

In the previous chapters the energy equations for non-flow and flow processes are derived, the concepts of reversibility and irreversibility introduced, and the properties of vapours and perfect gases discussed. It is the purpose of this chapter to consider processes in practice, and to combine this with the concepts of the previous chapters.

### 3.1 Reversible non-flow processes

#### Constant volume process

In a constant volume process the working substance is contained in a rigid vessel, hence the boundaries of the system are immovable and no work is done on or by the system, other than paddle-wheel work input. It is assumed that 'constant volume' implies zero work unless stated otherwise.

From the non-flow energy equation, (1.4), for unit mass,

$$Q + W = u_2 - u_1$$

Since no work is done, we therefore have

$$Q = u_2 - u_1$$

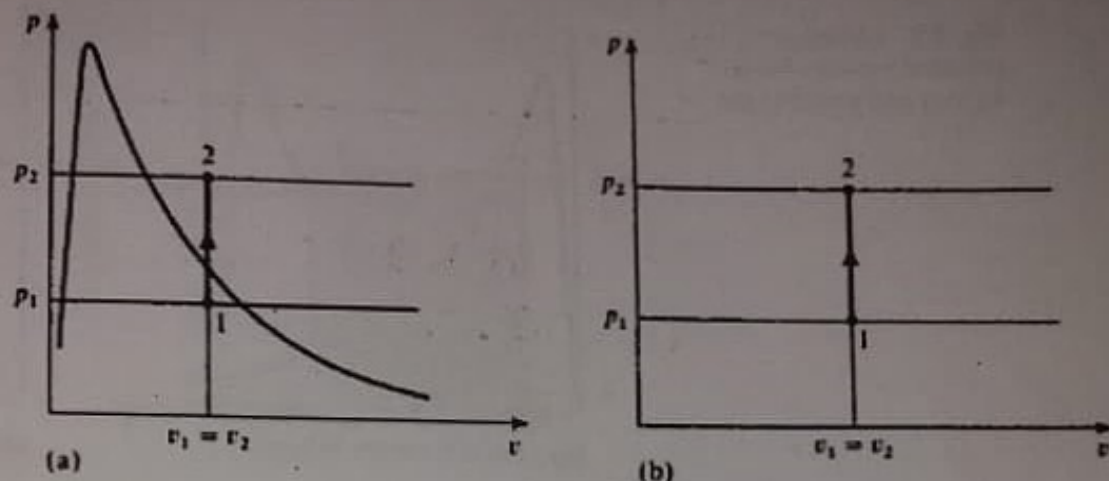
or for mass,  $m$ , of the working substance

$$Q = U_2 - U_1$$

All the heat supplied in a constant volume process goes to increasing the internal energy.

A constant volume process for a vapour is shown on a  $p-v$  diagram in Fig. 3.1(a). The initial and final states have been chosen to be in the wet steam region and superheat region respectively.

Fig. 3.1 Constant volume process for a vapour and a perfect gas



### Constant pressure process

It can be seen from Figs 3.1(a) and 3.1(b) that when the boundary of the system is inflexible as in a constant volume process, then the pressure rises when heat is supplied. Hence for a constant pressure process the boundary must move against an external resistance as heat is supplied; for instance a fluid in a cylinder behind a piston can be made to undergo a constant pressure process. Since the piston is pushed through a certain distance by the force exerted by the fluid, then work is done by the fluid on its surroundings.

From equation (1.2) for unit mass

$$W = - \int_{v_1}^{v_2} p \, dv \quad \text{for any reversible process}$$

Therefore, since  $p$  is constant,

$$W = -p \int_{v_1}^{v_2} dv = -p(v_2 - v_1)$$

From the non-flow energy equation, (1.4),

$$Q + W = u_2 - u_1$$

Hence for a reversible constant pressure process

$$Q = (u_2 - u_1) + p(v_2 - v_1) = (u_2 + pv_2) - (u_1 + pv_1)$$

Now from equation (1.9), enthalpy,  $h = u + pv$ , hence,

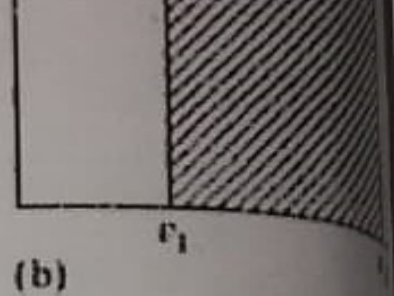
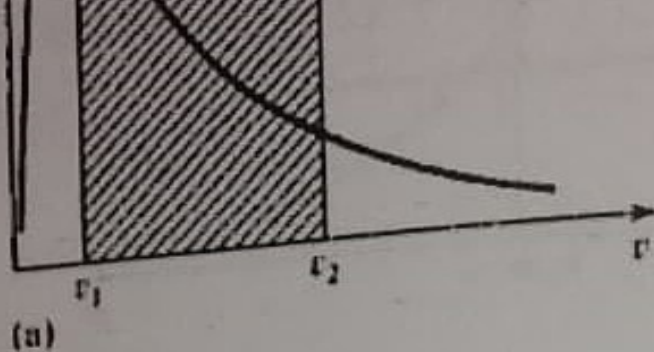
$$Q = h_2 - h_1 \quad (3.3)$$

or for mass,  $m$ , of a fluid,

$$Q = H_2 - H_1 \quad (3.4)$$

A constant pressure process for a vapour is shown on a  $p$ - $v$  diagram in Fig. 3.2(a). The initial and final states have been chosen to be in the wet region and the superheat region respectively. In Fig. 3.2(b) a constant pressure process for a perfect gas is shown on a  $p$ - $v$  diagram. For a perfect gas we have from





equation (2.12),

$$Q = mc_p(T_2 - T_1)$$

Note that in Figs 3.2(a) and 3.2(b) the shaded areas represent the work done by the fluid,  $p(v_2 - v_1)$ .

### Example 3.1

A mass of 0.05 kg of a fluid is heated at a constant pressure of 2 bar. The volume occupied is 0.0658 m<sup>3</sup>. Calculate the heat supplied and work done:

- when the fluid is steam, initially dry saturated;
- when the fluid is air, initially at 130°C.

(i) Initially the steam is dry saturated at 2 bar, hence,

$$h_1 = h_g \text{ at 2 bar} = 2707 \text{ kJ/kg}$$

Finally the steam is at 2 bar and the specific volume is given by

$$v_2 = \frac{0.0658}{0.05} = 1.316 \text{ m}^3/\text{kg}$$

Hence the steam is superheated finally. From superheat tables at 2 bar and 1.316 m<sup>3</sup>/kg the temperature of the steam is 300°C, and the enthalpy  $h_2 = 3072 \text{ kJ/kg}$ .

