Chapter 1   Tension, Compression, and Shear

1.1 Introduction

Mechanics of Materials :   to understand the behavior of solid bodies subjected to various types of loading

This course have several names : "Strength of Materials", "Mechanics of Deformable Bodies"

Two main aspects:

1. stress analysis : analysis of bodies under the action of external force, to determine the internal stress and their deformation
2. mechanical properties of materials : consideration of such things as material strength, stability, fatigue and brittle fracture etc.

The principal objective of this analysis is to determine the stresses, strains, and displacements in structures and their components due to loads acting on them, we use the physical properties of the materials as well as numerous theoretical laws and concepts to analysis

Theoretical analysis and experimental results have equally important roles in this study

In the analyses, we will make logical derivations to obtain formulas and equations for predicting mechanical behavior, but we must recognize that this formula cannot be used in a realistic way unless certain properties of the materials are known, also, many practical problems of great important in engineering cannot be handled efficiently by theoretical means, experimental measurements becomes a necessity.

In the study of this course, you will find that your efforts are divided naturally into two parts

1. understanding the logical development of the concepts
2. applying those concepts to practical situations

Finally, sometime use the symbolic solution, it provides the opportunity to check the dimensions at any stage of the work, and the most important reason is to obtain a general formula that can be programmed on a computer

Both systems of unit are used

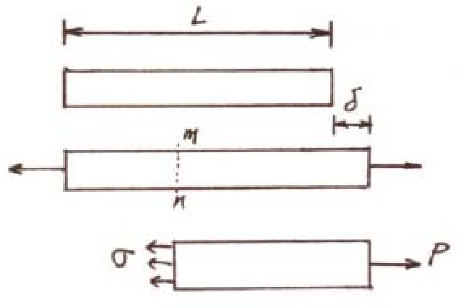
1. International System of Units (SI unit) : basic units are kg, sec, m
2. U.S. Customary System (USCS) : basic units are lb, sec, ft

Relative Courses:

Statics (Dynamics) --> Mechanics of Materials --> Advanced Mechanics of Materials --> Elasticity --> Plasticity, Viscoelasticity, Structural Stability, Plate and Shell, Experimental Stress Analysis etc.

1.2 Normal Stress and Strain

Consider a prismatic bar, the axial forces produce a uniform stretching of the bar, it is called the bar is in tension



mn : cross section $\perp$ the longitudinal axis

A : cross section area

the intensity of the force (force per unit area) is called stress, assuming that the stress has uniform distribution, then

$$\sigma = \frac{P}{A} \qquad \text{force equilibrium}$$

when the bar is stretched, the resulting stress are tensile stress, if the bar is compressed, the stress are compressive stress

the stress σ acts in the direction perpendicular to the cut surface, it is referred as normal stress, another type of stress is called shear stress

sign convention of the normal stresses are : tensile stress as positive and compressive stress as negative

Unit of stress :

SI unit : N / m^2 (Pa, Pascal), N / mm^2 (MPa)

1 MPa = 10^6 Pa, 1 GPa = 10^9 Pa

USCS : lb / in^2 (psi), kip / in^2 (ksi) 1 ksi = 10^3 psi

1 psi = 6,895 Pa 1 ksi = 6.895 MPa

for a force P = 27 kN acts on a round bar with d = 50 mm, the stress is

$$\sigma = \frac{P}{A} = \frac{P}{\pi d^2/4} = \frac{27 \text{ kN}}{\pi (50 \text{ mm})^2/4} = 13.8 \text{ MPa}$$

The equation $\sigma = P / A$ to be valid only for the stress must be uniformly distributed over the cross section of the bar, this condition is realized if the axial force P acts through the centroid of the cross section

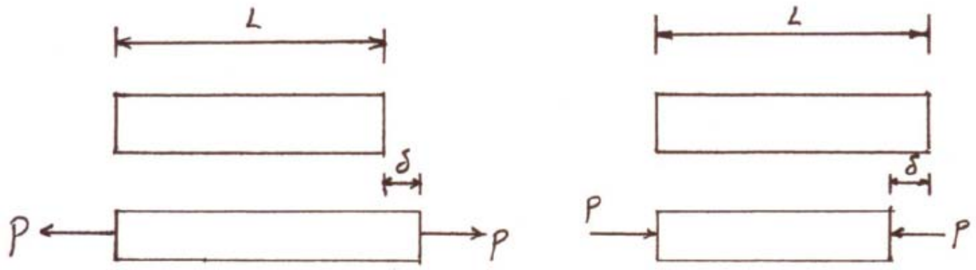
the uniform stress condition exists throughout the length of the member except near the ends (end effect or Saint Venant's principle)

if the load itself is distributed uniformly over the end, the stress at the end is the same as elsewhere, for the concentrated load acts at the end (over a small area), resulting in high localized stressed and nonuniform stress distribution

the formula $\sigma = P / A$ may be used with good accuracy at any point that is at least a distant d away from the end, where d is the largest transverse dimension of the bar

the formula $\sigma = P / A$ is the average normal stress when the stress distribution is nonuniform

An axial load bar becoming longer when in tension and shorter when in compression



the elongation δ is the cumulative of the stretching throughout the length L of the bar, then the elongation per unit length is called strain ε

$$\varepsilon = \frac{\delta}{L} \quad \text{(normal strain : associated with normal stress)}$$

tensile strain : positive compressive strain : negative

strain is a dimensionless quantity, it is recorded in the form as mm/m, μm/m, in/in

e.g. $L = 2$ m, $\delta = 1.4$ mm

$$\text{then} \quad \varepsilon = \frac{1.4 \times 10^{-3}\ \text{m}}{2\ \text{m}} = 0.0007 = 0.7\ \text{mm/m}$$

