SECTION 15–3 CHECKUP

1. How many poles does a second-order low-pass filter have? How many resistors and how many capacitors are used in the frequency-selective circuit?
2. Why is the damping factor of a filter important?
3. What is the primary purpose of cascading low-pass filters?

15–4 ACTIVE HIGH-PASS FILTERS

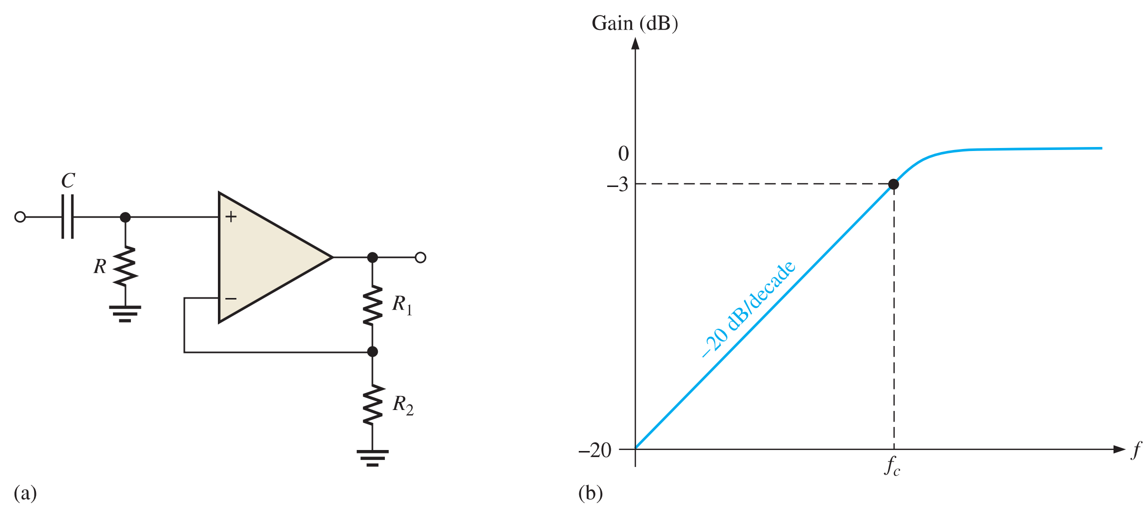
In high-pass filters, the roles of the capacitor and resistor are reversed in the *RC* circuits. Otherwise, the basic parameters are the same as for the low-pass filters.

After completing this section, you should be able to

- **Identify and analyze active high-pass filters**
- Identify a single-pole high-pass filter circuit
  - Explain limitations at higher pass-band frequencies
- Identify a Sallen-Key high-pass filter circuit
  - Describe the filter operation
  - Calculate component values
- Discuss cascaded high-pass filters
  - Describe a six-pole filter

A Single-Pole Filter

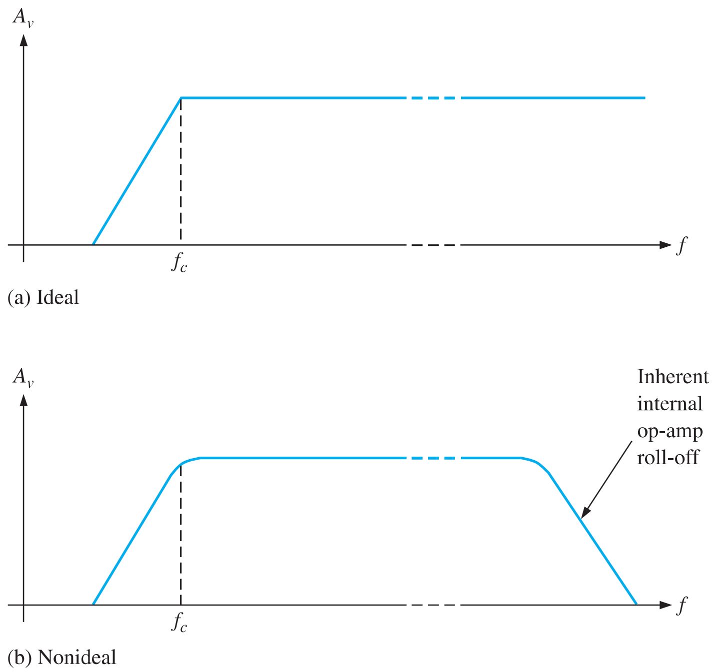
A high-pass active filter with a −20 dB/decade roll-off is shown in Figure 15–13(a). Notice that the input circuit is a single high-pass *RC* circuit. The negative feedback circuit is the same as for the low-pass filters previously discussed. The high-pass response curve is shown in Figure 15–13(b).



