CONCLUSION: If Z (calculated) $\geq$ Z (tabulated) at chosen level of significance, the null hypothesis is rejected. Otherwise, it is accepted.

EXAMPLE: A random sample of 90 poultry farms of one variety gave an average production of 240 eggs per bird/year with a S.D. of 18 eggs. Another random sample of 60 poultry farms of another variety gave an average production of 195 eggs per bird/year with a S.D. of 15 eggs. Distinguish between two varieties of birds with respect to their egg production.

Null hypothesis: $\mu_1 = \mu_2$

$$Z = \frac{|240-195|}{\sqrt{\frac{(18)^2}{90} + \frac{(15)^2}{60}}} = 16.61$$

CONCLUSION: Z (calculated) > Z (tabulated), 1.96 at 5 per cent level of significance. Here there is significant difference between two varieties of birds with respect to egg production.

10.4. Student's t-distribution

In small samples drawn from a normal population, the ratio of the difference between the sample and population means to its estimated standard error follows a distribution known as t-distribution, where

$$t = \frac{|\bar{X}-\mu|}{\frac{s}{\sqrt{n}}} \quad \text{where } s^2 = \frac{1}{n-1} \Sigma (X_i - \bar{X})^2$$

10.4.1. Properties of t-distribution: This distribution is symmetrical about the origin, and unimodel and extends from $-\infty$ to $+\infty$ in both directions. It is known as student's t-distribution, the name 'Student' being the Pen name of W.S. Gosset.

For large n, this distribution tends to standard normal distribution having zero mean and unit variance.

The moments μ_r^1 of the distribution exist only for $r < (n-1)$. All odd order moments are zero by symmetry. The even moments are given by

$$\mu_{2r} = \frac{(n-1)^r \, \Gamma\left(r+1/2\right) \, \Gamma\left(\frac{n-1}{2}-r\right)}{\Gamma\left(\frac{1}{2}\right) \, \Gamma\left(\frac{n-1}{2}\right)} \; ; \; 2r < (n-1)$$

The Skewness and Kurtosis coefficients are

$\gamma_1 = 0, \gamma_2 = 6/(n-5), (n-1) > 4.$

The values of 't' were given at different levels of significance and presented in the t-table. The tabulated values of 't' would differ for different degrees of freedom unlike in the case of Normal distribution. For normal distribution, the tabulated value would be entered into t-table at ∞ degrees of freedom.

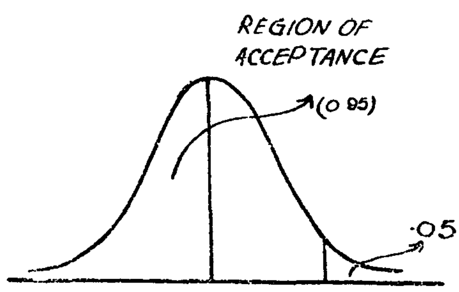

Fig. 10.4. Student's t-distribution.

Just as in the case of standard normal deviate test, student's t-test also plays an important role in tests of hypothesis in the case of small samples when the S.D. in the population is not known.

$$\text{Student's } t = \frac{\text{A standard normal deviate}}{\sqrt{\dfrac{\text{A chi-square variate}}{\text{d.f.}}}} = \frac{|\bar{X}-\mu|}{\sigma/\sqrt{n}} \Bigg/ \sqrt{\frac{(n-1)s^2}{\sigma^2} \Big/ (n-1)}$$

The numerator follows normal distribution with zero mean and unit S.D. and the denominator follows chi-square distribution with (n—1) d.f. where n is the size of the sample.

Simplifying, we have

$$t = \frac{|\bar{X} - \mu|}{\frac{s}{\sqrt{n}}} \qquad \text{where } s = \sqrt{\frac{1}{n-1} \Sigma(X_i - \bar{X})^2}$$

The confidence limits for the population mean, μ based on students' t-test is given as

$$\bar{X} \pm t_{tab} \ (n-1) \text{ d.f. } X \frac{s}{\sqrt{n}}$$

These limits can easily be derived from the expression of 't' by solving for 'μ'

10.5. One Sample t-test

Assumptions: 1. Population is normal.

2. Sample is drawn at random.

Conditions: 1. σ is not known.

2. Size of the sample is small.

Null hypothesis: $\mu = \mu_0$

$$t = \frac{|\bar{X} - \mu_0|}{s/\sqrt{n}} \qquad \text{where } s^2 = \frac{1}{n-1} \Sigma (X_i - \bar{X})^2$$

and is the unbiased estimate of σ^2

n = size of the sample.

