2

Force Systems and Resultants

2.1. INTRODUCTION

The study of bodies at rest or in equilibrium under the action of forces requires the concept of force, moment and couple.

2.2. FORCE

Force is a vector quantity hence it is described by magnitude and direction. In addition, the force has point of application.

The characteristics of a force can be stated as:

(i) Magnitude – F
(ii) Direction
(iii) Point of application

which have been shown in Fig. 2.1.

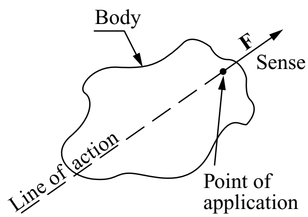


Fig. 2.1 Force designation.

When we say the direction of a force, it implies the line of action along which the force acts and the sense indicated by the arrow. Let us consider a bar acted upon by forces F_1 and F_2 as shown in Fig. 2.2 and the bar is in equilibrium.

When we consider the equilibrium of a rigid body, the point of application force is not important as evident from Figs. 2.2(a) and (b).

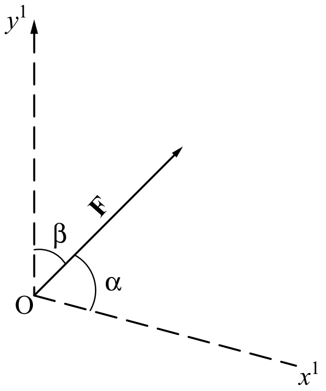


Fig. 2.4 Force components in specified directions.

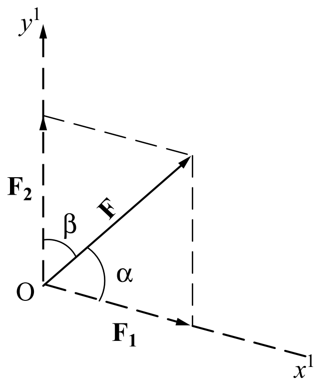


Fig. 2.5 Parallelogram of forces.

(ii) By constructing a triangle using the directions along which components are required, may be obtained as shown in Fig. 2.6.

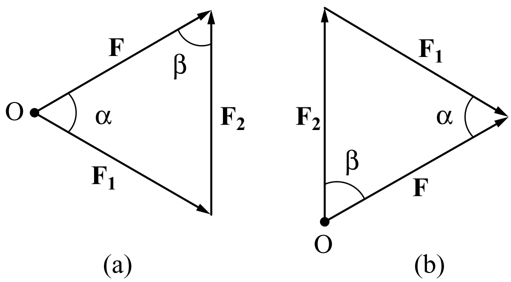


Fig. 2.6 Triangle of forces.

In this method, the line of action of $\mathbf{F_2}$ is not preserved as shown in Fig. 2.6. (a) as well as $\mathbf{F_1}$ which can be seen in Fig. 2.6 (b).

$$\frac{F_1}{\sin\beta} = \frac{F_2}{\sin\alpha} = \frac{F}{\sin\left[180 - (\alpha + \beta)\right]} \tag{2.6}$$

$$F = \sqrt{F_1{}^2 + F_2{}^2 - 2F_1F_2\cos\left[180 - (\alpha + \beta)\right]} \tag{2.7}$$

Resolution of forces is the inverse operation of composition of forces. Resolution and composition of forces are two essential concepts which find frequent applications for the solution of mechanics problems.

2.4. MOMENT OF A FORCE

Let us calculate the moment of a force F about an axis passing through the point '**A**'. The tendency of a moment is to cause rotation about the point '**A**'. The moment is sometimes referred to as torque.

2.5. PRINCIPLE OF MOMENTS

It states that moment of a force is equal to the sum of moments of its components. This is known as **Varignon's** theorem.