Then from equation (3.4)

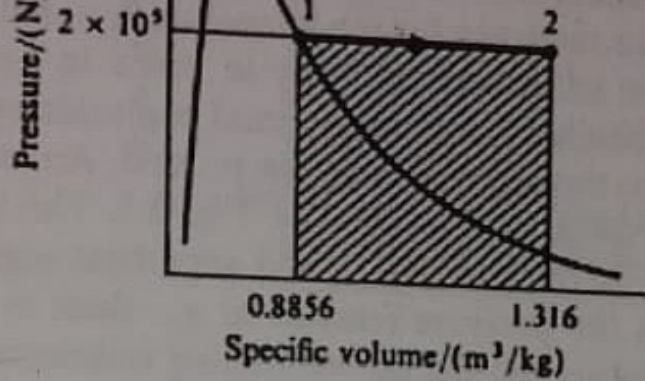
$$Q = H_2 - H_1 = m(h_2 - h_1) = 0.05(3072 - 2707)$$

i.e. Heat supplied = 0.05 × 365 = 18.25 kJ

The process is shown on a p-v diagram in Fig. 3.3

0.05 kg  
2  
0.0658 m<sup>3</sup>  
v = 0.0658 / 0.05  
= 1.316 m<sup>3</sup>/kg

Solution



$$P \frac{V}{m} = RT$$

$$PV = mRT$$

i.e. Work done by the total mass present =  $0.05 \times 86080$   
 $= 4304 \text{ N m} = 4.304 \text{ kJ}$

(ii) Using equation (2.6),

$$T_2 = \frac{p_2 V_2}{mR} = \frac{2 \times 10^5 \times 0.0658}{0.05 \times 0.287 \times 10^3} = 917 \text{ K}$$

For a perfect gas undergoing a constant pressure process we have, from equation (2.12),

$$Q = mc_p(T_2 - T_1)$$

i.e. Heat supplied =  $0.05 \times 1.005(917 - 403)$

where  $T_1 = 130 + 273 = 403 \text{ K}$ ,

i.e. Heat supplied =  $0.05 \times 1.005 \times 514 = 25.83 \text{ kJ}$

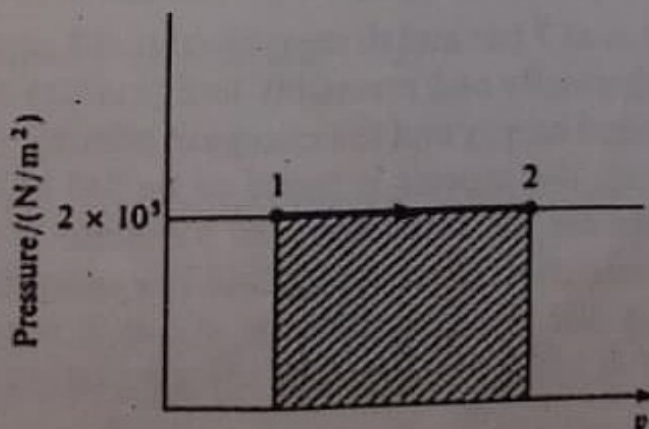
The process is shown on a  $p$ - $v$  diagram in Fig. 3.4, i.e.

$$-W = p(v_2 - v_1) = \text{shaded area}$$

From equation (2.5),  $pv = RT$ , therefore

$$\text{Work done} = -R(T_2 - T_1) = -0.287(917 - 403) \text{ kJ/kg}$$

i.e. Work done by the mass of gas present =  $0.05 \times 0.287 \times 514$   
 $= 7.38 \text{ kJ}$

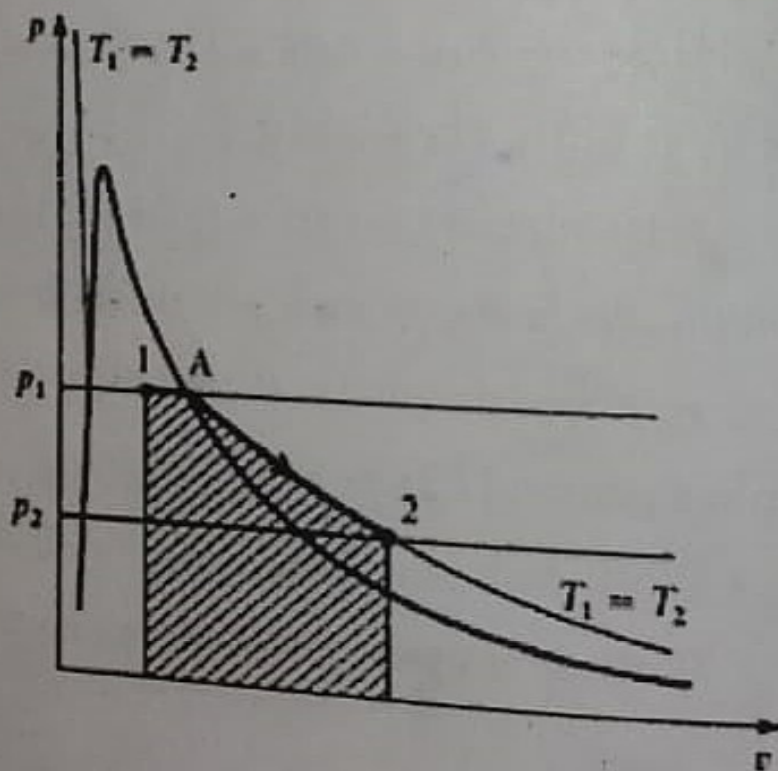




value. Similarly in an isothermal process for a fluid continuously during the process. An isothermal process for a fluid is shown on a  $p-v$  diagram in Fig. 3.5. The initial and final states are chosen in the wet region and superheat region respectively. From state A the pressure remains at  $p_1$ , since in the wet region the temperature are the corresponding saturation values. It can be seen that an isothermal process for wet steam is also at constant pressure. Equations (3.3) and (3.4) can be used (e.g. heat supplied from state 1 to 2 per kilogram of steam =  $h_A - h_1$ ). In the superheat region the procedure is not so simple. When  $p_1$  and  $p_2$  are fixed then the internal energies  $u_1$  and  $u_2$  may be obtained from tables. When the property entropy,  $s$ , is introduced in Chapter 4, a new way of evaluating the heat supplied will be shown. When the heat supplied is calculated the work done can then be obtained using the non-flow equation, (1.4), for unit mass