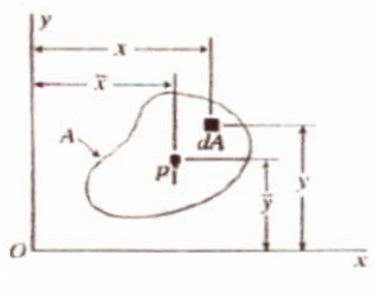
$$= 700 \times 10^{-6} = 700\ \mu\text{m/m} = 0.07\%$$

Uniaxial Stress and Strain

for a prismatic bar, the loads act through the centroid of the cross sections, and the material be homogeneous, the resulting state of stress and strain is called uniaxial stress and strain

Line of Action of the Axial Forces for a Uniform Stress Distribution

consider a prismatic bar of arbitrary cross-sectional shape subjected to axial forces P that produce uniformly distributed stresses σ, let $p_1(x, y)$ represent the point in the cross section where the line of action of the forces intersects the cross section



the moments of the force P are

$$M_x = P\bar{y} \qquad M_y = -P\bar{x}$$

for an element of area dA, the force acts on the element is $\sigma\, dA$, and the moments of this elemental force about the x and y axes are $\sigma\, y\, dA$ and $-\sigma\, x\, dA$, the total moments are obtained by integrating over the cross sectional area

$$M_x = \int \sigma\, y\, dA \qquad M_y = -\int \sigma\, x\, dA$$

then $$P\bar{y} = \int \sigma\, y\, dA \qquad P\bar{x} = \int \sigma\, x\, dA$$

because the stress σ is uniformly distributed, $\sigma = P / A$, then

$$\bar{y} = \int y\, dA / A \qquad \text{and} \qquad \bar{x} = \int x\, dA / A$$

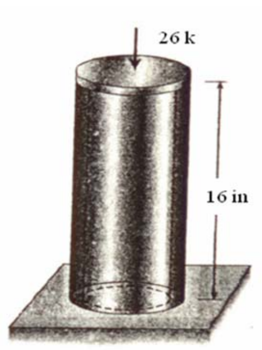
these equations are the same as the equations of the centroid of and area, thus we and concluded that : in order to have uniform tension or compression in a prismatic bar, the axial force must act through the centroid of the cross-sectional area

Example 1-1

for a hollow circular tube of aluminum supports a compressive load of

240 kN, with d_1 = 90 mm and d_2 = 130 mm, its length is 1 m, the shortening of the tube is 0.55 mm, determine the stress and strain

$$A = \frac{\pi}{4}(d_2^2 - d_1^2) = \frac{\pi}{4}(130^2 - 90^2) = 6{,}912 \text{ mm}^2$$



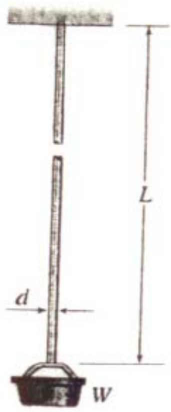
$$\sigma = \frac{P}{A} = \frac{240{,}000 \text{ N}}{6{,}912 \text{ mm}^2} = 34.7 \text{ MPa (comp.)}$$

the compressive strain is

$$\varepsilon = \frac{\delta}{L} = \frac{0.55 \text{ mm}}{1{,}000 \text{ mm}} = 550 \times 10^{-6} = 550\,\mu \text{ m/m}$$

Example 1-2

a circular steel rod of length L and diameter d hangs and holds a weight W at its lower end



(a) find $\sigma_{\max}$ of the rod, included its own weight

(b) L = 40 m, d = 8 mm, W = 1.5 kN, calculate $\sigma_{\max}$

(a) the maximum force F_{max} occurs at the upper end

$$F_{max} = W + W_0 = W + \gamma V = W + \gamma A L$$

$$\sigma_{\max} = \frac{F_{max}}{A} = \frac{W + \gamma A L}{A} = \frac{W}{A} + \gamma L$$

(b) the weight density γ of steel is 77 kN/m^3, the maximum stress is

$$\sigma_{\max} = \frac{1{,}500 \text{ N}}{\pi (8 \text{ mm})^2 / 4} + (77 \text{ kN/m}^3)(40 \text{ m})$$

$$= 29.84 \text{ MPa} + 3.08 \text{ MPa} = 32.92 \text{ MPa}$$

1.3 Mechanical Properties of Materials

mechanical properties of materials used in engineering design are determined by tests of small specimen according standards

ASTM : American Society for Testing of Materials

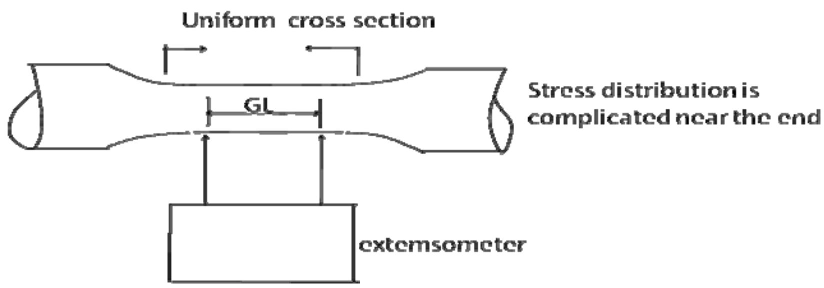
JIS : Japanese Industrial Standards

CNS : Chinese National Standards

etc.

the most common test is tension test for metals, to obtain the stress-strain diagram of materials (compression test are most used for rock and concrete)

cylindrical specimen are used



ASTM standard specimen for tension test (round bar)

$$d = 0.5 \text{ in } (12.7 \text{ mm}) \qquad GL = 2.0 \text{ in } (50 \text{ mm})$$

when the specimen is mounted on a testing system (MTS, Instron etc.), the load P and the elongation between GL are measured simultaneously

static test : the load is applied very slowly

dynamic test : load rate is very high

in the test $\sigma = P / A$ A : initial area of the cross section

$\varepsilon = \delta / GL$

σ : nominal stress (engineering stress)