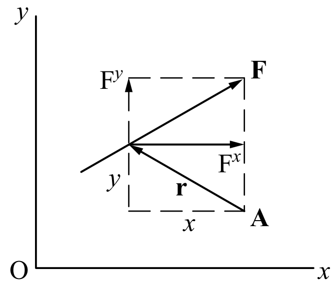


Fig. 2.9 Moment.

$$\text{Position vector} \qquad \mathbf{r} = -x\mathbf{i} + y\mathbf{j} \tag{2.14}$$

$$\text{Force} \qquad \mathbf{F} = F^x\mathbf{i} + F^y\mathbf{j} \tag{2.15}$$

$$\mathbf{M_A}^{\mathbf{F}} = \mathbf{r} \times \mathbf{F} = (-x\,\mathbf{i} + y\,\mathbf{j}) \times (F^x\mathbf{i} + F^y\mathbf{j}) \tag{2.16}$$

$$M_A{}^F = (-yF^x - xF^y)\mathbf{k} \tag{2.17}$$

$$M_A{}^F = (M^x + M^y)(-\mathbf{k}) \tag{2.18}$$

where $M^x = yF^x$ and $M^y = xF^y$.

The moment is a clockwise moment indicated by (–**k**).

2.6. PRINCIPLE OF TRANSMISSIBILITY

The external effect of a force on a rigid body is independent of where it is applied along its line of action. This can be demonstrated by calculating the moment of a force about a point by considering the effect of a force at different points on the line of action.

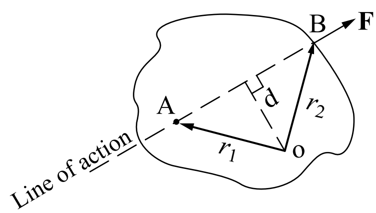


Fig. 2.10 Transmissibility of force.

(i) Considering the point of application of force F at A, the moment about 'o'

$$\mathbf{M_o}^{\mathbf{F}} = \mathbf{r_1} \times \mathbf{F} = dF(\circlearrowright) \tag{2.19}$$

(ii) Considering the point of application of force **F** at B, the moment about 'o'

$$\mathbf{M_o}^{\mathbf{F}} = \mathbf{r_2} \times \mathbf{F} = dF(\circlearrowright) \tag{2.20}$$

Since $\mathbf{M_o}^{\mathbf{F}} = \mathbf{r_1} \times \mathbf{F} = \mathbf{r_2} \times \mathbf{F}$, principle of transmissibility is proved.

2.7. REPRESENTATION OF FORCE IN THREE-DIMENSIONAL SPACE

Force vector **F** in three-dimensional space is shown in Fig. 2.11 using right hand coordinate system.

The force F is written as

$$\mathbf{F} = F^x\mathbf{i} + F^y\mathbf{j} + F^z\mathbf{k} \tag{2.21}$$

where F^x, F^y, F^z are rectangular components of the force F and **i**, **j**, **k** are unit vectors along x, y, z axes respectively.

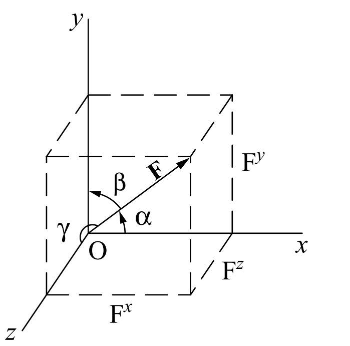


Fig. 2.11 Three-dimensional force.

α is the angle between force vector F and x-axis.

β is the angle between force vector F and y-axis.

γ is the angle between force vector F and z-axis.

$$F^x = F\cos\alpha, \quad \cos\alpha = \frac{x}{L} = l$$

$$F^y = F\cos\beta, \quad \cos\beta = \frac{y}{L} = m$$

$$F^z = F\cos\gamma, \quad \cos\gamma = \frac{z}{L} = n \tag{2.22}$$

$$F = \sqrt{\left(F^x\right)^2 + \left(F^y\right)^2 + \left(F^z\right)^2} \tag{2.23}$$

If **F** is represented by length L to some scale and similarly F^x by x, F^y by y and F^z by z and l, m, n are called direction cosines.

$$l^2 + m^2 + n^2 = 1 \tag{2.24}$$

$$\frac{F^x}{x} = \frac{F^y}{y} = \frac{F^z}{z} = \frac{F}{L} \tag{2.25}$$

$\frac{F}{L}$ is known as force multiplier.

