3. A certain RC circuit has an $f_{cl} = 235$ Hz, above which the attenuation is 0 dB. What is the dB attenuation at 23.5 Hz?
4. What is the amount of phase shift contributed by an input circuit when $X_C = 0.5R_{in}$ at a certain frequency below f_{cl1}?
5. What is the critical frequency when $R_D = 1.5\ k\Omega$, $R_L = 5\ k\Omega$, and $C_2 = 0.0022\ \mu F$ in a circuit like Figure 10–24?

10–4 HIGH-FREQUENCY AMPLIFIER RESPONSE

You have seen how the coupling and bypass capacitors affect the voltage gain of an amplifier at lower frequencies where the reactances of the coupling and bypass capacitors are significant. In the midrange of an amplifier, the effects of the capacitors are minimal and can be neglected. If the frequency is increased sufficiently, a point is reached where the transistor's internal capacitances begin to have a significant effect on the gain. The basic differences between BJTs and FETs are the specifications of the internal capacitances and the input resistance.

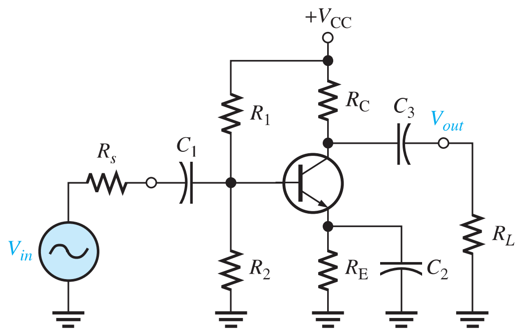
After completing this section, you should be able to

- **Analyze the high-frequency response of an amplifier**
- Analyze a BJT amplifier
  - Apply Miller's theorem
- Identify and analyze the BJT amplifier's input RC circuit
  - Calculate the upper critical frequency and gain roll-off ◆ Determine the phase shift
- Identify and analyze the BJT amplifier's output RC circuit
  - Calculate the upper critical frequency ◆ Determine the phase shift
- Analyze a FET amplifier
- Identify and analyze a JFET amplifier
  - Determine internal capacitances on a datasheet ◆ Apply Miller's theorem
- Identify and analyze the JFET amplifier's input RC circuit
  - Calculate the upper critical frequency ◆ Determine the phase shift
- Identify and analyze the JFET amplifier's output RC circuit
  - Calculate the upper critical frequency ◆ Determine the phase shift
- Discuss the total high-frequency response of an amplifier
  - Use Bode plots to illustrate the high-frequency response

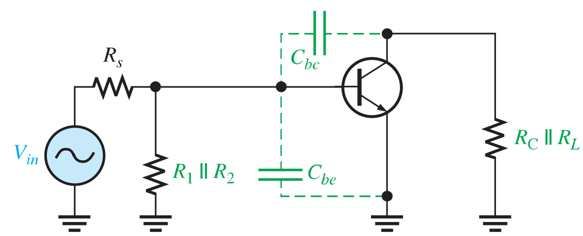
BJT Amplifiers

A high-frequency ac equivalent circuit for the BJT amplifier in Figure 10–31(a) is shown in Figure 10–31(b). Notice that the coupling and bypass capacitors are treated as effective shorts and do not appear in the equivalent circuit. The internal capacitances, C_{be} and C_{bc}, which are significant only at high frequencies, do appear in the diagram. As previously mentioned. C_{be} is sometimes called the input capacitance C_{ib}, and C_{bc} is sometimes called the output capacitance C_{ob}. C_{be} is specified on datasheets at a certain value of V_{BE}. Often, a datasheet will list C_{ib} as C_{ibo} and C_{ob} as C_{obo}. The *o* as the last letter in the subscript indicates the capacitance is measured with the base open. For example, a 2N2222A transistor has a C_{be} of 25 pF at $V_{BE} = 0.5$ V dc, $I_C = 0$, and $f = 1$ MHz. Also, C_{bc} is specified at a certain value of V_{BC}. The 2N2222A has a maximum C_{bc} of 8 pF at $V_{BC} = 10$ V dc.

Miller's Theorem in High-Frequency Analysis By applying Miller's theorem to the inverting amplifier in Figure 10–31(b) and using the midrange voltage gain, you have a circuit



(a) Capacitively coupled amplifier



(b) High-frequency equivalent circuit

▲ **FIGURE 10–31**

Capacitively coupled amplifier and its high-frequency equivalent circuit.