$$Q + W = u_2 - u_1$$

Isothermal  
for a vapour on  
diagram

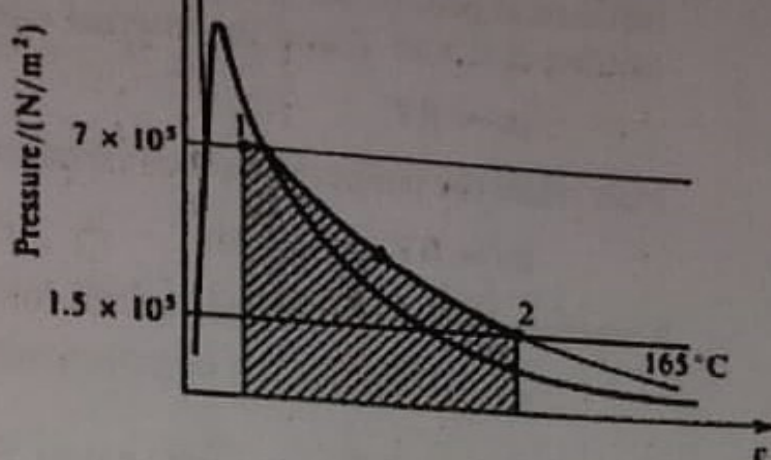


**Example 3.2**

Steam at 7 bar and dryness fraction 0.9 expands in a cylinder behind a piston isothermally and reversibly to a pressure of 1.5 bar. Calculate the change of internal energy and the change of enthalpy per kg of steam. The heat supplied during the process is found to be 547 kJ/kg, by the method of Chapter 3. Calculate the work done per kilogram of steam.

**Solution**

The process is a



therefore

$$u_1 = 69.6 + 2315.7 = 2385.3 \text{ kJ/kg}$$

Interpolating from superheat tables at 1.5 bar and 165°C, we have

$$u_2 = 2580 + \frac{15}{50}(2656 - 2580) = 2580 + 22.8$$

i.e.  $u_2 = 2602.8 \text{ kJ/kg}$

Therefore

$$\begin{aligned} \text{Gain in internal energy} &= u_2 - u_1 = 2602.8 - 2385.3 \\ &= 217.5 \text{ kJ/kg} \end{aligned}$$

$$h_1 = h_f + xh_{fg} = 697 + (0.9 \times 2067)$$

therefore

$$h_1 = 697 + 1860.3 = 2557.3 \text{ kJ/kg}$$

Interpolating from superheat tables at 1.5 bar and 165°C, we have

$$\begin{aligned} h_2 &= 2773 + \frac{15}{50}(2873 - 2773) = 2773 + 30 \\ &= 2803 \text{ kJ/kg} \end{aligned}$$

i.e.  $h_2 - h_1 = 2803 - 2557.3 = 245.7 \text{ kJ/kg}$

From the non-flow energy equation, (1.4),

$$Q + W = u_2 - u_1$$

therefore

$$W = (u_2 - u_1) - Q = 217.5 - 547 = -329.5 \text{ kJ/kg}$$

i.e.

$$\text{Work done by the system} = 329.5 \text{ kJ/kg}$$

217.5  
245.7  
547



relating  $p$  and  $v$ .  
 $pv = RT$   
 Now when the temperature is constant as in an isothermal process then

$$pv = RT = \text{constant}$$

Therefore for an isothermal process for a perfect gas

$$pv = \text{constant}$$

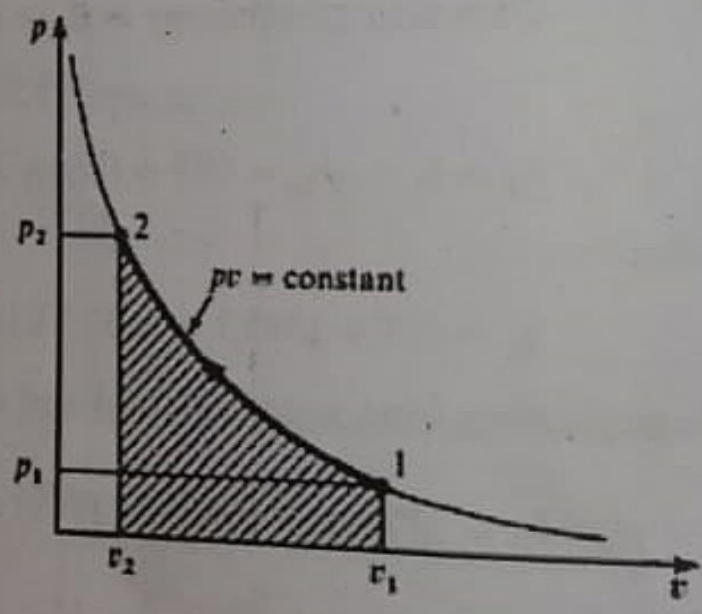
i.e.  $p_1v_1 = p_2v_2$

In Fig. 3.7 an isothermal compression process for a perfect gas is shown in a  $p$ - $v$  diagram. The equation of the process is  $pv = \text{constant}$ , which is the equation of a hyperbola. It must be stressed that an isothermal process is only for a perfect gas, because it is only for a perfect gas that an equation of state,  $pv = RT$ , can be applied.

From equation (1.2) we have for unit mass

$$W = - \int_1^2 p \, dv = (\text{shaded area in Fig. 3.7})$$

Fig. 3.7 Isothermal compression process for a perfect gas in a  $p$ - $v$  diagram



In this case,  $pv = \text{constant}$ , or  $p = c/v$ , where  $c = \text{constant}$ . Therefore

$$W = - \int_{v_1}^{v_2} c \frac{dv}{v} = -c [\ln v]_{v_1}^{v_2} = c \ln \left( \frac{v_1}{v_2} \right)$$

The constant  $c$  can either be written as  $p_1v_1$  or as  $p_2v_2$ , since  $p_1v_1 = p_2v_2 = \text{constant}$ ,  $c$ ,

$$W = p_1 V_1 \ln\left(\frac{v_1}{v_2}\right) \quad (3.7)$$

Also, since  $p_1 v_1 = p_2 v_2$ , then

$$\frac{v_1}{v_2} = \frac{p_2}{p_1}$$

Hence, substituting in equation (3.6)

$$W = p_1 v_1 \ln \frac{p_2}{p_1} \text{ per unit mass of gas} \quad (3.8)$$

or for mass,  $m$ , of the gas

$$W = p_1 V_1 \ln\left(\frac{p_2}{p_1}\right) \quad (3.9)$$

Using equation (2.5)

$$p_1 v_1 = RT$$

Hence, substituting in equation (3.8)

$$W = RT \ln\left(\frac{p_2}{p_1}\right) \text{ per unit mass of gas} \quad (3.10)$$

or for mass,  $m$ , of the gas

$$W = mRT \ln\left(\frac{p_2}{p_1}\right) \quad (3.11)$$

There are clearly a large number of equations for the work done, and no attempt should be made to memorize these since they can all be derived very simply from first principles.