ε : nominal strain (engineering strain)

σ_t : true stress (use actual area of the bar)

ε_t : true strain (actual distance of GL is used)

($\because$ during test, A and GL may vary)

specimens for compression test

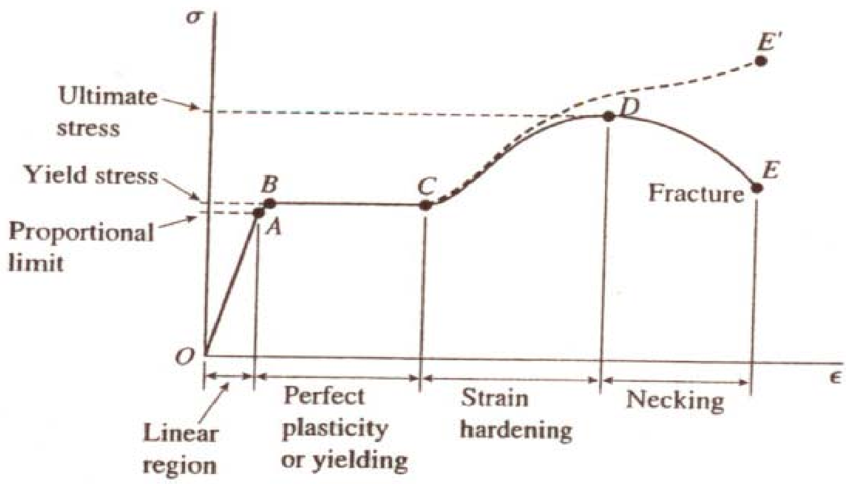
cubic : 2.0 in (50 mm) on one side

cylinder : d = 1 in (25 mm), L = 1 ~ 12 in (25 ~ 305 mm)

concrete : d = 6 in (152 mm), L = 12 in (305 mm), 28 days old

stress-strain diagram is characteristic of material

for mild-steel (low carbon steel)



A :

proportional limit (210 ~ 350 MPa, 30 ~ 50 ksi)

B : yield stress (more than 550 MPa [80 ksi], depends on the carbon content)

D : ultimate stress

E : fracture

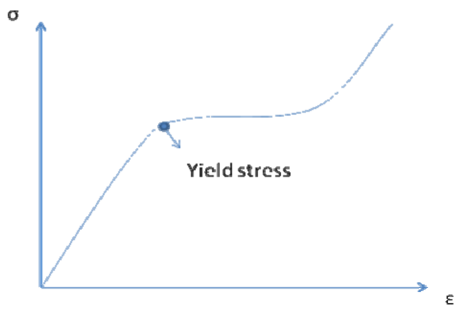
OA : linear region

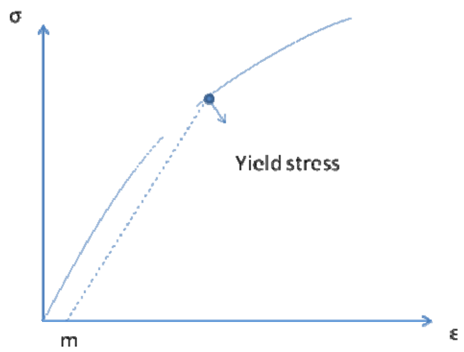
BC : perfect plasticity

CD : strain hardening

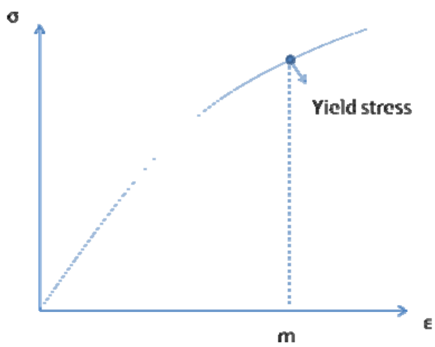
DE : necking

definitions of yield stress





om : specified offset, commonly 0.2 %



om : specified extension under load, commonly take 0.5%

lateral contraction occurs when specimen is in stretched, decrease in cross-sectional area $(A < A_0)$

for small strain, $\sigma \simeq \sigma_t$, for large strain, $\sigma < \sigma_t$

when contraction become large, necking occurs

in the stress-strain diagram

engineering σ-ε curve have a ultimate point *D*, and then decrease, this decrease is due to necking of the bar

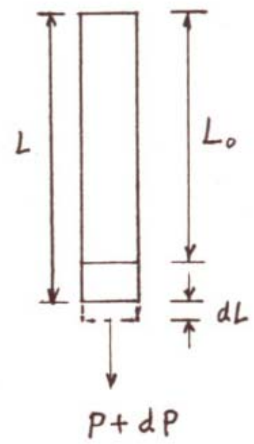
in true σ-ε curve, no decrease occur

definition of true strain : after a load P is applied to the specimen, the length changes from L_0 to L, an additional load dP produces an incremental length change dL, the strain de due to this increment is

$$d\varepsilon_t = \frac{dL}{L} \qquad \text{where } L \text{ is current length}$$

therefore, true strain ε_t can be expressed as an integral of the strain increment

$$\varepsilon_t = \int_{L_0}^{L} \frac{dL}{L} = \ln \frac{L}{L_0}$$



but $\quad \dfrac{L}{L_0} = 1 + \varepsilon$

$\therefore \quad \varepsilon_t = \ln(1 + \varepsilon)$

for small strain $\quad \varepsilon_t \simeq \varepsilon$

definition of true stress $\quad \sigma_t = P / A$

where A is current area, note that $A \neq A_0$

assume that under certain conditions of the volume of the material is the same before and after loading (it is true in plastic range)