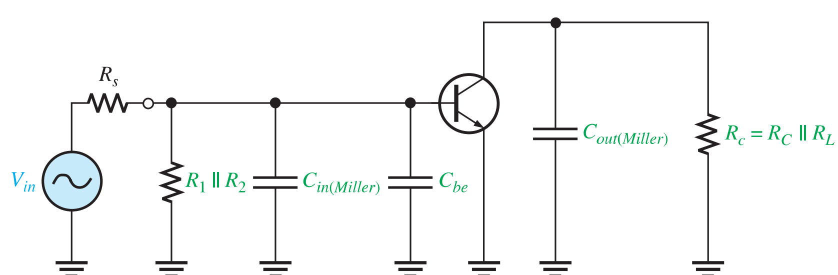
that can be analyzed for high-frequency response. Looking in from the signal source, the capacitance C_{bc} appears in the Miller input capacitance from base to ground.

$$C_{in(Miller)} = C_{bc}(A_v + 1)$$

C_{be} simply appears as a capacitance to ac ground, as shown in Figure 10–32, in parallel with $C_{in(Miller)}$. Looking in at the collector, C_{bc} appears in the Miller output capacitance from collector to ground. As shown in Figure 10–32, the Miller output capacitance appears in parallel with R_c.

$$C_{out(Miller)} = C_{bc}\left(\frac{A_v + 1}{A_v}\right)$$

These two Miller capacitances create a high-frequency input *RC* circuit and a high-frequency output *RC* circuit. These two circuits differ from the low-frequency input and output circuits, which act as high-pass filters, because the capacitances go to ground and therefore act as low-pass filters. The equivalent circuit in Figure 10–32 is an ideal model because stray capacitances that are due to circuit interconnections are neglected.



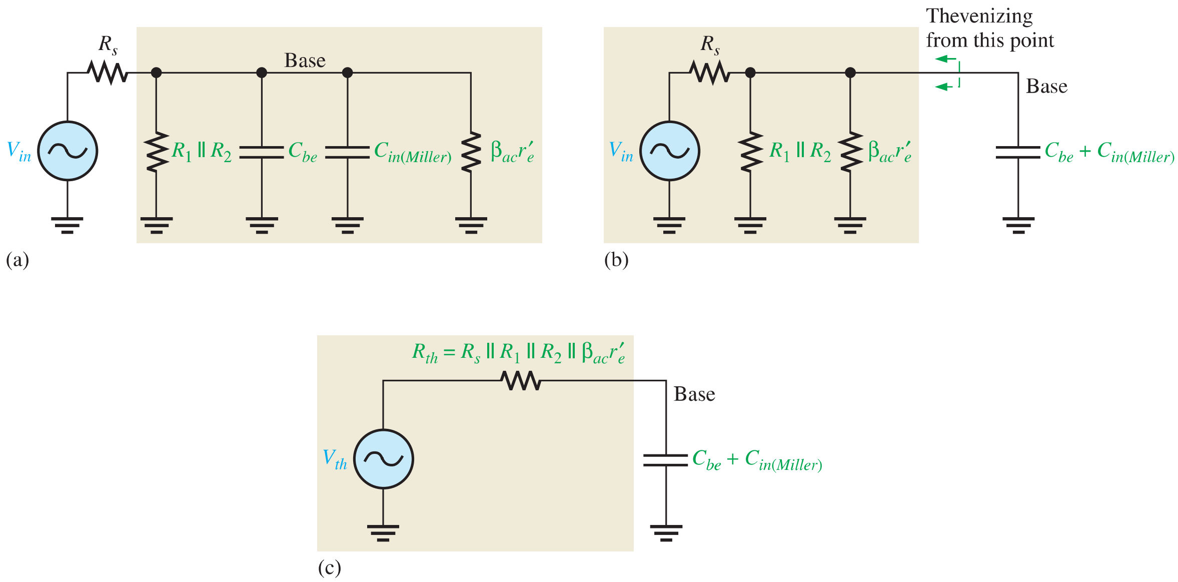
◀ **FIGURE 10–32**

High-frequency equivalent circuit after applying Miller's theorem.

The Input *RC* Circuit

At high frequencies, the input circuit is as shown in Figure 10–33(a), where $\beta_{ac}r'_e$ is the input resistance at the base of the transistor because the bypass capacitor effectively shorts the emitter to ground. By combining C_{be} and $C_{in(Miller)}$ in parallel and repositioning, you get the simplified circuit shown in Figure 10–33(b). Next, by thevenizing the circuit to the left of the capacitor, as indicated, the input *RC* circuit is reduced to the equivalent form shown in Figure 10–33(c).

As the frequency increases, the capacitive reactance becomes smaller. This causes the signal voltage at the base to decrease, so the amplifier's voltage gain decreases. The reason for this is that the capacitance and resistance act as a voltage divider and, as the frequency increases, more voltage is dropped across the resistance and less across the capacitance. At the critical frequency, the gain is 3 dB less than its midrange value. The upper critical high



▲ FIGURE 10–33

Development of the equivalent high-frequency input *RC* circuit.

frequency of the input circuit, $f_{cu(input)}$, is the frequency at which the capacitive reactance is equal to the total resistance.