▲ FIGURE 15–13

Single-pole active high-pass filter and response curve.

Ideally, a high-pass filter passes all frequencies above f_c without limit, as indicated in Figure 15–14(a), although in practice, this is not the case. As you have learned, all op-amps inherently have internal *RC* circuits that limit the amplifier's response at high frequencies.



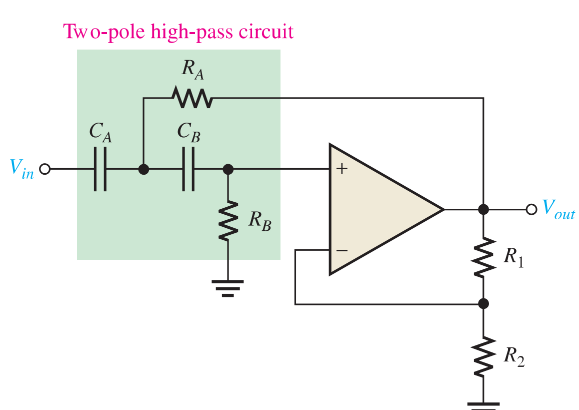
▲ FIGURE 15–14

High-pass filter response.

Therefore, there is an upper-frequency limit on the high-pass filter's response which, in effect, makes it a band-pass filter with a very wide bandwidth. In the majority of applications, the internal high-frequency limitation is so much greater than that of the filter's critical frequency that the limitation can be neglected. In some applications, discrete transistors are used for the gain element to increase the high-frequency limitation beyond that realizable with available op-amps.

The Sallen-Key High-Pass Filter

A high-pass Sallen-Key configuration is shown in Figure 15–15. The components R_A, C_A, R_B, and C_B form the two-pole frequency-selective circuit. Notice that the positions of the resistors and capacitors in the frequency-selective circuit are opposite to those in the low-pass configuration. As with the other filters, the response characteristic can be optimized by proper selection of the feedback resistors, R_1 and R_2.



◀ FIGURE 15–15

Basic Sallen-Key high-pass filter.

EXAMPLE 15–5

Choose values for the Sallen-Key high-pass filter in Figure 15–15 to implement an equal-value second-order Butterworth response with a critical frequency of approximately 10 kHz.

Solution

Start by selecting a value for R_A and R_B (R_1 or R_2 can also be the same value as R_A and R_B for simplicity).

$$R = R_A = R_B = R_2 = \mathbf{3.3\,k\Omega} \text{ (an arbitrary selection)}$$

Next, calculate the capacitance value from $f_c = 1/(2\pi RC)$.

$$C = C_A = C_B = \frac{1}{2\pi R f_c} = \frac{1}{2\pi(3.3\,\text{k}\Omega)(10\,\text{kHz})} = \mathbf{0.0048\,\mu F}$$

For a Butterworth response, the damping factor must be 1.414 and $R_1/R_2 = 0.586$.

$$R_1 = 0.586R_2 = 0.586(3.3\,\text{k}\Omega) = \mathbf{1.93\,k\Omega}$$

If you had chosen $R_1 = 3.3\,\text{k}\Omega$, then

$$R_2 = \frac{R_1}{0.586} = \frac{3.3\,\text{k}\Omega}{0.586} = 5.63\,\text{k}\Omega$$

