3} = 113400 \quad \text{v}$$

$$=113.4 \quad \text{kv}$$

- Phase difference between $\bar{V}_s$ and $\bar{I}_s$,

$$\phi_s = 5^\circ\ 11' - (-35^\circ\ 57')$$

$$= 41^\circ\ 8'$$

- Sending-end power factor ,

$$\cos\phi_s = \cos 41^\circ\ 8' = 0.7532$$

- Power loss in the line ,

$$= 3I_r^2\frac{R}{2} + 3I_s^2\frac{R}{2}$$

$$= 3\times(250)^2\times1.25 + 3\times(247)^2\times1.25$$

$$= 463.2\times10^3 \quad \text{v}$$

- Transmission efficiency ,

$$\eta_T = \frac{Power\ \ output}{Power\ \ output + Power\ \ loss}$$

$$= \frac{36\times10^6}{36\times10^6 + 463.2\times10^3}$$

$$= 0.9872 \quad or \quad 98.72 \ \ per \quad cent$$

- Alternatively , transmission efficiency may be calculated as follows :

$$\eta_T = \frac{3V_r I_r \cos\phi_r}{3V_s I_s \cos\phi_s}$$

$$= \frac{36\times10^6}{3\times65450\times247\times0.7532}$$

$$=0.986 \quad or \quad 98.6 \quad per \quad cent$$

Calculation by nominal π method

- The nominal circuit for the line is shown in Fig.(6.14)

$$\bar{I}_{ab} = \frac{\bar{Y}}{2}\bar{V}_r$$

$$= j\,39.3\times10^{-6}\times60\times10^{3} = j\,2.35$$

$$\bar{I} = \bar{I}_r + \bar{I}_{ab}$$

$$= (200 - j\,150) + j\,2.35$$

$$= 200 - j\,147.65$$

$$= [(200)^2 + (147.65)^2]^{1/2}\angle\tan^{-1}(-147.65/200)$$

$$= 245.6\angle-36^\circ\ 26'$$

- Voltage drop per phase ,

$$= \overline{I}\,\overline{Z} = (200 - j\,147.65)(2.5 + j\,31.4)$$

$$= 5136 + j\,5910$$

- Voltage at the sending-end per phase ,

$$\overline{V}_s = \overline{V}_{ab} + \overline{I}\,\overline{Z}$$

$$= 60 \times 10^3 + 5136 + j\,5910$$

$$= 65136 + j\,5910$$

$$= [(65136)^2 + (5910)^2]^{1/2} \angle \tan^{-1}(5910/65136)$$

$$= 65390 \angle 5^\circ\ 11' \qquad \text{v/phase}$$

- Sending-end line voltage ,

$$=65390\sqrt{3}=113.2 \qquad \textbf{kv}$$

$$\bar{I}_{cd} = \frac{\bar{Y}}{2}\bar{V}_{s}$$

$$= j\,39.3\times10^{-6}\,(65136 + j\,5190)$$

$$= -0.232+j\,2.557$$

Sending-end current ,

$$\overline{\mathbf{I}}_s = \overline{\mathbf{I}}_r + \overline{\mathbf{I}}_{cd}$$

$$= (200 - j\,150) + -0.232 + j\,2.557$$

$$= 199.8 - j\,145$$

$$= [(199.8)^2 + (145)^2]^{1/2} \angle \tan^{-1}(-145/199.8)$$

$$= 247 \angle -35^\circ\ 57' \quad \text{A}$$

Phase difference between $\overline{\mathbf{V}}_s$ and $\overline{\mathbf{I}}_s$,

$$\phi_s = 5^\circ\ 11' - (-35^\circ\ 57')$$

$$= 41^\circ\ 8'$$

- Sending-end power factor ,

$$\cos\phi_s = \cos 41° \;\; 8' = 0.7532$$

- Power loss in the line ,

$$= 3\, I^2\, R$$

$$= 3 \times (248.6)^2 \times 2.5$$

$$= 463.6 \quad \text{Kw}$$

- Transmission efficiency ,

$$\eta_T = \frac{36 \times 10^6}{36 \times 10^6 + 463.6 \times 10^3}$$

$$= 0.9873 \quad or \quad 98.73 \;\; per \quad cent$$

Example

- A three-phase , 50 Hz , 150 km line operates at 110 Kv between the lines at the sending-end. The total inductance and capacitance per phase are **(0.2 H)** and **(1.5 μF)** . Neglecting losses calculate the value of receiving-end load having a power factor of unity for which the voltage at the receiving-end will be the same as that at the sending-end . Assume one-half of the total capacitance of the line to be concentrated at each end .