$$X_{C_{tot}} = R_s \| R_1 \| R_2 \| \beta_{ac} r'_e$$

Therefore,

$$\frac{1}{2\pi f_{cu(input)} C_{tot}} = R_s \| R_1 \| R_2 \| \beta_{ac} r'_e$$

and

Equation 10–16

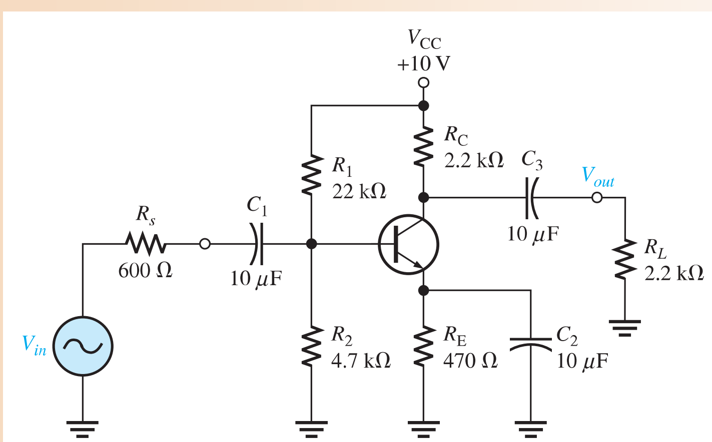
$$\boldsymbol{f_{cu(input)} = \frac{1}{2\pi(R_s \| R_1 \| R_2 \| \beta_{ac} r'_e)C_{tot}}}$$

where R_s is the resistance of the signal source and $C_{tot} = C_{be} + C_{in(Miller)}$. As the frequency goes above $f_{cu(input)}$, the input *RC* circuit causes the gain to roll off at a rate of -20 dB/decade just as with the low-frequency response.

EXAMPLE 10–11

Derive the equivalent high-frequency input *RC* circuit for the BJT amplifier in Figure 10–34. Use this to determine the upper critical frequency due to the input

▶ FIGURE 10–34



circuit. The transistor's datasheet provides the following: $\beta_{ac} = 125$, $C_{be} = 20$ pF, and $C_{bc} = 2.4$ pF.

Solution First, find r'_e as follows:

$$V_B = \left(\frac{R_2}{R_1 + R_2}\right)V_{CC} = \left(\frac{4.7\text{ k}\Omega}{26.7\text{ k}\Omega}\right)10\text{ V} = 1.76\text{ V}$$

$$V_E = V_B - 0.7\text{ V} = 1.06\text{ V}$$

$$I_E = \frac{V_E}{R_E} = \frac{1.06\text{ V}}{470\ \Omega} = 2.26\text{ mA}$$

$$r'_e = \frac{25\text{ mV}}{I_E} = 11.1\ \Omega$$

The total resistance of the input circuit is

$$R_{in(tot)} = R_s \| R_1 \| R_2 \| \beta_{ac}r'_e = 600\ \Omega \| 22\text{ k}\Omega \| 4.7\text{ k}\Omega \| 125(11.1\ \Omega) = 378\ \Omega$$

Next, in order to determine the capacitance, you must calculate the midrange gain of the amplifier so that you can apply Miller's theorem.

$$A_{v(mid)} = \frac{R_c}{r'_e} = \frac{R_C \| R_L}{r'_e} = \frac{1.1\text{ k}\Omega}{11.1\ \Omega} = 99$$

Apply Miller's theorem.

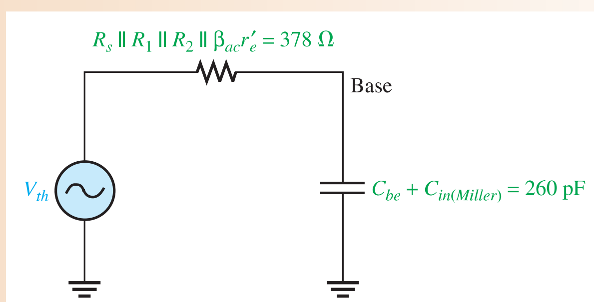
$$C_{in(Miller)} = C_{bc}(A_{v(mid)} + 1) = (2.4\text{ pF})(100) = 240\text{ pF}$$

The total input capacitance is $C_{in(Miller)}$ in parallel with C_{be}.

$$C_{in(tot)} = C_{in(Miller)} + C_{be} = 240\text{ pF} + 20\text{ pF} = 260\text{ pF}$$

The resulting high-frequency input *RC* circuit is shown in Figure 10–35. The upper critical frequency is

$$f_{cu(input)} = \frac{1}{2\pi(R_{in(tot)})(C_{in(tot)})} = \frac{1}{2\pi(378\ \Omega)(260\text{ pF})} = \mathbf{1.62\text{ MHz}}$$



▲ **FIGURE 10–35**

High-frequency equivalent input *RC* circuit for the amplifier in Figure 10–34.

Related Problem Determine the input *RC* circuit for Figure 10–34 and find its upper critical frequency if a transistor with the following specifications is used: $\beta_{ac} = 75$, $C_{be} = 15$ pF, $C_{bc} = 2$ pF.

Open the Multisim file E10-11 in the Examples folder on the companion website. Measure the critical frequency for the amplifier's high-frequency response and compare to the calculated result.