For a perfect gas from Joule's law, equation (2.14), we have

$$U_2 - U_1 = mc_v(T_2 - T_1)$$

Hence for an isothermal process for a perfect gas, since  $T_2 = T_1$ , then

$$U_2 - U_1 = 0$$

i.e. the internal energy remains constant in an isothermal process for a perfect gas.

From the non-flow energy equation (1.4) for unit mass

$$Q + W = u_2 - u_1$$

Therefore, since  $u_2 = u_1$ , then

$$Q + W = 0 \quad (3.12)$$



Example 3.3

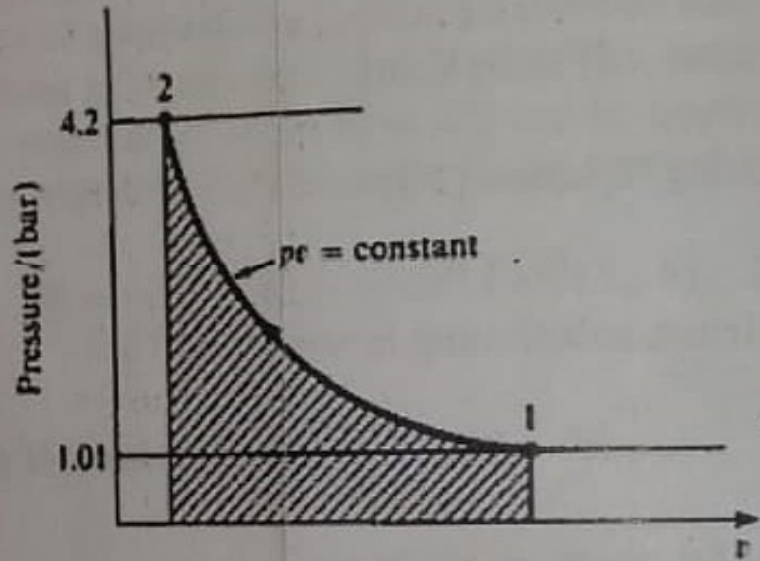
1 kg of nitrogen (molar mass 28) is compressed isothermally from 1.01 bar, 20°C to 4.2 bar. Calculate the work done and the heat flow during the process. Assume nitrogen to be a perfect gas.

Solution

From equation (2.9), for nitrogen,

$$R = \frac{\bar{R}}{\bar{m}} = \frac{8.3145}{28} = 0.297 \text{ kJ/kg K}$$

The process is shown on a  $p-v$  diagram in Fig. 3.8.



From equation (3.10)

$$W = RT \ln\left(\frac{p_2}{p_1}\right) = 0.297 \times 293 \times \ln\left(\frac{4.2}{1.01}\right) = 124 \text{ kJ/kg}$$

where  $T = 20 + 273 = 293 \text{ K}$ .

i.e. Work input = 124 kJ/kg

From equation (3.12), for an isothermal process for a perfect gas,

$$Q + W = 0$$

therefore

$$Q = -124 \text{ kJ/kg}$$

i.e. Heat rejected = 124 kJ/kg

$$Q = 0$$

Therefore we have

$$W = u_2 - u_1 \quad \text{for any adiabatic non-flow process} \quad (3.13)$$

Equation (3.13) is true for an adiabatic non-flow process whether or not the process is reversible. In an adiabatic compression process all the work done on the fluid goes to increasing the internal energy of the fluid. Similarly in an adiabatic expansion process, the work done by the fluid is at the expense of a reduction in the internal energy of the fluid. For an adiabatic process to take place, perfect thermal insulation for the system must be available.

For a vapour undergoing a reversible adiabatic process the work done can be found from equation (3.13) by evaluating  $u_1$  and  $u_2$  from tables. In order to fix state 2, one must be made of the fact that the process is reversible and adiabatic. When the property entropy,  $s$ , is introduced in Chapter 4 it will be shown that a reversible adiabatic process takes place at constant entropy, and this fact can be used to fix state 2.

For a perfect gas, a law relating  $p$  and  $v$  may be obtained for a reversible adiabatic process, by considering the non-flow energy equation in differential form. From equation (1.8) for unit mass

$$dq + dW = du$$

Also for a reversible process  $dW = -p dv$ , hence for a reversible adiabatic process

$$dq = du + p dv = 0 \quad (3.14)$$

Since  $h = u + pv$

then  $dh = du + p dv + v dp$

i.e.  $du + p dv = dh - v dp$

and hence, using equation (3.14),

$$dh - v dp = 0 \quad (3.15)$$

i.e.  $dh = v dp$

Also, using equations (2.5) and (3.14), we have

$$du + \frac{RT dv}{v} = 0$$

From equation (2.14)



Dividing through

$$c_v \frac{dT}{T} + \frac{R dv}{v} = 0$$

Integrating

$$c_v \ln T + R \ln v = \text{constant}$$

Using equation (2.5) we have  $T = (pv)/R$ , therefore substituting

$$c_v \ln \left( \frac{pv}{R} \right) + R \ln v = \text{constant}$$

Dividing through by  $c_v$

$$\ln \left( \frac{pv}{R} \right) + \frac{R}{c_v} \ln v = \text{constant}$$

Also, from equation (2.21),

$$c_v = \frac{R}{(\gamma - 1)} \quad \text{or} \quad \frac{R}{c_v} = \gamma - 1$$

Hence substituting

$$\ln \left( \frac{pv}{R} \right) + (\gamma - 1) \ln v = \text{constant}$$

or 
$$\ln \left( \frac{pv}{R} \right) + \ln(v^{\gamma-1}) = \text{constant}$$

therefore

$$\ln \left( \frac{pvv^{\gamma-1}}{R} \right) = \text{constant}$$

i.e. 
$$\ln \left( \frac{pv^\gamma}{R} \right) = \text{constant}$$

therefore

$$\frac{pv^\gamma}{R} = c^{(\text{constant})} = \text{constant}$$

i.e.  $pv = RT$

therefore

$$p = \frac{RT}{v}$$

Substituting in equation (3.16)

$$\frac{RT}{v} v^\gamma = \text{constant}$$

i.e.  $Tv^{\gamma-1} = \text{constant}$  (3.17)

Also,  $v = (RT)/p$ ; hence substituting in equation (3.16)

$$p \left( \frac{RT}{p} \right)^\gamma = \text{constant}$$

therefore

$$\frac{T^\gamma}{p^{\gamma-1}} = \text{constant}$$

or  $\frac{T}{p^{(\gamma-1)/\gamma}} = \text{constant}$  (3.18)

Therefore for a reversible adiabatic process for a perfect gas between states 1 and 2 we can write as follows. From equation (3.16)

$$p_1 v_1^\gamma = p_2 v_2^\gamma \quad \text{or} \quad \frac{p_1}{p_2} = \left( \frac{v_2}{v_1} \right)^\gamma \quad (3.19)$$