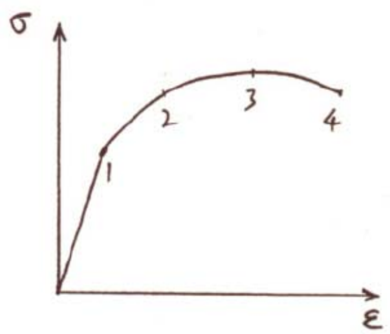
$$A_0 L_0 = A L$$

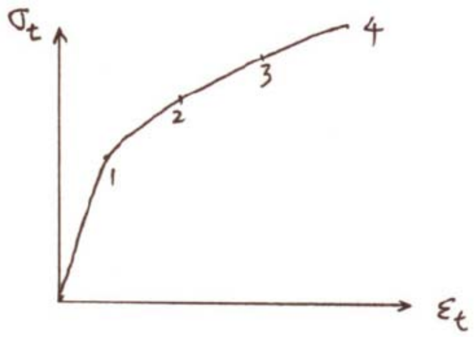
or $\quad \dfrac{L}{L_0} = \dfrac{A_0}{A} = 1 + \varepsilon$

$$\therefore \qquad \sigma_t = \frac{P}{A} = \frac{P}{A_0}\frac{A_0}{A} = \sigma(1+\varepsilon)$$

also for small strain $\sigma_t = \sigma$

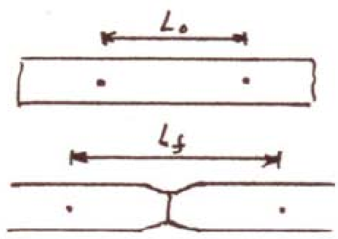
with known σ - ε curve, σ_t - ε_t curve can be obtained





Materials that undergo large strains before failure are classified as ductile (ε_f > 10%), such as mild steel, aluminum and its alloy, copper, lead etc.

aluminum alloy do not have a clearly definable yield point, gradual transition from linear to nonlinear region, yield stress always determined by offset method, in general, use 0.2% offset, it should be referred as offset yield stress, it is slightly above the proportional limit



The ductility of material in tension can be characterized by its elongation and by the reduction area

$$\text{percent elongation} = \frac{L_f - L_0}{L_0} \times 100\%$$

L_f : distance between gage mark in fracture

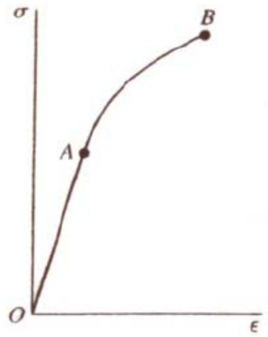
∵ the elongation is concentrated in the region of necking, thus the percent elongation depends upon the gage length and the dimensions of the specimen, when use percent elongation, the gage length should be given

$$\text{percent reduction in area} \quad = \quad \frac{A_0 - A_f}{A_0} \text{ x } 100\% \qquad \text{(RA)}$$

A_f : final area at fracture section

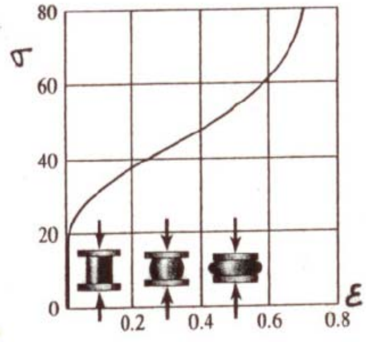
for ductile steel, $RA \simeq 50\%$

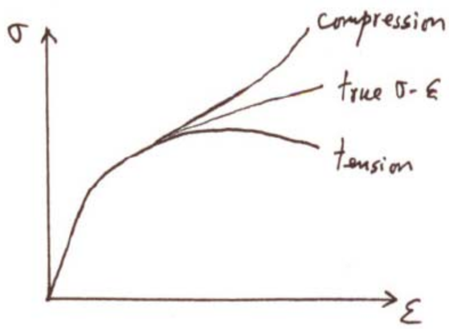
For material have small fracture strain, it is classified as brittle material, such as concrete, cast iron, rock, glass, etc.



B : fracture stress, same as ultimate stress

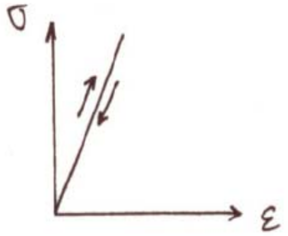
Stress-Strain diagram in compression have different shapes from those in tension, its cross-sectional will increase after yielding, and no necking occur