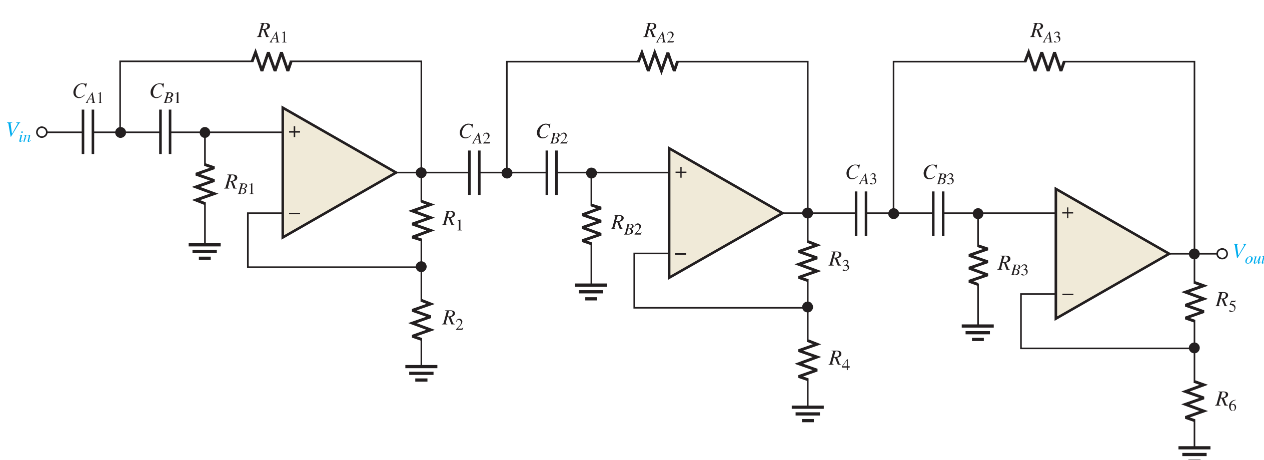
Either way, an approximate Butterworth response is realized by choosing the nearest standard values.

Related Problem

Select values for all the components in the high-pass filter of Figure 15–15 to obtain an $f_c = 300$ Hz. Use equal-value components with $R = 10\,\text{k}\Omega$ and optimize for a Butterworth response.

Cascading High-Pass Filters

As with the low-pass configuration, first- and second-order high-pass filters can be cascaded to provide three or more poles and thereby create faster roll-off rates. Figure 15–16 shows a six-pole high-pass filter consisting of three Sallen-Key two-pole stages. With this configuration optimized for a Butterworth response, a roll-off of −120 dB/decade is achieved.



▲ **FIGURE 15–16**

Sixth-order high-pass filter.

SECTION 15–4 CHECKUP

1. How does a high-pass Sallen-Key filter differ from the low-pass configuration?
2. To increase the critical frequency of a high-pass filter, would you increase or decrease the resistor values?
3. If three two-pole high-pass filters and one single-pole high-pass filter are cascaded, what is the resulting roll-off?

15–5 ACTIVE BAND-PASS FILTERS

As mentioned, band-pass filters pass all frequencies bounded by a lower-frequency limit and an upper-frequency limit and reject all others lying outside this specified band. A band-pass response can be thought of as the overlapping of a low-frequency response curve and a high-frequency response curve.

After completing this section, you should be able to

- **Analyze basic types of active band-pass filters**
- Describe how to cascade low-pass and high-pass filters to create a band-pass filter
  - Calculate the critical frequencies and the center frequency
- Identify and analyze a multiple-feedback band-pass filter
  - Determine the center frequency, quality factor (Q), and bandwidth ◆ Calculate the voltage gain
- Identify and describe the state-variable filter
  - Explain the basic filter operation ◆ Determine the Q
- Identify and discuss the biquad filter

Cascaded Low-Pass and High-Pass Filters

One way to implement a band-pass filter is a cascaded arrangement of a high-pass filter and a low-pass filter, as shown in Figure 15–17(a), as long as the critical frequencies are sufficiently separated. Each of the filters shown is a Sallen-Key Butterworth configuration so that the roll-off rates are −40 dB/decade, indicated in the composite response curve of Figure 15–17(b). The critical frequency of each filter is chosen so that the response curves overlap sufficiently, as indicated. The critical frequency of the high-pass filter must be sufficiently lower than that of the low-pass stage. This filter is generally limited to wide bandwidth applications.