Phase Shift of the Input RC Circuit Because the output voltage of a high-frequency input *RC* circuit is across the capacitor, the output of the circuit lags the input. The phase angle is expressed as

Equation 10–17

$$\boldsymbol{\theta = \tan^{-1}\left(\frac{R_s \| R_1 \| R_2 \| \beta_{ac} r'_e}{X_{C(tot)}}\right)}$$

At the critical frequency, the phase angle is 45° with the signal voltage at the base of the transistor lagging the input signal. As the frequency increases above f_c, the phase angle increases above 45° and approaches 90° when the frequency is sufficiently high.

The Output *RC* Circuit

The high-frequency output *RC* circuit is formed by the Miller output capacitance and the resistance looking in at the collector, as shown in Figure 10–36(a). In determining the output resistance, the transistor is treated as a current source (open) and one end of R_C is effectively ac ground, as shown in Figure 10–36(b). By rearranging the position of the capacitance in the diagram and thevenizing the circuit to the left, as shown in Figure 10–36(c), you get the equivalent circuit in Figure 10–36(d). The equivalent output *RC* circuit consists of a resistance equal to the parallel combination of R_C and R_L in series with a capacitance that is determined by the following Miller formula:

$$C_{out(Miller)} = C_{bc}\left(\frac{A_v + 1}{A_v}\right)$$

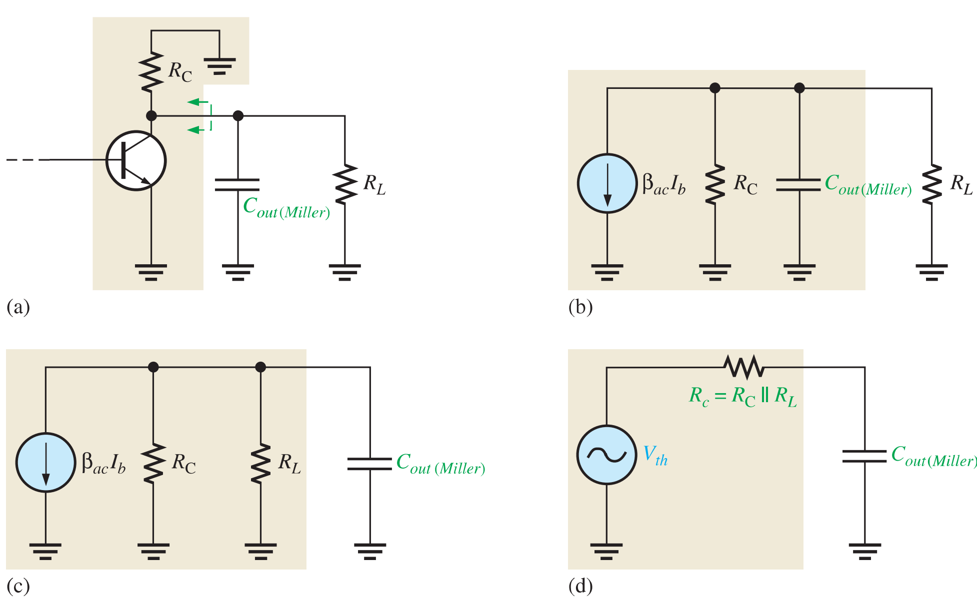
If the voltage gain is at least 10, this formula can be approximated as

$$C_{out(Miller)} \cong C_{bc}$$

The upper critical frequency for the output circuit is determined with the following equation, where $R_c = R_C \| R_L$.

Equation 10–18

$$\boldsymbol{f_{cu(output)} = \frac{1}{2\pi R_c C_{out(Miller)}}}$$



▲ **FIGURE 10–36**

Development of the equivalent high-frequency output *RC* circuit.

Just as in the input *RC* circuit, the output *RC* circuit reduces the gain by 3 dB at the critical frequency. When the frequency goes above the critical value, the gain drops at a −20 dB/decade rate. The phase angle introduced by the output *RC* circuit is

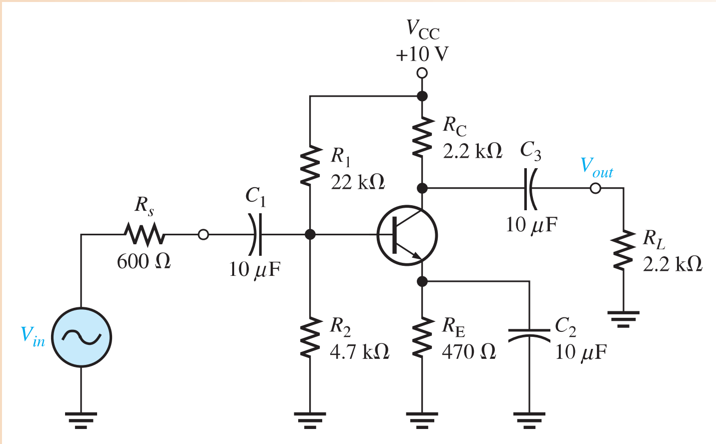
$$\theta = \tan^{-1}\left(\frac{R_c}{X_{C_{out(Miller)}}}\right)$$

Equation 10–19

EXAMPLE 10–12

Determine the upper critical frequency of the amplifier in Example 10–11 shown in Figure 10–37 due to its output *RC* circuit.