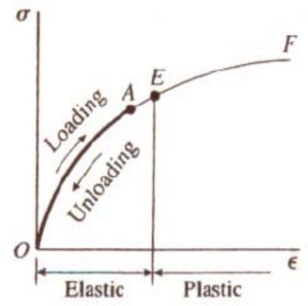




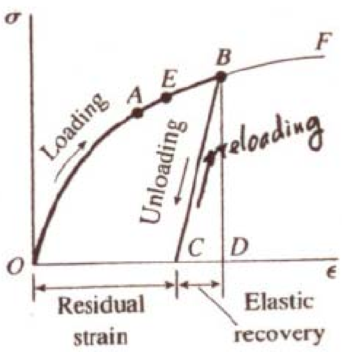
1.4 Elasticity, Plasticity and Creep

(a) linear elastic (b) nonlinear elastic





(c) elastoplastic

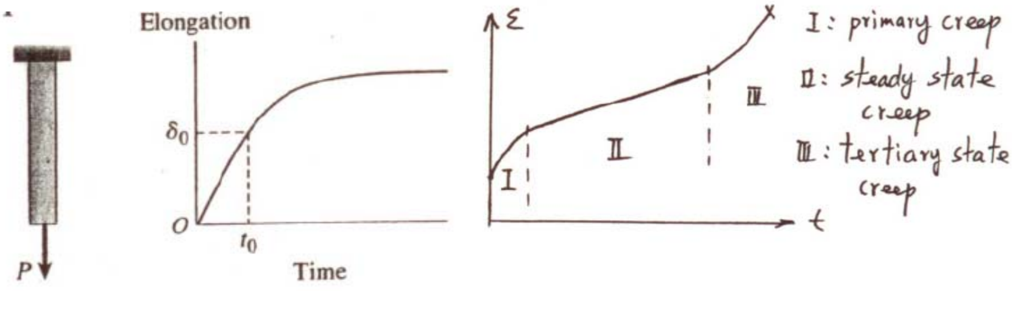


unloading is elastic

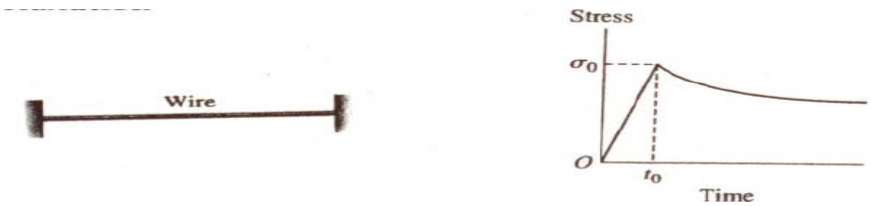
most of metallic materials have behave in this type

for reloading, the yield stress is increased to σ_B

(d) creep



(e) relaxation



other properties : fatigue strength, toughness, fracture toughness, notch toughness, resilience, etc.

1.5 Linear Elasticity, Hooke's Law and Poisson's Ratio

behavior of materials : linear and elastic

homogeneous : same composition throughout the body

isotropic : same elastic properties in all direction

linear relationship between stress and strain for linear elastic homogeneous and isotropic materials

$$\sigma = E\,\varepsilon \qquad \text{(Hooke's Law)}$$

E : modulus of elasticity (Young's modulus)

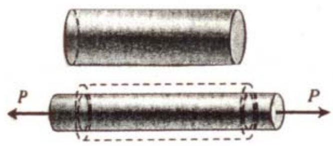
for steel E = 190 ~ 210 GPa or 28,000 ~ 30,000 ksi

for Al E = 70 ~ 73 GPa or 10,000 ~ 10,600 ksi

the value of E for most materials are listed in Appendix H

Poisson's ratio : when a bar is loaded in tension, the axial elongation is accompanied by lateral contraction, the lateral strain is proportional to the axial strain in linear elastic range for homogeneous and isotropic materials, the Poisson's ratio ν is defined

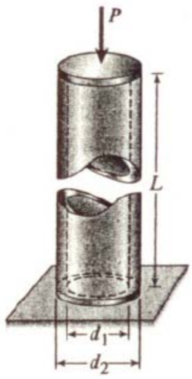
$$\nu = -\frac{\text{lateral strain}}{\text{axial strain}} = -\frac{\varepsilon'}{\varepsilon}$$



ε' : lateral strain

ν has a positive value for ordinary materials, and equal 0.25 ~ 0.35 for most materials, and it has almost the same value for both tension and compression

Example 1-3



A steel pipe with L = 1.2 m, d_2 = 150 mm, d_1 = 110 mm

P = 620 kN, E = 200 GPa, ν = 0.3

determine (a) δ, (b) ε', (c) $\triangle d_2$ and $\triangle d_1$

(d) $\triangle t$

$A = \pi\,(d_2^2 - d_1^2) / 4 = \pi\,(150^2 - 110^2) / 4 = 8{,}168\ \mathrm{mm}^2$

$\sigma = -P / A = -620\ \mathrm{kN} / 8{,}168\ \mathrm{mm}^2 = -75.9$ PMa (comp)

$\varepsilon = \sigma / E = -75.9\ \mathrm{MPa} / 200{,}000\ \mathrm{MPa} = -379.5 \times 10^{-6}$

(a) $\delta = \varepsilon L = (-379.5 \times 10^{-6})\,(1{,}200\ \mathrm{mm}) = -0.455\ \mathrm{mm}$

(b) $\varepsilon' = -\nu\,\varepsilon = -(0.3)\,(-379.5 \times 10^{-6}) = 113.9 \times 10^{-6}$

(c) $\triangle d_2 = \varepsilon'\,d_2 = (113.9 \times 10^{-6})\,(150\ \mathrm{mm}) = 0.0171\ \mathrm{mm}$

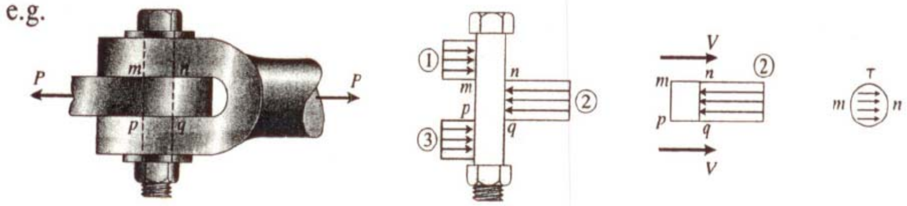
$\triangle d_1 = \varepsilon'\,d_1 = (113.9 \times 10^{-6})\,(110\ \mathrm{mm}) = 0.0125\ \mathrm{mm}$

(d) $\triangle t = \varepsilon'\,t = (113.2 \times 10^{-6})\,(0.75\mathrm{in}) = 0.000085\ \mathrm{in}$

1.6 Shear Stress and Strain

shear stress : parallel or tangent to the surface

e.g.



bearing stress : contact stress between clevis and bolt

1 the average bearing stress σ_b is

$$\sigma_b = F_b / A_b$$

F_b : bearing force A_b : bearing area

the shear force $V = P / 2$

the shear stress distributed over the cross sectional area of the bolt, the exact shear stress distribution is not known yet, it is highest near the middle and become zero on the edge, the average shear stress is

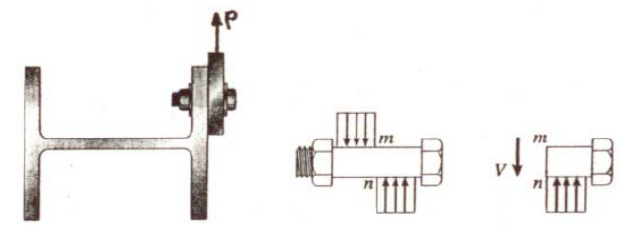
$$\tau_{\text{ave}} = V / A$$

direct shear (simple shear) : shear stresses are created by a direct action of the force in trying to cut through the material, such as of bolts, pins, rivets, keys, etc.

