$$\begin{bmatrix} -a_{n1} & -a_{n2} & \cdots & 1-a_{nn} \end{bmatrix}$$

Example 1:

For three industries the demand for good X is found when the matrix of technical coefficients (A) and the vector of final demand (H) are given. [(UOS: 2009-S) (UOH: 2012)]

$$A = \begin{bmatrix} 0.3 & 0.4 & 0.1 \\ 0.5 & 0.2 & 0.6 \\ 0.1 & 0.3 & 0.1 \end{bmatrix}, \quad H = \begin{bmatrix} 20 \\ 10 \\ 30 \end{bmatrix} \qquad \text{As: } X = (I-A)^{-1}H$$

$$I-A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.3 & 0.4 & 0.1 \\ 0.5 & 0.2 & 0.6 \\ 0.1 & 0.3 & 0.1 \end{bmatrix} = \begin{bmatrix} 0.7 & -0.4 & -0.1 \\ -0.5 & 0.8 & -0.6 \\ -0.1 & -0.3 & 0.9 \end{bmatrix}$$

To find $(I-A)^{-1}$, $|I-A|$ is calculated.

$$I-A = \begin{bmatrix} 0.7 & -0.4 & -0.1 \\ -0.5 & 0.8 & -0.6 \\ -0.1 & -0.3 & 0.9 \end{bmatrix}$$

$$|I-A| = 0.7\ [0.8(0.9)-(-0.3)(-0.6)] - (-0.4)[-0.5(0.9)-(-0.1)(-0.6)]$$
$$-0.1[-0.5(-0.3)-(-0.1)(0.8)]$$

$$|I-A| = 0.7(0.72-0.18)+0.4(-0.45-0.06)-0.1(0.15+0.08)$$

$$= 0.7(0.54)+0.4(-0.51)-0.1(0.23) = 0.378-0.204-0.023 = 0.151$$

The cofactor of matrix $(I-A)$ are calculated.

$$I-A = \begin{bmatrix} 0.7 & -0.4 & -0.1 \\ -0.5 & 0.8 & -0.6 \\ -0.1 & -0.3 & 0.9 \end{bmatrix}$$

$$[illegible] = \begin{vmatrix} 0.8 & -0.6 \\ -0.3 & 0.9 \end{vmatrix} = 0.8(0.9)-(-0.6)(-0.3) = 0.72-0.18 = 0.54$$

$$[illegible] = \begin{vmatrix} -0.5 & -0.6 \\ -0.1 & 0.9 \end{vmatrix} = -[-0.51] = 0.51$$

$$c_{13} = \begin{vmatrix} -0.5 & 0.8 \\ -0.1 & -0.3 \end{vmatrix} = 0.23, \qquad c_{21} = -\begin{vmatrix} -0.4 & -0.1 \\ -0.3 & 0.9 \end{vmatrix} = 0.39$$

$$c_{22} = \begin{vmatrix} 0.7 & -0.1 \\ -0.1 & 0.9 \end{vmatrix} = 0.62, \qquad c_{23} = -\begin{vmatrix} 0.7 & -0.4 \\ -0.1 & -0.3 \end{vmatrix} = 0.25$$

$$c_{31} = \begin{vmatrix} -0.4 & -0.1 \\ 0.8 & -0.6 \end{vmatrix} = 0.32; \qquad c_{32} = -\begin{vmatrix} 0.7 & -0.1 \\ -0.5 & -0.6 \end{vmatrix} = 0.47$$

$$c_{33} = \begin{vmatrix} 0.7 & -0.4 \\ -0.5 & 0.8 \end{vmatrix} = 0.36$$

$$C = \begin{bmatrix} c_{11} & c_{12} & c_{13} \\ c_{21} & c_{22} & c_{23} \\ c_{31} & c_{32} & c_{33} \end{bmatrix} = \begin{bmatrix} 0.54 & 0.51 & 0.23 \\ 0.39 & 0.62 & 0.25 \\ 0.32 & 0.47 & 0.36 \end{bmatrix}, \quad C' = \text{Adj}\,(I - A) = \begin{bmatrix} 0.54 & 0.39 & 0.32 \\ 0.51 & 0.62 & 0.47 \\ 0.23 & 0.25 & 0.36 \end{bmatrix}$$