▶ FIGURE 10–37



Solution Calculate the Miller output capacitance.

$$C_{out(Miller)} = C_{bc}\left(\frac{A_v + 1}{A_v}\right) = (2.4\text{ pF})\left(\frac{99 + 1}{99}\right) \cong 2.4\text{ pF}$$

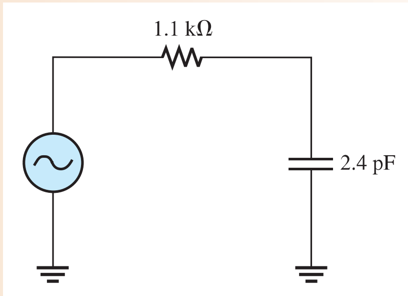
The equivalent resistance is

$$R_c = R_C \| R_L = 2.2\text{ k}\Omega \| 2.2\text{ k}\Omega = 1.1\text{ k}\Omega$$

The equivalent output *RC* circuit is shown in Figure 10–38. Determine the upper critical frequency as follows ($C_{out(Miller)} \cong C_{bc}$):

$$f_{cu(output)} = \frac{1}{2\pi R_c C_{bc}} = \frac{1}{2\pi(1.1\text{ k}\Omega)(2.4\text{ pF})} = \mathbf{60.3\text{ MHz}}$$

▶ FIGURE 10–38

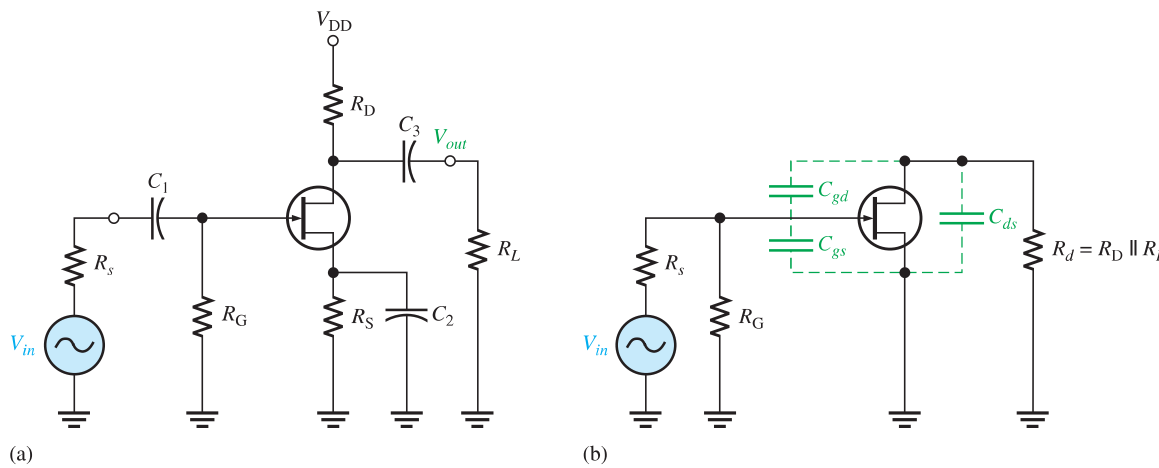


Related Problem If another transistor with $C_{bc} = 5$ pF is used in the amplifier, what is $f_{cu(output)}$?

FET Amplifiers

The approach to the high-frequency analysis of a FET amplifier is similar to that of a BJT amplifier. The basic differences are the specifications of the internal FET capacitances and the determination of the input resistance.

Figure 10–39(a) shows a JFET common-source amplifier that will be used to illustrate high-frequency analysis. A high-frequency equivalent circuit for the amplifier is shown in Figure 10–39(b). Notice that the coupling and bypass capacitors are assumed to have negligible reactances and are considered to be shorts. The internal capacitances C_{gs} and C_{gd} appear in the equivalent circuit because their reactances are significant at high frequencies.



▲ FIGURE 10–39

Example of a JFET amplifier and its high-frequency equivalent circuit.

Values of C_{gs}, C_{gd}, and C_{ds} FET datasheets do not normally provide values for C_{gs}, C_{gd}, or C_{ds}. Instead, three other values are usually specified because they are easier to measure. These are C_{iss}, the input capacitance; C_{rss}, the reverse transfer capacitance; and C_{oss}, the output capacitance. Because of the manufacturer's method of measurement, the following relationships allow you to determine the capacitor values needed for analysis.

Equation 10–20
$$C_{gd} = C_{rss}$$

Equation 10–21
$$C_{gs} = C_{iss} - C_{rss}$$

Equation 10–22
$$C_{ds} = C_{oss} - C_{rss}$$

C_{oss} is not specified as often as the other values on datasheets. Sometimes, it is designated as $C_{d(sub)}$, the drain-to-substrate capacitance. In cases where a value is not available, you must either assume a value or neglect C_{ds}.