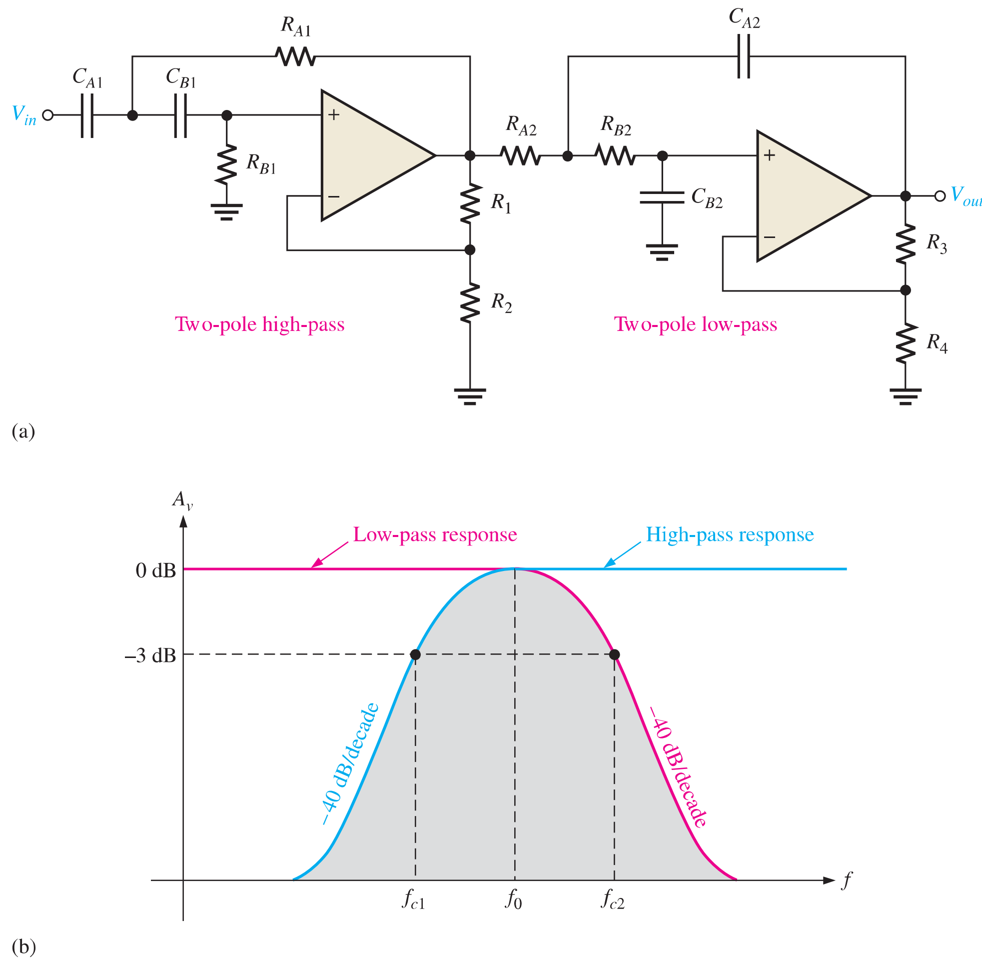
The lower frequency f_{c1} of the passband is the critical frequency of the high-pass filter. The upper frequency f_{c2} is the critical frequency of the low-pass filter. Ideally, as discussed earlier, the center frequency f_0 of the passband is the geometric mean of f_{c1} and f_{c2}. The following formulas express the three frequencies of the band-pass filter in Figure 15–17.

$$f_{c1} = \frac{1}{2\pi\sqrt{R_{A1}R_{B1}C_{A1}C_{B1}}}$$

$$f_{c2} = \frac{1}{2\pi\sqrt{R_{A2}R_{B2}C_{A2}C_{B2}}}$$

$$f_0 = \sqrt{f_{c1}f_{c2}}$$

Of course, if equal-value components are used in implementing each filter, the critical frequency equations simplify to the form $f_c = 1/(2\pi RC)$.