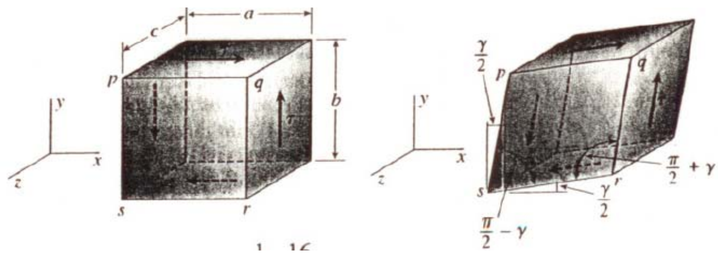
shear stresses also arise in an indirect manner when members are subjected to tension, torsion and bending etc.

double shear : in the above example, there are two planes to subject shear force

single shear : only one plane to subject shear force



consider a small element having sides of lengths a, b, and c in the x, y, and z directions, respectively



τ applied at the top and also at the bottom (force equilibrium)

τ applied at the top and also at the right surface (moment equilibrium)

$(\tau\ a\ c)\ b \;=\; (\tau\ b\ c)\ a$ moment about z-axis

shear stress has the properties :

1. shear stresses on opposite faces of an element are equal in magnitude and opposite in direction

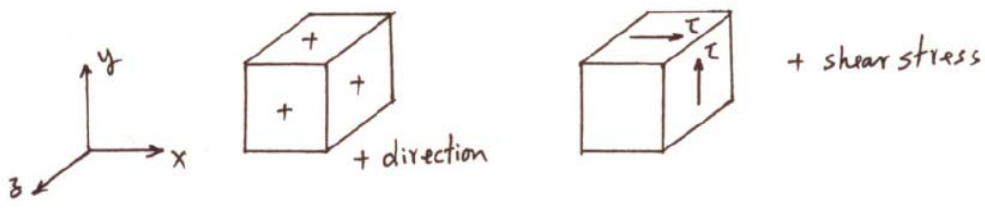
2. shear stresses on perpendicular faces of and element is equal in magnitude and have directions such that both stresses point toward or away from the corner

an element subjected to shear stress only is said to be pure shear, and the material will deform, resulting in shear strains

the angle γ is a measure of the distortion, it is called shear strain, its unit are in degrees or radians

Sign convention of shear stress :

positive surface : out normal directed in the positive direction



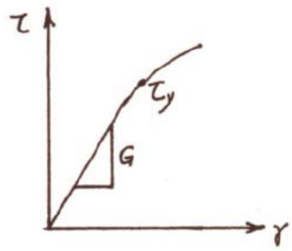
shear stress acting on a positive face is positive if it is acts in the positive direction of the coordinate axis, and negative if it acts in the negative direction of the axis

Sign convention for shear strain :

shear strain is positive when the angle between two positive (or negative) faces is reduced, it is negative when the angle between two positive (or negative) faces is increased

Hooke's Law in Shear

from experimental result, shear stress-strain diagram may be plotted



$$\tau_y \simeq 0.5 \sim 0.6\,\sigma_y$$

in the linear elastic region

$$\tau = G\,\gamma$$

G : shear modulus of elasticity (modulus of rigidity)

G have the same unit as E

G = 75 ~ 80 GPa or 10,800 ~ 11,800 ksi for steel

= 26 ~ 28 GPa or 3,800 ~ 4,300 ksi for Al and its alloy

E, ν, G are not independent, they are related by

$$G = \frac{E}{2\,(1 + \nu)}$$

this relation will derived in chapter 3

$\because\ \nu = 0 \sim 0.5$ $\qquad\therefore\ G = (\frac{1}{2} \sim \frac{1}{3})\,E$

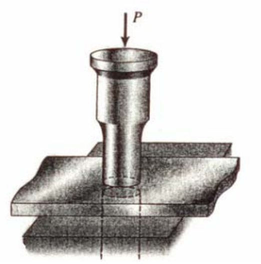
Example 1-4

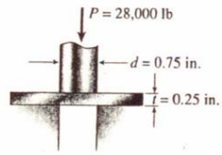
A punch for making hole

punch diameter d = 20 mm

plate thickness t = 800

punching force P = 110 kN





(a) average shear stress in the plate

the shear area A_s is

$$A_s = \pi\, d\, t = \pi\,(20\text{ mm})\,(8\text{ mm}) = 502.7\text{ mm}^2$$

$$\tau_{\text{aver}} = \frac{P}{A_s} = \frac{110\text{ kN}}{502.7\text{ mm}^2} = 219\text{ MPa}$$

(b) average compressive stress in the punch

$$\sigma_{\text{c}} = \frac{P}{A_{punch}} = \frac{P}{\pi\, d^2/\,4} = \frac{110\text{ kN}}{\pi\,(20\text{ mm})^2/4} = 350\text{ MPa}$$