Force is also written as

$$\mathbf{F} = F(l\mathbf{i} + m\mathbf{j} + n\mathbf{k}) \tag{2.26}$$

Let **n** be the unit vector in the direction force

$$\mathbf{n_F} = l\mathbf{i} + m\mathbf{j} + n\mathbf{k} \tag{2.27}$$

$$\mathbf{F} = F\mathbf{n_F} \tag{2.28}$$

Resultant force F given the rectangular components F^x, F^y, F^z:

(i) Let us take F^y and F^z and find the resultant of these components using parallelogram law in *YOZ* plane. (Fig. 2.12)

(ii) Adding F^x component to R vectorially the resultant force F is obtained (Fig. 2.13).

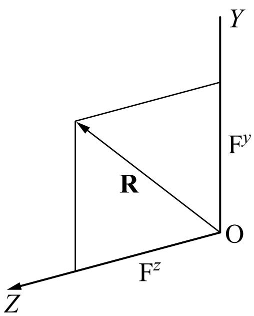


Fig. 2.12 Resultant in *YOZ* plane.

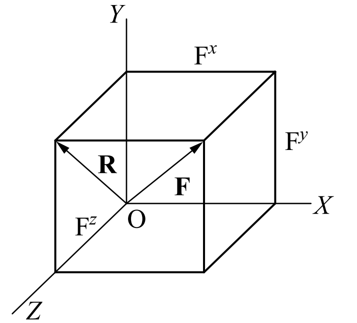


Fig. 2.13 Resultant in three-dimensional space.

2.8. RESULTANT OF COPLANAR CONCURRENT FORCES

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \mathbf{F}_4 \tag{2.29}$$

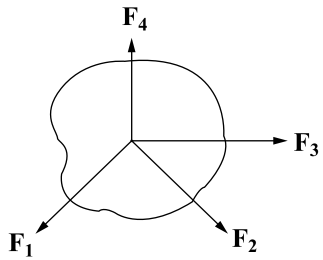


Fig. 2.14 Coplanar concurrent forces.

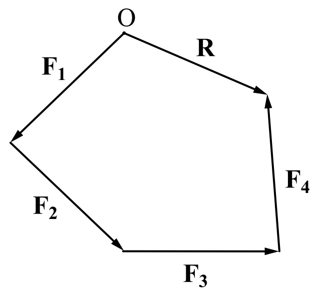


Fig. 2.15 Polygon of forces.

Resultant of forces with be obtained by adding the vectors graphically and the closing side of the polygon will be the resultant R.

2.9. RESULTANT OF NON-CONCURRENT COPLANAR FORCES

In general a system of forces acting in a plane on a body may be non-concurrent. The system of forces acting on the body must be reduced to a simple force system which does not alter the external effect of the original force system on the body. The equivalent of the forces acting on the body is the resultant force F acting at an arbitrary point and a couple. The choice of this arbitrary point will depend upon the convenience for the particular problem under consideration.

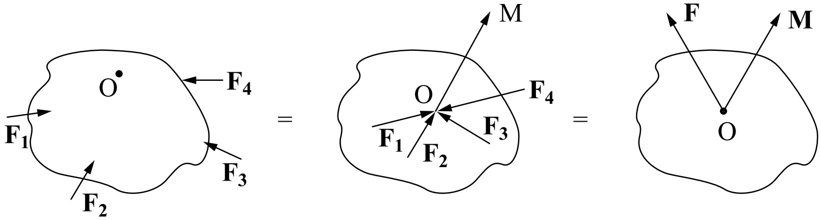


Fig. 2.16 Force system.

