Estimation

Interval estimation

Normal Pop With σ Unknown

When a random sample $X_{1,}X_2 \ldots\ldots X_n$ of size n is drawn from a normal pop with σ unknown, we estimate σ by the sample standard deviation, which is then used in place of σ.if the sample size is sufficiently large $(n \geq 30)$, then the central limit theorem allow us to assume that the sampling distribution of $\overline{X}$ is approximately normal with mean μ and a standard deviation of $\frac{S}{\sqrt{n}}$,where S is the sample standard deviation.

The Probability expression for estimating μ then becomes

$$P(\overline{X} - z\alpha/_2 \frac{S}{\sqrt{n}} < \mu < \overline{X} + z\alpha/_2 \frac{S}{\sqrt{n}}) = 1 - \alpha$$

Thus a 100(1-α) percent confidence interval for μ is given by

$$\overline{X} \pm z\alpha/_2 \frac{S}{\sqrt{n}}$$

When σ is unknow and sample size is small $(n < 30)$, the sampling distribution of $\overline{X}$ will not be normally distributed. The sampling distribution of $\overline{X}$ then follows a distribution, known as student's-t distribution.

Thus if $-t\alpha/_{2(v)}$ and $t\alpha/_{2(v)}$ denote the values of t for which an area equal to $\alpha/2$ lies in each tail of the student's t-distribution with v degree of freedom, then the probability of t lying between these two values is given by the relation

$$P(\overline{X} - t\alpha/_{2,(v)} \frac{S}{\sqrt{n}} < \mu < \overline{X} + t\alpha/_{2,(v)} \frac{S}{\sqrt{n}}) = 1 - \alpha$$

Thus a 100(1-α) percent confidence interval for μ (when pop σ is unknown) for particular random sample of size $(n < 30)$ is given by

$$\overline{X} \pm t\alpha/_{2,(v)} \frac{S}{\sqrt{n}}$$

Large Sample Confidence interval for Pop Mean μ when Pop Standard deviation σ is unknown.

Example #1

The mean and standard deviation of the maximum loads supported by 60 cables are 11.09 tons respectively. Find (a) 95% and (b) 99% confidence interval for the mean of the maximum loads of all cables produced by the company.

Solution

The Sample size n is large so that a normal approximation for the distribution of the sample mean is appropriate.

From the sample data,we have $\overline{X} = 11.09$, $S = 0.73$ and $n = 60$

(a)

With $1 - \alpha = 0.95$ we have $\alpha/2 = 0.025$ and $Z\alpha_{/2} = 1.96$

Hence a 95% Confidnce interval for Pop Mean μ is

$$(\overline{X} - z\alpha_{/2}\frac{S}{\sqrt{n}}, \overline{X} + z\alpha_{/2}\frac{S}{\sqrt{n}})$$

$$(11.09 - (1.96)\frac{0.73}{\sqrt{60}}, 11.09 + (1.96)\frac{0.73}{\sqrt{60}})$$

$$(11.09 - 0.18, \quad 11.09 + 0.18)$$

$$(10.91, 11.27)$$

Thus the 95% confidence interval estimate for μ is $(10.91, 11.27)$

(b) Do it Yourself

Example #2

Compute A 90% confidence interval for pop mean, if

n= 36 $\sum X$=5400, and $\sum(X - \overline{X})^2 = 1296.$