Example 1-5

A steel strut S serving as a grace for a bolt

P = 54 kN

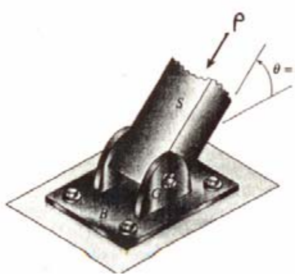
wall thickness t = 12 mm

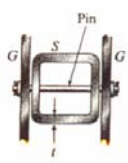
angle $\theta = 40^{\text{o}}$ d_{pin} = 18 mm

gussets thickness t_G = 15 mm

base plate thickness t_B = 8 mm

d_{bolt} = 12 mm





(a) bearing stress between strut and pin

$$\sigma_{\text{b1}} = \frac{1}{2}\,\frac{P}{t\, d_{pin}} = \frac{54\text{ kN}}{2\,(12\text{ mm})\,(18\text{ mm})} = 125\text{ MPa}$$

(b) shear stress in pin

$$\tau_{pin} = \frac{1}{2}\,\frac{P}{\pi\, d^2_{pin}/4} = \frac{54\text{ kN}}{2\,\pi\,(18\text{ mm})^2/4} = 106\text{ MPa}$$

(c) bearing stress between pin and gussets

$$\sigma_{b2} = \frac{1}{2}\,\frac{P}{t_G\, d_{pin}} = \frac{54\text{ kN}}{2\,(15\text{ mm})\,(18\text{ mm})} = 100\text{ MPa}$$

(d) bearing stress between anchor bolts and base plate

$$\sigma_{b3} = \frac{1}{4}\,\frac{P\cos 40^{\circ}}{t_B\, d_{bolt}} = \frac{(54\text{ kN})\cos 40^{\circ}}{4\,(8\text{ mm})\,(12\text{ mm})} = 108\text{ MPa}$$

(e) shear stress in anchor bolts

$$\tau_{bolt} = \frac{1}{4}\,\frac{P\cos 40^{\circ}}{\pi\, d^2_{bolt}/4} = \frac{(54\text{ kN})\cos 40^{\circ}}{4\,\pi\,(12\text{ mm})^2/4} = 91.4\text{ MPa}$$

Example 1-6

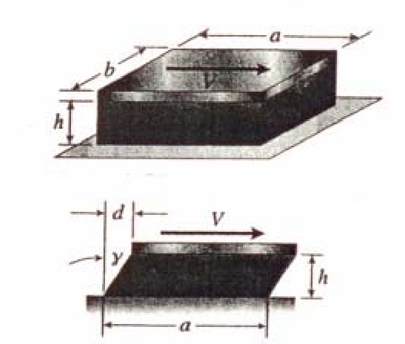
a elastomer of linear elastic material

capped by a steel plate

thickness of elastomer is h

dimension of the plate is a x b

subjected to a shear force V



$$\tau_{ave} = \frac{V}{a\,b} \qquad \gamma = \frac{\tau_{ave}}{G_e} = \frac{V}{a\,b\,G_e}$$

$$d = h\tan\gamma = h\tan\left(\frac{V}{a\,b\,G_e}\right)$$

in most practical situations the shear strain γ is very small, and is such cases we replace $\tan\gamma$ by γ and obtain

$$d = h\gamma = \frac{h\,V}{a\,b\,G_e}$$

1.7 Allowable Stresses and Allowable Loads

Strength : the ability of structure (or material) to resist loads

The actual strength of a structure (or material) must exceeded the required strength

$$\text{Factor of Safety} \quad n = \frac{\text{actual strength}}{\text{required strength}}$$

$$n \geqq 1 \qquad n = 1 \sim 10$$

Failure : fracture or complete collapse of structure or deformations have exceeded some limiting value

the determination of a factor of safety must also take into account such matter as follows :

- the accidental overloading
- the type of load (static, dynamic or repeated)
- possibility of fatigue failure
- inaccuracies in construction, quality of workmanship
- variation in properties of materials
- environmental effect (corrosion, thermal effect, etc.)
- accuracy of method of analysis
- failure is gradual or sudden

- others

if factor of safety too low, probability of failure will be high, if too high, waste materials

for many structure, it is important that the material remain within the linear elastic region, in order to avoid the permanent deformation, use yield stress and factor of safety to design, this is called allowable stress (or working stress)

$$\sigma_{\text{allow}} = \frac{\text{yield stress}}{\text{factor of safety}}$$

nowhere of the structure yields, for tension and shear stresses

$$\sigma_{\text{allow}} = \frac{\sigma_{\text{y}}}{n_1} \qquad \tau_{\text{allow}} = \frac{\tau_{\text{y}}}{n_2}$$

e.g. for building design, n takes 1.67 for mild steel

another method, use the ultimate stress instead of yield stress, this method is suitable for brittle materials, the allowable for tension and shear stresses are

$$\sigma_{\text{allow}} = \frac{\sigma_{\text{u}}}{n_3} \qquad \tau_{\text{allow}} = \frac{\tau_{\text{u}}}{n_4}$$

n is normally much greater then that when use σ_{y},

e.g. $n = 2.8$ for mild steel

in aircraft design, use margin of safety

margin of safety $= n - 1$

Allowable load concept is also used in design

Allowable load = (Allowable stress) (Area)

$$P_{\text{allow}} = \sigma_{\text{allow}} A \qquad \text{for tension}$$

$$P_{\text{allow}} = \tau_{\text{allow}} A \qquad \text{for shear}$$

$$P_{\text{allow}} = \sigma_{\text{b}} A_b \qquad \text{for bearing}$$

another method, use the service loads (working loads) and factor of safety to determine the ultimate loads

$$\text{factor of safety} = \frac{\text{ultimate load}}{\text{service load}}$$

or ultimate load = service load x load factor (factor of safety)