EXAMPLE 10–13

The datasheet for a 2N3823 JFET gives $C_{iss} = 6$ pF and $C_{rss} = 2$ pF. Determine C_{gd} and C_{gs}.

Solution

$$C_{gd} = C_{rss} = \mathbf{2\ pF}$$
$$C_{gs} = C_{iss} - C_{rss} = 6\ \text{pF} - 2\ \text{pF} = \mathbf{4\ pF}$$

Related Problem Although C_{oss} is not specified on the datasheet for the 2N3823 JFET, assume a value of 3 pF and determine C_{ds}.

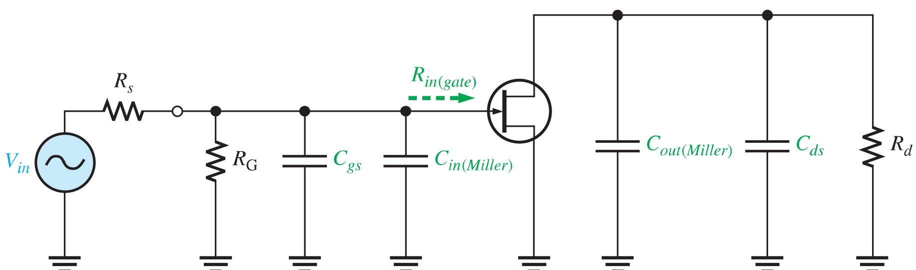
Using Miller's Theorem Miller's theorem is applied the same way in FET inverting amplifier high-frequency analysis as was done in BJT amplifiers. Looking in from the signal source in Figure 10–39(b), C_{gd} effectively appears in the Miller input capacitance, which was given in Equation 10–1, as follows:

$$C_{in(Miller)} = C_{gd}(A_v + 1)$$

C_{gs} simply appears as a capacitance to ac ground in parallel with $C_{in(Miller)}$, as shown in Figure 10–40. Looking in at the drain, C_{gd} effectively appears in the Miller output capacitance (from Equation 10–2) from drain to ground in parallel with R_d, as shown in Figure 10–40.

$$C_{out(Miller)} = C_{gd}\left(\frac{A_v + 1}{A_v}\right)$$

These two Miller capacitances contribute to a high-frequency input *RC* circuit and a high-frequency output *RC* circuit. Both are low-pass filters, which produce phase lag.



▲ **FIGURE 10–40**

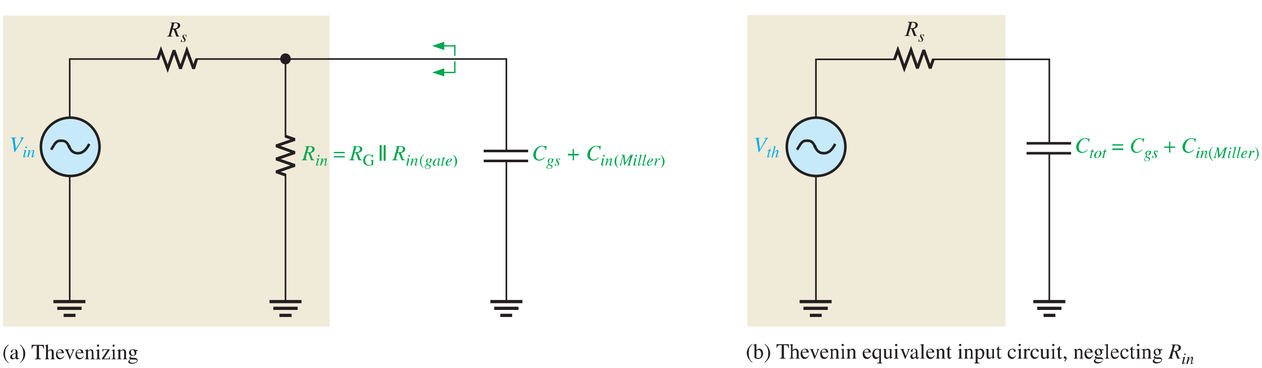
High-frequency equivalent circuit after applying Miller's theorem.

The Input *RC* Circuit

The high-frequency input circuit forms a low-pass type of filter and is shown in Figure 10–41(a). Because both R_G and the input resistance at the gate of FETs are extremely high, the controlling resistance for the input circuit is the resistance of the input source as long as $R_s \ll R_{in}$. This is because R_s appears in parallel with R_{in} when Thevenin's theorem is applied. The simplified input *RC* circuit appears in Figure 10–41(b). The upper critical frequency for the input circuit is

$$f_{cu(input)} = \frac{1}{2\pi R_s C_{tot}}$$

Equation 10–23



▲ **FIGURE 10–41**

Input *RC* circuit.

where $C_{tot} = C_{gs} + C_{in(Miller)}$. The input RC circuit produces a phase angle of