CONCLUSION: If t (calculated) $<$ t (tabulated) with (n—1) d.f. at chosen level of significance, the null hypothesis is accepted. That is, there is no significant difference between sample mean and population mean. Otherwise, the null hypothesis is rejected.

EXAMPLE: The heights of plants in a particular field were assumed to follow normal distribution. A random sample of 10 plants were selected and whose heights (in cms) were recorded as 96, 100, 102, 99, 104, 105, 99, 98, 100 and 101. Discuss in the light of the above data the mean height of plants in the population is 100.

Null hypothesis: $\mu = \mu_0 = 100$

CONCLUSION: t (calculated) $<$ t (tabulated), (2.262) with 9 d.f. at 5 per cent level of significance. Therefore, the null hypothesis is accepted. In other words, the sample may belong to the population whose mean height is 100 cm.

TABLE 10.2

X	d_i	d_i^2
96	−4	16
100	0	0
102	2	4
99	−1	1
104	4	16
105	5	25
99	−1	1
98	−2	4
100	0	0
101	1	1
	4	68

$d_i=(X_i-A)$ where $A=100$

$$\bar{X}=A+\Sigma\frac{d_i}{n}=100+\frac{4}{10}=100.4$$

$$s=\sqrt{\frac{1}{n-1}\left[\Sigma d_i^2-\frac{(\Sigma d_i)^2}{n}\right]}$$

$$=\sqrt{\frac{1}{9}(68-16/10)}=2.72$$

$$t=\frac{|100.4-100|}{2.72/\sqrt{10}}=0.46$$

10.6. Two Sample t-test

Assumptions: 1. Populations are normal.
2. Samples are drawn independently and at random.

Conditions: 1. S.D.'s in the populations are same and not known.
2. Sizes of the samples are small.

Null hypothesis: $\mu_1=\mu_2$ where μ_1, μ_2 are the means of 1st and 2nd populations respectively.

$$t=\frac{|\bar{X}_1-\bar{X}_2|}{\sqrt{s_c^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$$

where $s_c^2=\dfrac{(n_1-1)\,s_1^2+(n_2-1)\,s_2^2}{n_1+n_2-2}=\dfrac{\Sigma(X_{1i}-\bar{X}_1)^2+\Sigma(X_{2i}-\bar{X}_2)^2}{n_1+n_2-2}$

and $s_1^2=\dfrac{1}{n_1-1}\Sigma(X_{1i}-\bar{X}_1)^2$ and $s_2^2=\dfrac{1}{n_2-1}\Sigma(X_{2i}-\bar{X}_2)^2$

CONCLUSION : If t (calculated) $\leq$ t (tabulated) with $(n_1 + n_2 - 2)$ d.f. at chosen levelof significance, the null hypothesis is accepted. That is there is no significant difference between the two samples means. Otherwise, the null hypothesis is rejected.

EXAMPLE : Two types of diets were administered to two groups of pre school going children for increase in weight and the following increases in weight (100 gm) were recorded after a month.

Increases in weight

Diet A	4	3	2	2	1	0	5	6	3
Diet B	5	4	4	2	3	2	6	1	

Test whether there is any significant difference between the two diets with respect to increase in weight.

Null hypothesis: $\mu_1 = \mu_2$

TABLE 10.3

X_1	X_2	X_1^2	X_2^2
4	5	16	25
3	4	9	16
2	4	4	16
2	2	4	4
1	3	1	9
0	2	0	4
5	6	25	36
6	1	36	1
3		9	
26	27	104	111

$\bar{X}_1 = 2.89,\ \bar{X}_2 = 3.38$

$$S_c^2 = \frac{\left[104 - \frac{(26)^2}{9}\right] + \left[111 - \frac{(27)^2}{8}\right]}{9+8-2} = 3.25$$

$$t = \frac{|\,2.89 - 3.38\,|}{\sqrt{3.25\,(1/9 + 1/8)}} = 0.56$$

CONCLUSION: t (calculated) $<$ t (tabulated), (2.131) with 15 d.f. at 5 per cent level of significance. Therefore, the null hypothesis is accepted. That is, there is no significant difference between the two diets with respect to increases in weight.

10.7. Paired t-test

When the two small samples of equal size are drawn from two populations and the samples are dependent on each other then the paired t-test is used in preference to independent t-test. The same patients for the comparison of two drugs with some time interval; the neighbouring plots of a field for comparison of two fertilizers with respect to yield assuming that the neighbouring plots will have the same soil composition; rats from the same litter for comparison of two diets; branches of same plant for comparison of the nitrogen uptake, etc., are some of the situations where paired t-test can be used.