Solution

- The circuit for the given line is shown in fig.(6.17) . It is a nominal representation .

$$V_r = V_s = \frac{110 \times 1000}{\sqrt{3}} = 63510 \quad v$$

- Inductive reactance per phase ,

$$X_L = 2\pi fL = 2\times 3.14\times 50\times 0.2 = 62.8 \quad \Omega$$

- Series impedance per phase ,

$$\bar{Z} = jX_L = j\,62.8 \quad \Omega$$

- Shunt admittance per phase ,

$$Y = 2\pi fC = 2\times 3.14\times 50\times 1.5\times 10^{-6}$$

$$= 4.71\times 10^{-4} \quad \text{siemins}$$

$$\bar{Y} = j\,4.71\times 10^{-4}$$

Taking $\bar{\mathbf{V}}_r$ as the reference phasor ,

$$\bar{\mathbf{V}}_r = V_r + j\,0$$

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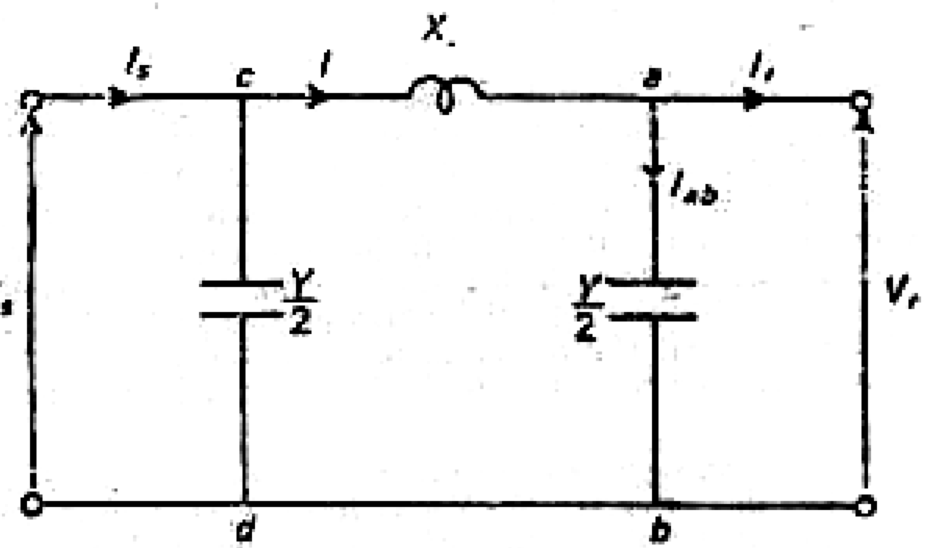

- Current in the load-end capacitor ,

$$\bar{\mathbf{I}}_{\mathbf{ab}} = \frac{\bar{\mathbf{Y}}}{2}\bar{\mathbf{V}}_r = j\,\frac{\mathbf{4.71}}{\mathbf{2}}\times 10^{-4}\times 63510 = j\,14.96 \qquad A$$

- Let the load current be I_r . Since the load power factor is unity ,

$$\bar{\mathbf{I}}_{\mathbf{r}} = I_s\angle 0^\circ = I_s + j\,0$$

- Current through the inductive reactance ,

$$\bar{\mathbf{I}} = \bar{\mathbf{I}}_{\mathbf{r}} + \bar{\mathbf{I}}_{\mathbf{ab}}$$

$$= \bar{\mathbf{I}}_{\mathbf{r}} + j\,14.96$$

- Sending-end voltage ,

$$\bar{V}_s = \bar{V}_r + \bar{I}\,\bar{Z}$$

$$= V_r + j\,0 + (\bar{I}_r + j\,14.96)(j\,62.8)$$

$$= (V_r - 939.5) + j\,62.8\,I_r$$

$$V_s^2 = (V_r - 939.5)^2 + (62.8\,I_r)^2$$

$$(63510)^2 = (63510 - 939.5)^2 + (62.8\,I_r)^2$$

$$(62.8\,I_r)^2 = 118 \times 10^6$$

$$\therefore \quad I_r = \frac{10862}{62.8} = 173 \qquad A$$