Equation 10–24

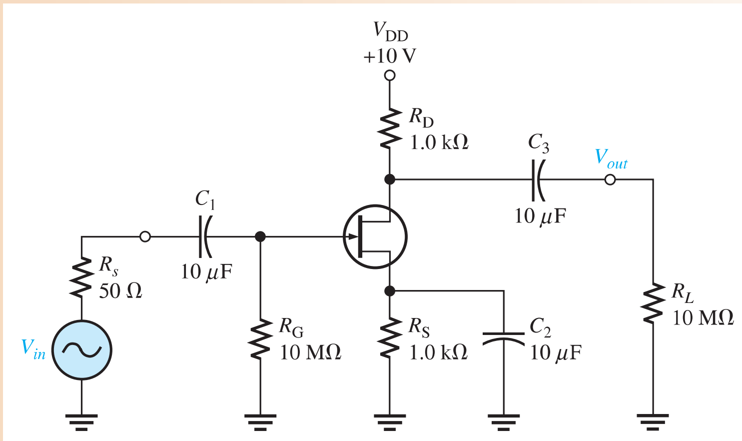
$$\theta = \tan^{-1}\left(\frac{R_s}{X_{C_{tot}}}\right)$$

The effect of the input RC circuit is to reduce the midrange gain of the amplifier by 3 dB at the critical frequency and to cause the gain to decrease at −20 dB/decade above f_c.

EXAMPLE 10–14

Find the upper critical frequency of the input RC circuit for the FET amplifier in Figure 10–42. $C_{iss} = 8$ pF, $C_{rss} = 3$ pF, and $g_m = 6500\ \mu$S.

▶ FIGURE 10–42



Solution Determine C_{gd} and C_{gs}.

$$C_{gd} = C_{rss} = 3\text{ pF}$$
$$C_{gs} = C_{iss} - C_{rss} = 8\text{ pF} - 3\text{ pF} = 5\text{ pF}$$

Determine the upper critical frequency for the input RC circuit as follows:

$$A_v = g_m R_d = g_m(R_D \| R_L) \cong (6500\ \mu\text{S})(1\text{ k}\Omega) = 6.5$$
$$C_{in(Miller)} = C_{gd}(A_v + 1) = (3\text{ pF})(7.5) = 22.5\text{ pF}$$

The total input capacitance is

$$C_{in(tot)} = C_{gs} + C_{in(Miller)} = 5\text{ pF} + 22.5\text{ pF} = 27.5\text{ pF}$$

The upper critical frequency is

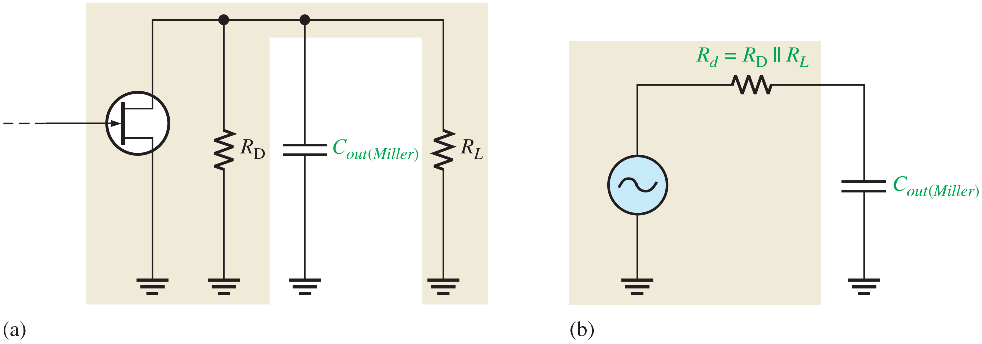
$$f_{cu(input)} = \frac{1}{2\pi R_s C_{in(tot)}} = \frac{1}{2\pi(50\ \Omega)(27.5\text{ pF})} = \mathbf{116\ MHz}$$

Related Problem If the gain of the amplifier in Figure 10–42 is increased to 10, what happens to f_c?

The Output *RC* Circuit

The high-frequency output RC circuit is formed by the Miller output capacitance and the output resistance looking in at the drain, as shown in Figure 10–43(a). As in the case of the BJT, the FET is treated as a current source. When you apply Thevenin's theorem, you get an equivalent output RC circuit consisting of R_D in parallel with R_L and an equivalent output capacitance.

$$C_{out(Miller)} = C_{gd}\left(\frac{A_v + 1}{A_v}\right)$$



◄ **FIGURE 10–43**

Output *RC* circuit.

This equivalent output circuit is shown in Figure 10–43(b). The critical frequency of the output *RC* lag circuit is

$$f_{cu(output)} = \frac{1}{2\pi R_d C_{out(Miller)}}$$ **Equation 10–25**

The output circuit produces a phase shift of

$$\theta = \tan^{-1}\left(\frac{R_d}{X_{C_{out(Miller)}}}\right)$$ **Equation 10–26**

EXAMPLE 10–15

Determine the upper critical frequency of the output *RC* circuit for the FET amplifier in Figure 10–42. What is the phase shift introduced by this circuit at the critical frequency? Which *RC* circuit is dominant, that is, which one has the lower value of upper critical frequency?