From equation (3.17)

$$T_1 v_1^{\gamma-1} = T_2 v_2^{\gamma-1} \quad \text{or} \quad \frac{T_1}{T_2} = \left( \frac{v_2}{v_1} \right)^{\gamma-1} \quad (3.20)$$

From equation (3.18)

$$\frac{T_1}{p_1^{(\gamma-1)/\gamma}} = \frac{T_2}{p_2^{(\gamma-1)/\gamma}} \quad \text{or} \quad \frac{T_1}{T_2} = \left( \frac{p_1}{p_2} \right)^{(\gamma-1)/\gamma} \quad (3.21)$$

From equation (3.13) the work done in an adiabatic process per unit mass of gas is given by  $W = (u_2 - u_1)$ . The gain in internal energy of a perfect gas is given by equation (2.16),

i.e. for unit mass  $u_2 - u_1 = c_v(T_2 - T_1)$

therefore



Also, from equation (2.21),

$$c_v = \frac{R}{(\gamma - 1)}$$

Hence substituting

$$W = \frac{R(T_2 - T_1)}{(\gamma - 1)}$$

Using equation (2.5),  $pv = RT$ ,

$$W = \frac{p_2 v_2 - p_1 v_1}{\gamma - 1}$$

A reversible adiabatic process for a perfect gas is shown on a  $p-v$  diagram in Fig. 3.9. We have

$$-W = \int_{v_1}^{v_2} p \, dv = \text{shaded area}$$

Fig. 3.9 Reversible adiabatic process for a perfect gas on a  $p-v$  diagram

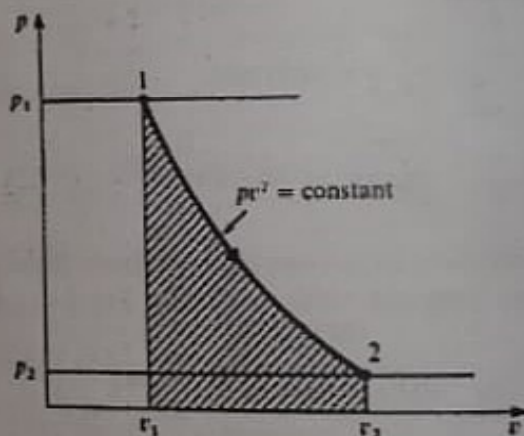


Fig. 3.10 Reversible adiabatic process for steam on a  $p-v$  diagram for Example 3.4

Therefore, since  $pv^\gamma = \text{constant}$ ,  $c$ , then

$$W = - \int_{v_1}^{v_2} \frac{c \, dv}{v^\gamma}$$

$$\begin{aligned} \text{i.e. } W &= -c \int_{v_1}^{v_2} \frac{dv}{v^\gamma} = -c \left[ \frac{v^{-\gamma+1}}{-\gamma+1} \right]_{v_1}^{v_2} \\ &= -c \left( \frac{v_2^{-\gamma+1} - v_1^{-\gamma+1}}{1-\gamma} \right) = -c \left( \frac{v_1^{-\gamma+1} - v_2^{-\gamma+1}}{\gamma-1} \right) \end{aligned}$$

The constant in this equation can be written as  $p_1 v_1^\gamma$  or as  $p_2 v_2^\gamma$ . Hence

$$W = \frac{p_2 v_2^\gamma v_2^{1-\gamma} - p_1 v_1^\gamma v_1^{1-\gamma}}{\gamma-1} = \frac{p_2 v_2 - p_1 v_1}{\gamma-1}$$

This is the same expression obtained before as equation (3.23).

## Example 3.4

Solution

**Example 3.4** 1 kg of steam at 100 bar and 375°C expands reversibly in a perfectly thermally insulated cylinder behind a piston until the pressure is 38 bar and the steam is then dry saturated. Calculate the work done.

**Solution** From superheat tables at 100 bar and 375°C,

$$h_1 = 3017 \text{ kJ/kg and } v_1 = 0.02453 \text{ m}^3/\text{kg}$$

Using equation (1.9)

$$u = h - pv$$

therefore

$$u_1 = 3017 - \frac{100 \times 10^5 \times 0.02453}{10^3} = 2771.7 \text{ kJ/kg}$$

Also,  $u_2 = u_g$  at 38 bar = 2602 kJ/kg

Since the cylinder is perfectly thermally insulated then no heat flows to or from the steam during the expansion; the process is therefore adiabatic. Using equation (3.13),

$$W = u_2 - u_1 = 2602 - 2771.7$$

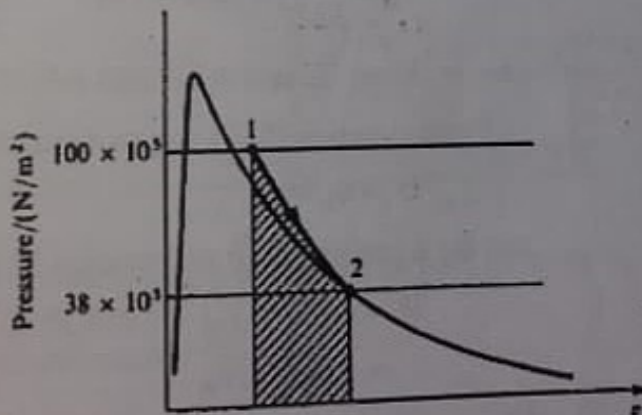
therefore

$$W = -169.7 \text{ kJ/kg}$$

i.e. Work done by the steam = +169.7 kJ/kg

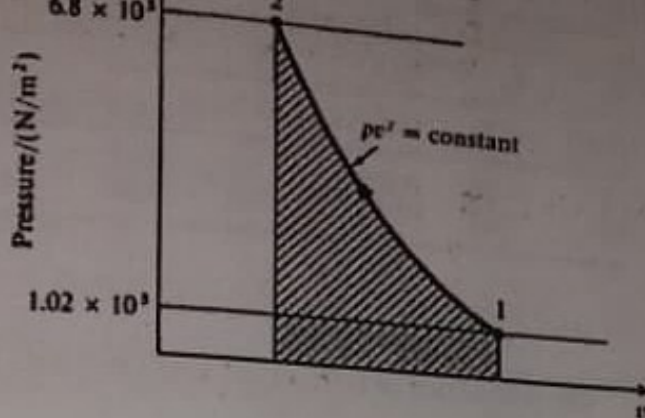
The process is shown on a  $p-v$  diagram in Fig. 3.10, the shaded area representing the work done by the steam.