Thus $$(I - A)^{-1} = \frac{1}{|I - A|}\text{Adj}\,(I - A) = \frac{1}{0.151}\begin{bmatrix} 0.54 & 0.39 & 0.32 \\ 0.51 & 0.62 & 0.47 \\ 0.23 & 0.25 & 0.36 \end{bmatrix}$$

$H = \begin{bmatrix} 20 \\ 10 \\ 30 \end{bmatrix}$ As $X = (I - A)^{-1} H$, hence putting their values.

$$X = \frac{1}{0.151}\begin{bmatrix} 0.54 & 0.39 & 0.32 \\ 0.51 & 0.62 & 0.47 \\ 0.23 & 0.25 & 0.36 \end{bmatrix}\begin{bmatrix} 20 \\ 10 \\ 30 \end{bmatrix} = \frac{1}{0.151}\begin{bmatrix} 24.3 \\ 30.3 \\ 17.9 \end{bmatrix}$$

$$= \begin{bmatrix} 24.3/0.151 \\ 30.3/0.151 \\ 17.9/0.151 \end{bmatrix} = \begin{bmatrix} 160.93 \\ 201.99 \\ 118.54 \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

Example 2.

We suppose that the...

outputs are gi...

Example 2.

We suppose that there are three industries A, B and C in the economy. Their inputs and outputs are given by the schedule. Construct the technology matrix and find vector of production.

Producers	USERS			Final demand	Total output
	A	B	C		
A	90	150	225	75	540
B	135	150	300	15	600
C	270	200	300	130	900

Technology matrix is constructed. We know that:

$$a_{11} = \frac{b_{11}}{x_1}, \quad a_{12} = \frac{b_{12}}{x_2}, \quad a_{13} = \frac{b_{13}}{x_3}$$

$$a_{22}x_2 = b_{22} \quad \text{or} \quad a_{22} = \frac{b_{22}}{x_2}, \quad a_{21}x_1 = b_{21} \quad \text{or} \quad a_{21} = \frac{b_{21}}{x_1}$$

$$a_{23}x_3 = b_{23} \quad \text{or} \quad a_{23} = \frac{b_{23}}{x_3} \quad a_{32}x_2 = b_{32} \quad \text{or} \quad a_{32} = \frac{b_{32}}{x_2},$$

$$a_{31}x_1 = b_{31} \quad \text{or} \quad a_{31} = \frac{b_{31}}{x_1} \quad a_{33}x_3 = b_{33} \quad \text{or} \quad a_{33} = \frac{b_{33}}{x_3}$$

Technology matrix with above values.

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} = \begin{bmatrix} \frac{90}{540} & \frac{150}{600} & \frac{225}{900} \\ \frac{135}{540} & \frac{150}{600} & \frac{300}{900} \\ \frac{270}{540} & \frac{200}{600} & \frac{300}{900} \end{bmatrix} = \begin{bmatrix} \frac{1}{6} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{3} \end{bmatrix}$$

$$I - A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} \frac{1}{6} & \frac{1}{4} & \frac{1}{4} \\ \frac{1}{4} & \frac{1}{4} & \frac{1}{3} \\ \frac{1}{2} & \frac{1}{3} & \frac{1}{3} \end{bmatrix} = \begin{bmatrix} \frac{5}{6} & -\frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{3}{4} & -\frac{1}{3} \\ -\frac{1}{2} & -\frac{1}{3} & \frac{2}{3} \end{bmatrix}$$

$$|I - A| = \begin{vmatrix} \frac{5}{6} & -\frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{3}{4} & -\frac{1}{3} \\ -\frac{1}{2} & -\frac{1}{3} & \frac{2}{3} \end{vmatrix} = \frac{5}{6}\left[\left(\frac{3}{4}\right)\left(\frac{2}{3}\right) - \left(-\frac{1}{3}\right)\left(-\frac{1}{3}\right)\right] - \left(-\frac{1}{4}\right)\left[\left(-\frac{1}{4}\right)\left(\frac{2}{3}\right) - \left(-\frac{1}{2}\right)\left(-\frac{1}{3}\right)\right] - \frac{1}{4}\left[\left(-\frac{1}{4}\right)\left(-\frac{1}{3}\right) - \left(-\frac{1}{2}\right)\left(\frac{3}{4}\right)\right]$$

$$= \frac{5}{6}\left[\frac{1}{2} - \frac{1}{9}\right] + \frac{1}{4}\left[-\frac{1}{6} - \frac{1}{6}\right] - \frac{1}{4}\left[\frac{1}{12} + \frac{3}{8}\right] = \frac{109}{864}$$