▶ **FIGURE 15–17**

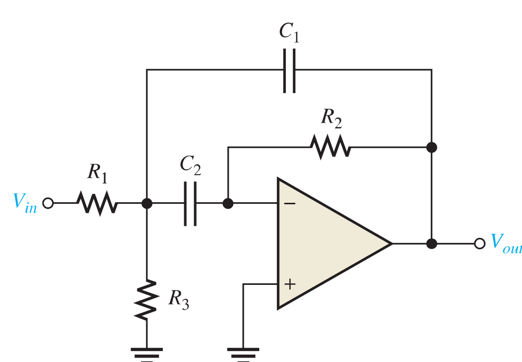
Band-pass filter formed by cascading a two-pole high-pass and a two-pole low-pass filter (it does not matter in which order the filters are cascaded).

Multiple-Feedback Band-Pass Filter

Another type of filter configuration, shown in Figure 15–18, is a multiple-feedback band-pass filter. The two feedback paths are through R_2 and C_1. Components R_1 and C_1 provide the low-pass response, and R_2 and C_2 provide the high-pass response. The maximum gain, A_0, occurs at the center frequency. Q values of less than 10 are typical in this type of filter.

▶ **FIGURE 15–18**

Multiple-feedback band-pass filter.



An expression for the center frequency is developed as follows, recognizing that R_1 and R_3 appear in parallel as viewed from the C_1 feedback path (with the V_{in} source replaced by a short).

$$f_0 = \frac{1}{2\pi\sqrt{(R_1 \parallel R_3)R_2C_1C_2}}$$

Making $C_1 = C_2 = C$ yields

$$f_0 = \frac{1}{2\pi\sqrt{(R_1 \| R_3)R_2C^2}} = \frac{1}{2\pi C\sqrt{(R_1 \| R_3)R_2}}$$

$$= \frac{1}{2\pi C}\sqrt{\frac{1}{R_2(R_1 \| R_3)}} = \frac{1}{2\pi C}\sqrt{\left(\frac{1}{R_2}\right)\left(\frac{1}{R_1R_3/R_1 + R_3}\right)}$$

$$\mathbf{f_0 = \frac{1}{2\pi C}\sqrt{\frac{R_1 + R_3}{R_1R_2R_3}}}$$

Equation 15–8

A value for the capacitors is chosen and then the three resistor values are calculated to achieve the desired values for f_0, BW, and A_0. As you know, the Q can be determined from the relation $Q = f_0/BW$. The resistor values can be found using the following formulas (stated without derivation):

$$R_1 = \frac{Q}{2\pi f_0 CA_0}$$

$$R_2 = \frac{Q}{\pi f_0 C}$$

$$R_3 = \frac{Q}{2\pi f_0 C(2Q^2 - A_0)}$$

To develop a gain expression, solve for Q in the R_1 and R_2 formulas as follows:

$$Q = 2\pi f_0 A_0 CR_1$$

$$Q = \pi f_0 CR_2$$

Then,

$$2\pi f_0 A_0 CR_1 = \pi f_0 CR_2$$

Cancelling yields

$$2A_0R_1 = R_2$$

$$\mathbf{A_0 = \frac{R_2}{2R_1}}$$

Equation 15–9

In order for the denominator of the equation $R_3 = Q/[2\pi f_0 C(2Q^2 - A_0)]$ to be positive, $A_0 < 2Q^2$, which imposes a limitation on the gain.

EXAMPLE 15–6

Determine the center frequency, maximum gain, and bandwidth for the filter in Figure 15–19.

▶ FIGURE 15–19