Let $\mathbf{F}_1$, $\mathbf{F}_2$, $\mathbf{F}_3$, $\mathbf{F}_4$ are the forces acting on the body. The system of forces are moved to an arbitrary point O so that it can be reduced to a resultant force **F** and corresponding resultant couple **M**.

2.10. COUPLES

A couple is formed by two equal and opposite forces acting along two non-coincident parallel straight lines in a body produces a moment. A couple has no resultant force.

Couple is characterized by the following qualities:

1. Forces F.
2. Arm of the couple which is the perpendicular distance between their lines of action.
3. Plane of the couple which is the plane containing the lines action of force.
4. Moment of the couple M is represented by a vector directed normal to the plane of the couple and sense of M is determined with respect to the notation used with right hand coordinate system.

Moment vector is the cross product of the relative position vector R and the force vector **F**.

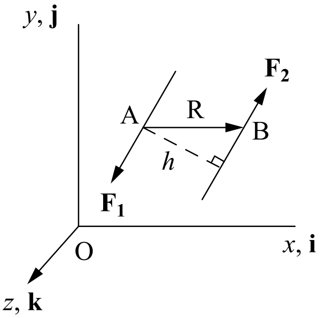


Fig. 2.17 Couple.

$$\mathbf{F}_2 = -\mathbf{F}_1 \tag{2.30}$$

$$\mathbf{M} = \mathbf{R} \times \mathbf{F}_2 \tag{2.31}$$

$$\mathbf{R} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k}$$

$$\mathbf{F}_2 = F^x\mathbf{i} + F^y\mathbf{j} + F^z\mathbf{k}$$

Moment vector
$$\mathbf{M} = \mathbf{R} \times \mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ x & y & z \\ F^x & F^y & F^z \end{vmatrix}$$

$$M^x = yF^z - zF^y$$
$$M^y = zF^x - xF^z$$
$$M^z = xF^y - yF^x$$
$$M = M^x\mathbf{i} + M^y\mathbf{j} + M^z\mathbf{k} \qquad (2.32)$$

Magnitude $M = hF_2 = \sqrt{(M^x)^2 + (M^y)^2 + (M^z)^2}$ where h is the perpendicular distance between the forces and produces a counter-clockwise moment.

The following observations emphasize important characteristics of couple.

(i) The magnitude of moment is independent of the choice of the centre about which moments are taken.
(ii) The moment vector **M** is independent of any particular origin and is thus considered a free vector.
(iii) The forces of a couple may be rotated within their plane provided their magnitudes and distance between their lines of action are kept constant.

It can be shown that the relative positive vector **R** can be chosen connecting any two points on the line of action.

Resolving **R** into two components, $\mathbf{R_1}$ perpendicular to the line of action of the forces and $\mathbf{R_2}$ parallel to the line of action of **F**.

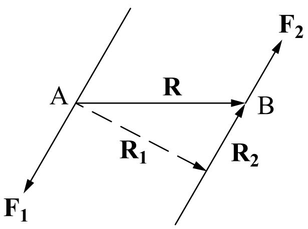


Fig. 2.18 Position vector and forces forming couple.

$$\mathbf{F_1} = -\mathbf{F_2} \qquad (2.33)$$

$$\mathbf{M} = \mathbf{R} \times \mathbf{F} = (\mathbf{R_1} + \mathbf{R_2}) \times \mathbf{F_2}$$
$$= \mathbf{R_1} \times \mathbf{F_2} + \mathbf{R_2} \times \mathbf{F_2} \qquad (2.34)$$

$\mathbf{R_2} \times \mathbf{F_2} = 0$ since $\mathbf{R_2}$ and $\mathbf{F_2}$ are collinear vectors.

$$\mathbf{M} = \mathbf{R_1} \times \mathbf{F_2} \qquad (2.35)$$

2.11. EQUIVALENT FORCE SYSTEMS

In the study of mechanics, we are concerned many a time with equivalent force systems acting on a rigid body. The following considerations will form the basis for force equivalences for rigid bodies.