Fig. 3.10 Reversible adiabatic process for steam on a  $p-v$  diagram for Example 3.4



2 v₂. Hence





### 3.3 Polytropic processes

It is found that many processes in practice approximate to a reversible law of the form  $pv^n = \text{constant}$ , where  $n$  is a constant. Both vapours and perfect gases obey this type of law closely in many non-flow processes. Such processes are internally reversible.

From equation (1.2) for any reversible process,

$$W = - \int p \, dv$$

For a process in which  $pv^n = \text{constant}$ , we have  $p = c/v^n$ , where  $c$  is a constant. Therefore

$$W = -c \int_{v_1}^{v_2} \frac{dv}{v^n} = -c \left[ \frac{v^{-n+1}}{-n+1} \right] = -c \left( \frac{v_2^{-n+1} - v_1^{-n+1}}{-n+1} \right)$$

i.e. 
$$W = c \left( \frac{v_2^{1-n} - v_1^{1-n}}{n-1} \right) = \frac{p_2 v_2 v_2^{1-n} - p_1 v_1 v_1^{1-n}}{n-1}$$

since the constant,  $c$ , can be written as  $p_1 v_1^n$  or as  $p_2 v_2^n$ ,

i.e. 
$$\text{Work input} = \frac{p_2 v_2 - p_1 v_1}{n-1} \quad (3.24)$$

Equation (3.24) is true for any working substance undergoing a reversible polytropic process. It follows also that for any polytropic process we can write

$$\frac{p_1}{p_2} = \left( \frac{v_2}{v_1} \right)^n \quad (3.25)$$

#### Example 3.6

At the commencement of compression in the reciprocating compressor of a refrigeration plant the refrigerant is dry saturated at 1 bar. The compression process follows the law  $pv^{1.1} = \text{constant}$  until the pressure is 10 bar. Using

reversible adiabatic expansion; 1 to D' is constant volume cooling. Note that, since  $\gamma$  is always greater than unity, then process 1 to C must lie between processes 1 to B and 1 to D; similarly, process 1 to C' must lie between processes 1 to B' and 1 to D'.

For a vapour a generalization such as the above is not possible. A vapour may undergo a process according to a law  $pv = \text{constant}$ . In this case, since the characteristic equation of state,  $pv = RT$ , does not apply to a vapour, then the process is not isothermal. Tables must be used to find the properties at the end states, making use of the fact that  $p_1 v_1 = p_2 v_2$ . Expansion of steam in a reciprocating engine is found to approximate to a hyperbolic expansion ( $pv = \text{constant}$ ); such engines are rarely used nowadays.

### 3.4 Reversible flow processes

Although flow processes in practice are usually highly irreversible, it is sometimes convenient to assume that a flow process is reversible in order to provide an ideal comparison. An observer travelling with the flowing fluid would appear to see a change in thermodynamic properties as in a non-flow process. For example, in a reversible adiabatic process for a perfect gas, an observer travelling with the gas would appear to see a process  $pv^\gamma = \text{constant}$  taking place, but the work input would not be given by  $-\int p dv$ , or by the change in internal energy as given by equation (3.13). Some work is done by virtue of the forces acting between the moving gas and its surroundings. For example, for a reversible adiabatic flow process for a perfect gas, from the flow equation (1.10), for unit mass flow rate

$$\left(h_1 + \frac{C_1^2}{2}\right) + Q + W = \left(h_2 + \frac{C_2^2}{2}\right)$$

Then since  $Q = 0$

$$W = (h_2 - h_1) + \left(\frac{C_2^2}{2} - \frac{C_1^2}{2}\right)$$

Also, since the process is assumed to be reversible, then for a perfect gas,  $pv^\gamma = \text{constant}$ . This equation can be used to fix the end states. Note that even if the kinetic energy terms are negligibly small the work input in a reversible adiabatic flow process between two states is not equal to the work input in a reversible adiabatic non-flow process between the same states (given by equation (3.13) as  $W = u_2 - u_1$ ).

#### Example 3.8

A gas turbine receives gases from the combustion chamber at 7 bar and 650°C, with a velocity of 9 m/s. The gases leave the turbine at 1 bar with a velocity of 45 m/s. Assuming that the expansion is adiabatic and reversible in the ideal case, calculate the power output per unit mass flow rate. For the gases take  $\gamma = 1.333$  and  $c_p = 1.11 \text{ kJ/kg K}$ .



*Solution* Using the flow equation for an adiabatic process

$$W = \dot{m} \left\{ (h_2 - h_1) + \left( \frac{C_2^2 - C_1^2}{2} \right) \right\}$$

For a perfect gas from equation (2.18),  $h = c_p T$ , therefore,

$$W = \dot{m} \left\{ c_p (T_2 - T_1) + \left( \frac{C_2^2 - C_1^2}{2} \right) \right\}$$

To find  $T_2$  use equation (3.21),

$$\frac{T_1}{T_2} = \left( \frac{p_1}{p_2} \right)^{(1-\gamma)/\gamma}$$

i.e.  $\frac{T_1}{T_2} = \left( \frac{7}{1} \right)^{(1.333-1)/1.333} = 1.626$

therefore

$$T_2 = \frac{T_1}{1.626} = \frac{923}{1.626} = 567.7 \text{ K}$$

where  $T_1 = 650 + 273 = 923 \text{ K}$ .

Hence substituting for unit mass flow rate

$$\dot{W} = 1 \times 1.11(567.7 - 923) + \left( \frac{45^2 - 9^2}{2 \times 10^3} \right)$$

therefore

$$\dot{W} = -394.4 + 0.97 = -393.4 \text{ kW}$$

i.e. Power output per kilogram per second = 393.4 kW

Note that in Example 3.8 the kinetic energy change is small compared with the enthalpy change. This is often the case in problems on flow processes. The change in kinetic energy can sometimes be taken to be negligible.

For a vapour undergoing a reversible adiabatic flow process the entropy is fixed by equating the initial and final entropies (see Ch. 4).

### 3.5 Irreversible processes

The criteria of reversibility are stated in section 1.4. The equations of sections 3.1, 3.2, and 3.3 can only be used when the process obeys the criteria of reversibility to a close approximation. In processes in which a fluid is contained in a cylinder behind a piston, friction effects can be assumed to be negligible. However, in order to satisfy criterion (c) in section 1.4 heat must be transferred to or from the system through a finite temperature difference. In an isothermal process this is conceivable, since in all other processes the temperature of the system is continually changing during the process. To satisfy criterion (c) the temperature of the cooling or heating medium must be

Reversible and

Fig. 3.13 insulated vessels



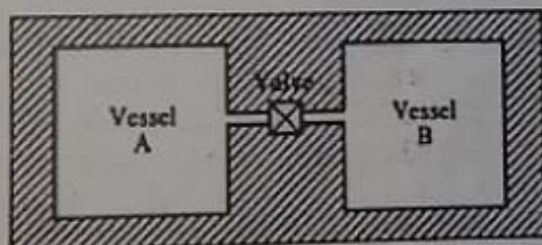
to the system would be required to change correspondingly. Ideally a way of achieving reversibility can be imagined, but in practice it cannot even be approached as an approximation. Nevertheless, if we accept inevitable irreversibilities in the surroundings, we can still have processes which are internally reversible. That is, the system undergoes a process which can be reversed, but the surroundings undergo an irreversible change. Most processes occurring in a cylinder behind a piston can be assumed to be internally reversible to a close approximation, and the equations of sections 3.1, 3.2, and 3.3 can be used where applicable. Certain processes cannot be assumed to be internally reversible, and the important cases will now be considered.