Cofactors are calculated.

$$c_{11} = (-1)^{1+1}\begin{vmatrix} \frac{3}{4} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} \end{vmatrix} = \frac{7}{18}, \quad c_{12} = (-1)^{1+2}\begin{vmatrix} -\frac{1}{4} & -\frac{1}{3} \\ -\frac{1}{2} & \frac{2}{3} \end{vmatrix} = \frac{1}{3}$$

$$c_{13} = (-1)^{1+3}\begin{vmatrix} -\frac{1}{4} & \frac{3}{4} \\ -\frac{1}{2} & -\frac{1}{3} \end{vmatrix} = \frac{11}{24}, \quad c_{21} = (-1)^{2+1}\begin{vmatrix} -\frac{1}{4} & -\frac{1}{4} \\ -\frac{1}{3} & \frac{2}{3} \end{vmatrix} = \frac{1}{4}$$

$$c_{22} = (-1)^{2+2}\begin{vmatrix} \frac{5}{6} & -\frac{1}{4} \\ -\frac{1}{2} & \frac{2}{3} \end{vmatrix} = \frac{31}{72}, \quad c_{23} = (-1)^{2+3}\begin{vmatrix} \frac{5}{6} & -\frac{1}{4} \\ -\frac{1}{2} & -\frac{1}{3} \end{vmatrix} = \frac{29}{72}$$

$$c_{31} = (-1)^{3+1}\begin{vmatrix} -\frac{1}{4} & -\frac{1}{4} \\ \frac{3}{4} & -\frac{1}{3} \end{vmatrix} = \frac{13}{48}, \quad c_{32} = (-1)^{3+2}\begin{vmatrix} \frac{5}{6} & -\frac{1}{4} \\ -\frac{1}{4} & -\frac{1}{3} \end{vmatrix} = \frac{49}{144}$$

$$c_{33} = (-1)^{3+3} \begin{vmatrix} \frac{5}{6} & -\frac{1}{4} \\ -\frac{1}{4} & \frac{3}{4} \end{vmatrix} = \frac{9}{16}$$

$$C = \begin{bmatrix} \frac{7}{18} & \frac{1}{3} & \frac{11}{24} \\ \frac{1}{4} & \frac{31}{72} & \frac{29}{72} \\ \frac{13}{48} & \frac{49}{144} & \frac{9}{16} \end{bmatrix}, \qquad C' = \begin{bmatrix} \frac{7}{18} & \frac{1}{4} & \frac{13}{48} \\ \frac{1}{3} & \frac{31}{72} & \frac{49}{144} \\ \frac{11}{24} & \frac{29}{72} & \frac{9}{16} \end{bmatrix} = \text{Adj}\,(I-A)$$

$$(I-A)^{-1} = \frac{1}{|I-A|}\text{Adj}\,(I-A) = \frac{1}{109/864} \begin{bmatrix} \frac{7}{18} & \frac{1}{4} & \frac{13}{48} \\ \frac{1}{3} & \frac{31}{72} & \frac{49}{144} \\ \frac{11}{24} & \frac{29}{72} & \frac{9}{16} \end{bmatrix}$$

$H = \begin{bmatrix} 75 \\ 15 \\ 130 \end{bmatrix}$ As $X = (I-A)^{-1} H$, hence putting their values.

$$X = \frac{864}{109} \begin{bmatrix} \frac{7}{18} & \frac{1}{4} & \frac{13}{48} \\ \frac{1}{3} & \frac{31}{72} & \frac{49}{144} \\ \frac{11}{24} & \frac{29}{72} & \frac{9}{16} \end{bmatrix} \begin{bmatrix} 75 \\ 15 \\ 130 \end{bmatrix} = \begin{bmatrix} 540 \\ 600 \\ 900 \end{bmatrix}$$

If the vector of final demand changes, as it is 50 for A, 10 for B and 100 for C. Putting them.

$$X = \frac{864}{109} \begin{bmatrix} \frac{7}{18} & \frac{1}{4} & \frac{13}{48} \\ \frac{1}{3} & \frac{31}{72} & \frac{49}{144} \\ \frac{11}{24} & \frac{29}{72} & \frac{9}{16} \end{bmatrix} \begin{bmatrix} 50 \\ 10 \\ \end{bmatrix} = \begin{bmatrix} 388.62 \\ 435.96 \\ \end{bmatrix}$$