(i) The sum of a system of concurrent forces is a single force that is equivalent to the original system.
(ii) A force may be moved along its line of action, i.e., forces are transmissible vectors.

(iii) The effect of a couple on a rigid body is to create a moment which is described by a free vector.

Translation of a force to a parallel position:

Let us consider a force acting in a plane and try to move it to a parallel position at point A as shown in Fig. 2.19 without changing the effect of force **F** on the rigid body.

This can be achieved by applying parallel forces **F** and **−F** passing through A. The net effect of the above operation leads to a force **F** and a moment **M** caused by couple whose magnitude is **F**d and sense is counter-clockwise. The moment vector **M** is perpendicular to the plane of the couple.

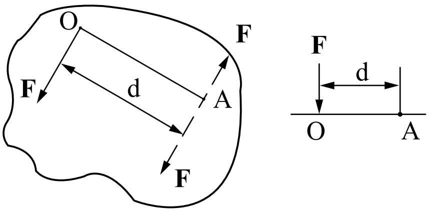


Fig. 2.19 Translation of force.

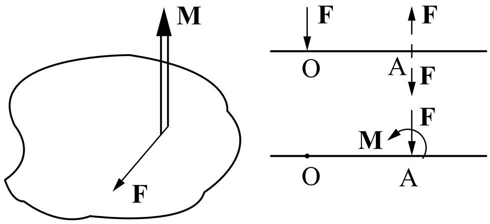


Fig. 2.20 Force and couple.

It can also be inferred that a force **F** and couple moment **M** (Fig. 2.20) can be replaced by a single force **F** (Fig. 2.19).

2.12. THE WRENCH

If the force and the couple acting on the body are not coplanar, these can be replaced by an equivalent system which consist of a force and a couple vector lying along same line of action. This equivalent system consisting of the force F and a collinear couple C_F is known as wrench.

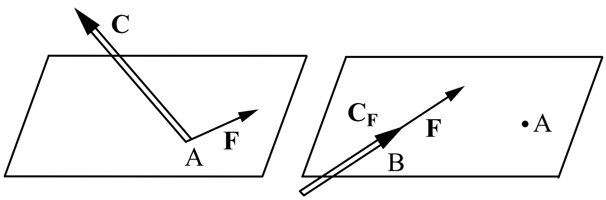


Fig. 2.21 Wrench.

When the moment representation of the couple and the force are in the same direction, the wrench is positive and otherwise negative if they are opposite in direction.

The following calculations are being suggested to calculate the equivalent system called wrench.

(i) Calculate the unit vector $\mathbf{n_F}$ in the direction of the force.
(ii) Calculate the dot product of couple vector **C** and unit vector $\mathbf{n_F}$, i.e., $(\mathbf{C}\cdot\mathbf{n_F})$

(iii) Compute the couple vector collinear with the force vector

$$\mathbf{C_F} = (\mathbf{C} \cdot \mathbf{n_F})\,\mathbf{n_F} \tag{2.36}$$

(iv) Determine the couple vector perpendicular to the force vector F

$$\mathbf{C_P} = (\mathbf{C} - \mathbf{C_F}) \tag{2.37}$$

(v) Find the position vector connecting the point A to B where the force vector is to be translated parallel to its own direction such that moment caused by $\mathbf{C_P}$ will be nullified.

(vi) Calculate the cross product $\mathbf{r_{AB}} \times \mathbf{F}$

(vii) Equate the above cross product to $\mathbf{C_P}$.

$$\mathbf{r_{AB}} \times \mathbf{F} = \mathbf{C_P}$$

which will be used to calculate the coordinates of point B knowing the Force vector **F**, couple vector **C** and the coordinates of point A.