### Unresisted, or free, expansion

This process was mentioned in section 1.5 in order to show that in an irreversible process the work done is not given by  $-\int p \, dv$ . Consider two vessels A and B, interconnected by a short pipe with a valve, and perfectly thermally insulated (see Fig. 3.13). Initially let the vessel A be filled with a fluid at a certain pressure, and let B be completely evacuated. When the valve is opened the fluid in A will expand rapidly to fill both vessels A and B. The pressure finally will be lower than the initial pressure in vessel A. This is known as an unresisted expansion or a free expansion. The process is not reversible, since external work would have to be done to restore the fluid to its initial condition. The non-flow energy equation, (1.4), can be applied between the initial and final states,

$$\text{i.e. } Q + W = u_2 - u_1$$

Fig. 3.13 Two perfectly insulated interconnected vessels



Now in this process no work is done on or by the fluid, since the boundary of the system does not move. No heat flows to or from the fluid since the system is well lagged. The process is therefore adiabatic, but irreversible,

$$\text{i.e. } u_2 - u_1 = 0 \quad \text{or} \quad u_2 = u_1$$

In a free expansion therefore the internal energy initially equals the internal energy finally.

For a perfect gas, we have, from equation (2.14),

$$u = c_v T$$

Therefore for a free expansion of a perfect gas

$$c_v T_1 = c_v T_2$$

$$\text{i.e. } T_1 = T_2$$

That is, for a perfect gas undergoing a free expansion, the initial temperature is equal to the final temperature.

**Example 3.9**

Air at 20 bar is initially contained in vessel A of Fig. 3.13, the volume of which can be assumed to be  $1 \text{ m}^3$ . The valve is opened and the air expands to fill vessels A and B. Assuming that the vessels are of equal volume, find the final pressure of the air.

**Solution**

For a perfect gas for a free expansion,  $T_1 = T_2$ . Also from equation  $pV = mRT$ , hence  $p_1 V_1 = p_2 V_2$ .

Now  $V_2$  is the combined volumes of vessels A and B,

$$\text{i.e. } V_2 = V_A + V_B = 1 + 1 = 2 \text{ m}^3 \quad \text{and} \quad V_1 = 1 \text{ m}^3$$

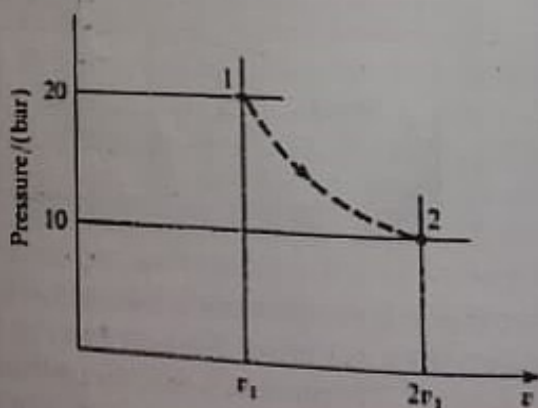
Therefore we have

$$p_2 = p_1 \times \frac{V_1}{V_2} = 20 \times \frac{1}{2} = 10 \text{ bar}$$

i.e. Final pressure = 10 bar

The process is shown on a  $p$ - $v$  diagram in Fig. 3.14. State 1 is fixed at 20 bar and  $1 \text{ m}^3$  when the mass of gas is known; state 2 is fixed at 10 bar and  $2 \text{ m}^3$  for the same mass of gas. The process between these states is irreversible and must be drawn dotted. The points 1 and 2 lie on an isothermal line, but the process between 1 and 2 cannot be called isothermal, since the intermediate temperatures are not the same throughout the process. There is no work done during the process, and the area under the dotted line does not represent work done.

Fig. 3.14 Irreversible process on a  $p$ - $v$  diagram for Example 3.9

**Throttling**

A flow of fluid is said to be throttled when there is some restriction to the flow, when the velocities before and after the restriction are either equal or nearly equal, and when there is a negligible heat loss to the surroundings. The restriction to flow can be a partly open valve, an orifice, or any other sudden change in the cross-section of the flow.

Fig. 3.15 Throttling process

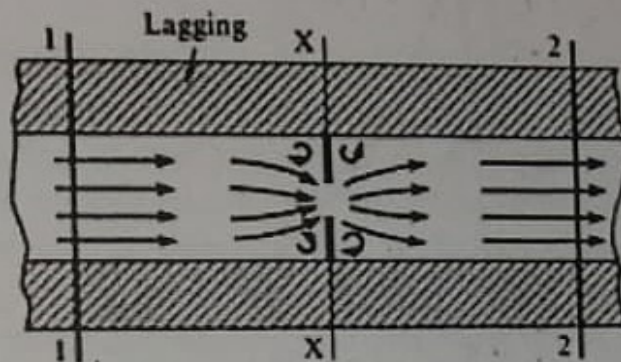


An example of throttling is shown in Fig. 3.15. The fluid, flowing steadily along a well-lagged pipe, passes through an orifice at section X-X. Since the pipe is well lagged it can be assumed that no heat flows to or from the fluid. The flow equation (1.10) can be applied between any two sections of the flow,

$$\text{i.e.} \quad \dot{m} \left( h_1 + \frac{C_1^2}{2} \right) + \dot{Q} + \dot{W} = \dot{m} \left( h_2 + \frac{C_2^2}{2} \right)$$

Now since  $\dot{Q} = 0$ , and  $\dot{W} = 0$ , then

$$h_1 + \frac{C_1^2}{2} = h_2 + \frac{C_2^2}{2}$$



When the velocities  $C_1$  and  $C_2$  are small, or when  $C_1$  is approximately equal to  $C_2$ , then the kinetic energy terms may be neglected. (Note that sections 1-1 and 2-2 can be chosen well upstream and well downstream of the disturbance to the flow, so that this latter assumption is justified.) Then  $h_1 = h_2$ . Therefore for a throttling process, the enthalpy initially is equal to the enthalpy finally.