Q.1. If the matrix of technical coefficients *(A)* and the vector of final demand *(d)* are given, find the level of output for three industries.

$$A=\begin{bmatrix}0.3 & 0.4 & 0.1\\ 0.5 & 0.2 & 0.6\\ 0.1 & 0.3 & 0.1\end{bmatrix},\quad d=\begin{bmatrix}20\\ 10\\ 30\end{bmatrix}\qquad A=\begin{bmatrix}0.50 & 0.00 & 0.30\\ 0.20 & 0.50 & 0.20\\ 0.00 & 0.25 & 0.20\end{bmatrix}\quad d=\begin{bmatrix}10\\ 8\\ 4\end{bmatrix}$$

(UOH: 2012) (UOPR: 2008) (UOS: 2013)

Q.2. Find the level of output for three industries, if followings are given.

$$A=\begin{bmatrix}0.2 & 0.3 & 0.2\\ 0.4 & 0.1 & 0.3\\ 0.3 & 0.5 & 0.2\end{bmatrix},\quad d=\begin{bmatrix}150\\ 200\\ 210\end{bmatrix}$$

Q.3. Find the level of output for three industries in the presence of followings.

$$A=\begin{bmatrix}0.4 & 0.3 & 0.1\\ 0.2 & 0.2 & 0.3\\ 0.2 & 0.4 & 0.2\end{bmatrix},\quad d=\begin{bmatrix}140\\ 220\\ 180\end{bmatrix}$$

(UAJK: 2011)
(UOP: 2010-S, 2011-A)
(GUDIKHAN: 2014)

Q.4. (a) Find the level of output for three industries.

(b) If the vector of final demand changes: For *A* 120, for *B* 40 and for *C* 10, then find the level of output.

Producers	USERS			Final demand	Total output
	A	B	C		
A	80	100	100	40	320
B	80	200	60	60	400
C	80	100	100	20	300

Example 1:

For three industries the demand for good X is found when the matrix of technical coefficients (A) and the vector of final demand (H) are given. **[(UOS: 2009-S) (UOH: 2012)]**

$$A = \begin{bmatrix} 0.3 & 0.4 & 0.1 \\ 0.5 & 0.2 & 0.6 \\ 0.1 & 0.3 & 0.1 \end{bmatrix}, \quad H = \begin{bmatrix} 20 \\ 10 \\ 30 \end{bmatrix} \qquad \text{As: } X = (I - A)^{-1} H$$

$$I - A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} - \begin{bmatrix} 0.3 & 0.4 & 0.1 \\ 0.5 & 0.2 & 0.6 \\ 0.1 & 0.3 & 0.1 \end{bmatrix} = \begin{bmatrix} 0.7 & -0.4 & -0.1 \\ -0.5 & 0.8 & -0.6 \\ -0.1 & -0.3 & 0.9 \end{bmatrix}$$

To find $(I - A)^{-1}$, $|I - A|$ is calculated.

$$I - A = \begin{bmatrix} 0.7 & -0.4 & -0.1 \\ -0.5 & 0.8 & -0.6 \\ -0.1 & -0.3 & 0.9 \end{bmatrix}$$

$$|I - A| = 0.7\ [0.8\,(0.9) - (-0.3)\,(-0.6)] - (-0.4)\,[-0.5\,(0.9) - (-0.1)\,(-0.6)]$$
$$- 0.1\,[-0.5\,(-0.3) - (-0.1)\,(0.8)]$$

$$|I - A| = 0.7\,(0.72 - 0.18) + 0.4\,(-0.45 - 0.06) - 0.1\,(0.15 + 0.08)$$

$$= 0.7\,(0.54) + 0.4\,(-0.51) - 0.1\,(0.23) = 0.378 - 0.204 - 0.023 = 0.151$$

The cofactor of matrix $(I - A)$ are calculated.

$$I - A = \begin{bmatrix} 0.7 & -0.4 & -0.1 \\ -0.5 & 0.8 & -0.6 \\ -0.1 & -0.3 & 0.9 \end{bmatrix}$$

$$c_{11} = \begin{vmatrix} 0.8 & -0.6 \\ -0.3 & 0.9 \end{vmatrix} = 0.8(0.9) - (-0.6)\,(-0.3) = 0.72 - 0.18 = 0.54$$

$$c_{12} = \begin{vmatrix} -0.5 & -0.6 \\ -0.1 & 0.9 \end{vmatrix} = -[-0.51] = 0.51$$