Solution Since R_L is very large compared to R_D, it can be neglected, and the equivalent output resistance is

$$R_d \cong R_D = 1.0\,\text{k}\Omega$$

The equivalent output capacitance is

$$C_{out(Miller)} = C_{gd}\left(\frac{A_v + 1}{A_v}\right) = (3\text{ pF})\left(\frac{7.5}{6.5}\right) = 3.46\text{ pF}$$

Therefore, the upper critical frequency is

$$f_{cu(output)} = \frac{1}{2\pi R_d C_{out(Miller)}} = \frac{1}{2\pi(1.0\,\text{k}\Omega)(3.46\text{ pF})} = \mathbf{46\text{ MHz}}$$

Although it has been neglected, any stray wiring capacitance could significantly affect the frequency response because $C_{out(Miller)}$ is very small.

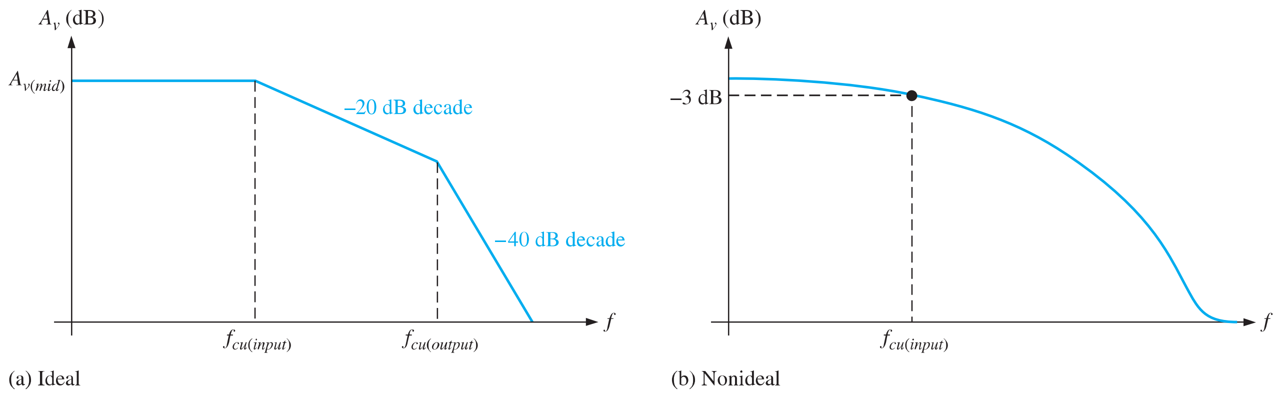
The phase angle is always **45°** at f_c for an *RC* circuit and the output lags.

In Example 10–14, the upper critical frequency of the input *RC* circuit was found to be 116 MHz. Therefore, the upper critical frequency for the output circuit is dominant because it is the lower of the two.

Related Problem If A_v of the amplifier in Figure 10–42 is increased to 10, what is the upper critical frequency of the output circuit?

Total High-Frequency Response of an Amplifier

As you have seen, the two *RC* circuits created by the internal transistor capacitances influence the high-frequency response of both BJT and FET amplifiers. As the frequency increases and reaches the high end of its midrange values, one of the *RC* circuits will cause the amplifier's gain to begin dropping off. The frequency at which this occurs is the dominant upper critical frequency; it is the lower of the two upper critical high frequencies. An ideal high-frequency Bode plot is shown in Figure 10–44(a). It shows the first break point at $f_{cu(input)}$ where the voltage gain begins to roll off at −20 dB/decade. At $f_{cu(output)}$, the gain begins dropping at −40 dB/decade because each *RC* circuit is providing a −20 dB/decade roll-off. Figure 10–44(b) shows a nonideal Bode plot where the voltage gain is actually −3 dB/decade below midrange at $f_{cu(input)}$. Other possibilities are that the output *RC* circuit is dominant or that both circuits have the same critical frequency.



▲ **FIGURE 10–44**

High-frequency Bode plots.

SECTION 10–4 CHECKUP

1. What determines the high-frequency response of an amplifier?
2. If an amplifier has a midrange voltage gain of 80, the transistor's C_{bc} is 4 pF, and C_{be} = 8 pF, what is the total input capacitance?
3. A certain amplifier has $f_{cu(input)}$ = 3.5 MHz and $f_{cu(output)}$ = 8.2 MHz. Which circuit dominates the high-frequency response?
4. What are the capacitances that are usually specified on a FET datasheet?
5. If C_{gs} = 4 pF and C_{gd} = 3 pF, what is the total input capacitance of a FET amplifier whose voltage gain is 25?

10–5 TOTAL AMPLIFIER FREQUENCY RESPONSE

In the previous sections, you learned how each *RC* circuit in an amplifier affects the frequency response. In this section, we will bring these concepts together and examine the total response of typical amplifiers and the specifications relating to their performance.

After completing this section, you should be able to

- **Analyze an amplifier for total frequency response**
- Discuss bandwidth
  - Define the dominant critical frequencies
- Explain gain-bandwidth product
  - Define *unity-gain frequency*