The process is adiabatic, but is highly irreversible because of the eddying of the fluid round the orifice at X-X. Between sections 1-1 and X-X the enthalpy decreases and the kinetic energy increases as the fluid accelerates through the orifice. Between sections X-X and 2-2 the enthalpy increases as the kinetic energy is destroyed by fluid eddies.

For a perfect gas, from equation (2.18),  $h = c_p T$ , therefore,

$$c_p T_1 = c_p T_2 \quad \text{or} \quad T_1 = T_2$$

For throttling of a perfect gas, therefore, the temperature initially equals the temperature finally.

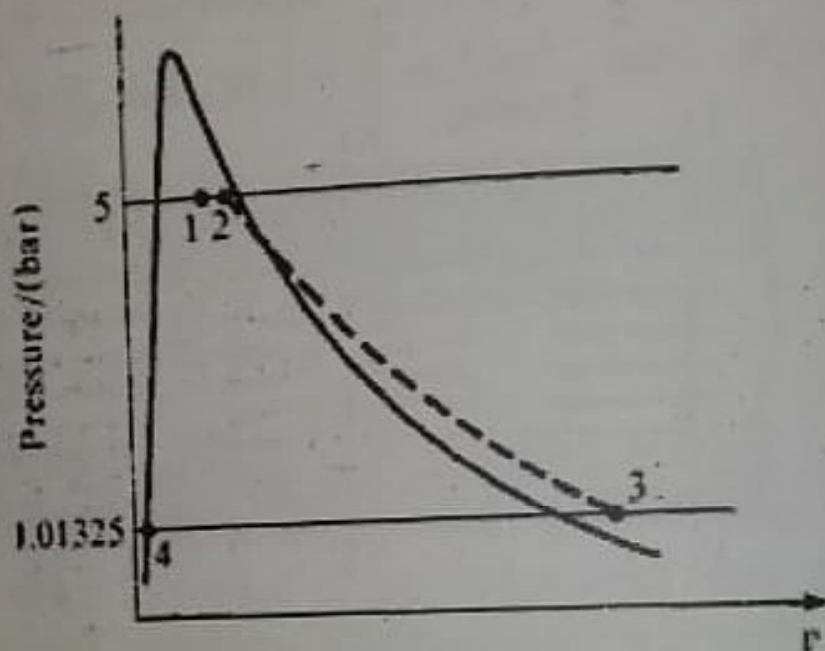
The process of throttling can be used to find the dryness fraction of steam. A sample of steam is drawn off the steam main, passed through a mechanical separator, then through a throttle valve, and finally through a condenser; the water separated from the mechanical separator and the water from the condenser are weighed and the dryness fraction calculated as shown in the following example.

### Example 3.10

The dryness fraction of wet steam in a main is determined using a separating and throttling calorimeter. The pressure in the main is 5 bar; after throttling, the steam pressure and temperature are 1.013 25 bar and  $120^\circ\text{C}$ ; the water collected from the separator is at the rate of 0.5 kg/h, and that from the condenser at the rate of 9 kg/h. Making suitable assumptions, calculate the dryness fraction of the steam in the main.



the separating process. Process 2-3 is shown dotted since the process is isenthalpic, no work is done during the process and the area under line 2-3 is equal to work done.



The enthalpy after throttling is obtained by interpolating from steam

$$h_3 = 2676 + \frac{(120 - 100)}{(150 - 100)} \times (2777 - 2676)$$

$$= 2716.4 \text{ kJ/kg}$$

For an adiabatic throttling process neglecting kinetic energy changes, therefore using equation (2.2)

$$h_2 = h_3 = h_{f2} + x_2 h_{fg2}$$

therefore

$$x_2 = \frac{2716.4 - 640}{2109} = 0.985$$

where  $h_{f2} = 640 \text{ kJ/kg}$ , and  $h_{fg2} = 2109 \text{ kJ/kg}$ , are read from saturation tables at 5 bar.

The mass flow rate of

therefore given

Fig. 3.  
process

The mass flow rate of dry vapour in the sample is therefore the total mass flow rate of  $(0.5 + 9)$  kg/h minus  $\dot{m}_{w2}$ , ( $= 0.635$  kg/h)

i.e. Mass flow rate of dry vapour in sample  
 $= 0.5 + 9 - 0.635 = 8.865$  kg/h

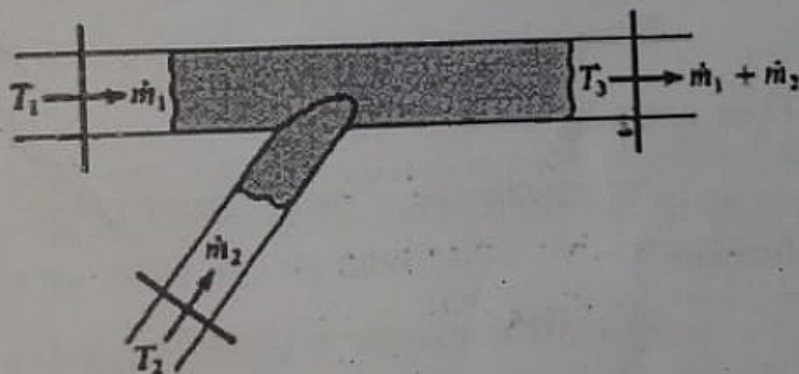
Then the dryness fraction in the main is the mass flow rate of dry vapour divided by the total mass flow rate,

$$\text{i.e. } x_1 = \frac{8.865}{(0.5 + 9)} = 0.933$$

## Adiabatic mixing

The mixing of two streams of fluid is quite common in engineering practice, and can usually be assumed to occur adiabatically. Consider two streams of a fluid mixing as shown in Fig. 3.17. Let the streams have mass flow rates  $\dot{m}_1$  and  $\dot{m}_2$  and temperatures  $T_1$  and  $T_2$ . Let the resulting mixed stream have a temperature  $T_3$ . There is no heat flow to or from the fluid, and no work is done, hence from the flow equation, we have, neglecting changes in kinetic energy,

$$\dot{m}_1 h_1 + \dot{m}_2 h_2 = \dot{m}_3 h_3 \quad (3.33)$$



For a perfect gas, from equation (2.18),  $h = c_p T$ , hence,

$$\dot{m}_1 c_{p1} T_1 + \dot{m}_2 c_{p2} T_2 = (\dot{m}_1 c_{p1} + \dot{m}_2 c_{p2}) T_3$$

Or, assuming that the two streams 1 and 2 are of the same fluid with the same specific heat capacity,

$$\dot{m}_1 T_1 + \dot{m}_2 T_2 = (\dot{m}_1 + \dot{m}_2) T_3 \quad (3.34)$$

The mixing process is highly irreversible due to the large amount of eddying and churning of the fluid that takes place.