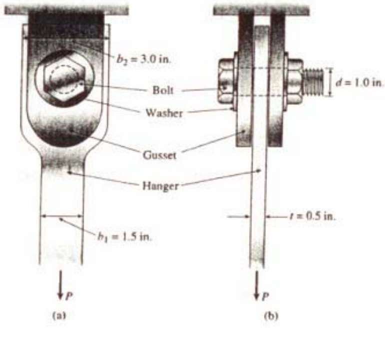
this is known as strength design or ultimate load design

e.g. for reinforced concrete structure

$$\text{load factor} = \begin{cases} 1.4 & \text{for dead load} \\ 1.7 & \text{for live load} \end{cases}$$

Example 1-7

a steel bar serving as a vertical hanger



b_1 = 38 mm $\qquad$ t = 13 mm

b_2 = 75 mm $\qquad$ d = 25 mm

determine the allowable load P

(a) σ_{allow} = 110 MPa in the main part

$$P_1 = \sigma_{\text{allow}} A = \sigma_{\text{allow}} b_1 t$$

$= \quad (110 \text{ MPa})\ (38 \text{ mm} \times 13 \text{ mm})$

$= \quad 53.4 \text{ kN}$

(b) $\sigma_{\text{allow}} = 75$ MPa in the hanger at the cross section through the bolt

$$P_2 \quad = \quad \sigma_{\text{allow}} A \quad = \quad \sigma_{\text{allow}} (b_2 - d)\, t$$

$$= \quad (75 \text{ MPa})\ (75 \text{ mm} - 25 \text{ mm})\ (13 \text{ mm}) \quad = \quad 48.7 \text{ kN}$$

(c) $(\sigma_{\text{b}})_{\text{allow}} = 180$ MPa between hanger and bolt

$$P_3 \quad = \quad \sigma_{\text{b}} A_b \quad = \quad \sigma_{\text{b}}\, d\, t \quad = \quad (180 \text{ MPa})\ (25 \text{ mm})\ (13 \text{ mm})$$

$$= \quad 58.5 \text{ kN}$$

(d) $\tau_{\text{allow}} = 45$ MPa for the bolt

$$P_4 \quad = \quad \tau_{\text{allow}} A \quad = \tau_{\text{allow}}\, 2\, (\pi\, d^2) / 4 = (45 \text{ MPa})\ 2\, \pi\, (25 \text{ mm})^2 / 4$$

$$= \quad 44.1 \text{ kN}$$

comparing the four preceding results

$$P_{\text{allow}} \quad = \quad \text{minimum}\ [P_1, P_2, P_3, P_4] \quad = \quad 44.1 \text{ kN}$$

failure by shear on the bolt first

1.8 Design for Axial Loads and Direct Shear

Design : to determine the properties of the structure in order that the structure will support the loads

$$\text{Required area} \quad = \quad \frac{\text{Load to be transmitted}}{\text{Allowable stress}}$$

analysis : the calculation of such quantities as stresses, strains, and deformations of loaded members or structures design : the determination of the geometric configuration of a structure in order that it will fulfill a prescribed function

optimization is the task of designing the "best" structure to meet a particular goal, such as least weight

analysis, design and optimization are closely linked

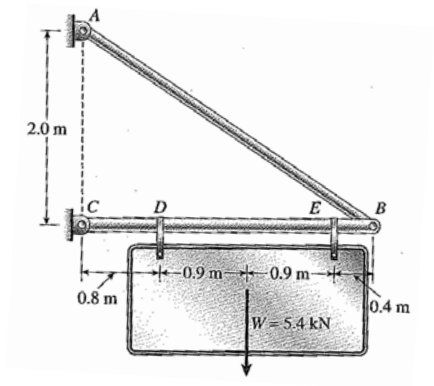
Example 1-8

a two-bar truss *ABC* as shown

σ_{allow} for AB is 125 MPa

τ_{allow} for pin C is 45 MPa

determine A_{AB} and d_C



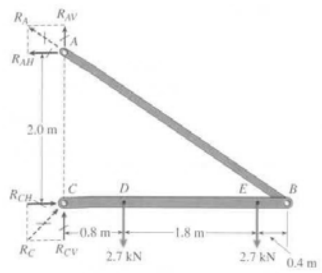
$\Sigma M_C = 0$ for the whole structure

R_{AH} x 2 - 2.7 x 0.8 - 2.7 x 2.6 = 0

$R_{AH} = 4.59$ kN

$\Sigma F_x = 0$ for the whole structure

$R_{CH} = R_{AH} = 4.59$ kN

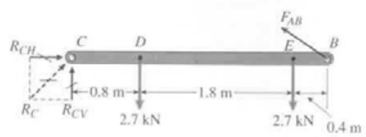


(a)

$\Sigma M_B = 0$ for the member BC

$- R_{CV}$ x 3 + 2.7 x 2.2 + 2.7 x 0.4 = 0

$R_{CV} = 2.34$ kN



(b)

$\Sigma F_y = 0$ for the whole structure

$R_{AV} + R_{CV} - 2.7 - 2.7 = 0$

$R_{AV} = 3.06$ kN

$$F_{AB} = R_A = \sqrt{(\mathrm{R}_{AH})^2 + (\mathrm{R}_{AV})^2} = 5.516 \text{ kN}$$

$$V_C = R_C = \sqrt{(\mathrm{R}_{CH})^2 + (\mathrm{R}_{CV})^2} = 5.152 \text{ kN}$$

the required area for member AB is

$$A_{AB} = \frac{F_{AB}}{\sigma_{\text{allow}}} = \frac{5.516 \text{ kN}}{125 \text{ MPa}} = 44.1 \text{ mm}^2$$

the required area for pin C (double shear) is

$$A_C = \frac{V_C}{2\,\tau_{\text{allow}}} = \frac{5.152 \text{ kN}}{2\,(45 \text{ MPa})} = 57.2 \text{ mm}^2$$

$$d_{pin} = \sqrt{4A_{pin}/\pi} = 8.54 \text{ mm}$$