2.13. DISTRIBUTED FORCES

Distributed forces occur in the study of mechanics in three forms, namely, acting along lines, over areas and over volumes. Examples of these forces are distributed force acting on a beam, surface forces due to wind loads on the wall of a building and gravitational forces acting over the volume. Intensity of distributed are specified as force per unit length (N/m), force per unit area (N/m^2 (pascal)), force per unit volume (N/m^3).

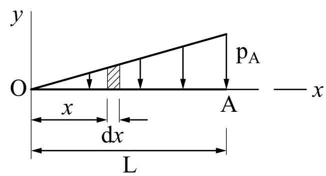


Fig. 2.22 Linearly varying force.

Let us consider a linearly varying distributed force acting on a beam, the intensity is p N/m. Instead of dealing with the distributed force directly, we can find the resultant force which is an equivalent distributed force and its location.

The equivalent force F is obtained by using the integral.

$$F = \int_{o}^{L} p\,dx \tag{2.38}$$

and

$$p = \frac{p_A \cdot x}{L} \tag{2.39}$$

Substituting Eq. (2.38) into Eq. (2.39)

$$F = \frac{p_A}{L}\int_{o}^{L} x\,dx = \frac{p_A}{L}\cdot\frac{L^2}{2} = \frac{p_A \cdot L}{2}$$

$$\bar{x} \cdot F = \int_{o}^{L} p\,x\,dx = \frac{p_A}{L}\cdot\frac{L^3}{3} = \frac{p_A \cdot L^2}{3}$$

The location of the equivalent concentrated force F is

$$\bar{x} = \frac{p_A L^2}{3F} = \frac{p_A L^2}{3} \cdot \frac{2}{p_A L} = \frac{2L}{3} \tag{2.40}$$

$$\bar{x} = \frac{2}{3} L$$

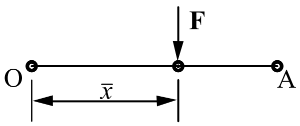


Fig. 2.23 Resultant force.

Determination of the resultant force and its location involves the use of integral. Similar calculations are carried out in the case of distributed forces over areas and volumes.

Summary

In this chapter, basic knowledge of mechanics which includes understanding of force, moment and couple has been discussed. Determination of resultants is an essential element in the solution of mechanics problems. Resultant of a force system is a force acting at an arbitrary point and a couple. In addition, mathematical treatment of concentrated and distributed forces acting on a body has been presented.

EXAMPLES

EXAMPLE 2.1

Resolve the force 100 N into rectangular components

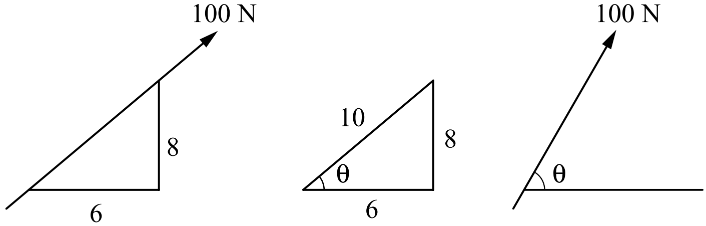
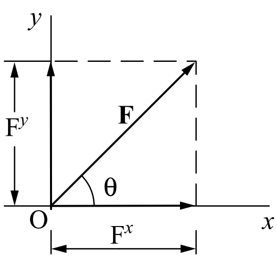


Fig. 2.24



Force is specified by the slope with vertical distance 8 and horizontal 6.

The rectangular components are

$$F^x = F\cos\theta = 100 \times \frac{6}{10} = 60 \text{ N}$$

$$F^y = F\sin\theta = 100 \times \frac{8}{10} = 80 \text{ N}$$

EXAMPLE 2.2

Determine the rectangular components of the given force F.

Force is given in polar notation $R = r\angle\theta = 50\angle 30^\circ$

$$R^x = r\cos\theta = 50\cos(30^\circ) = 43.3\text{ N}$$

$$R^y = r\sin\theta = 50\sin(30^\circ) = 25$$

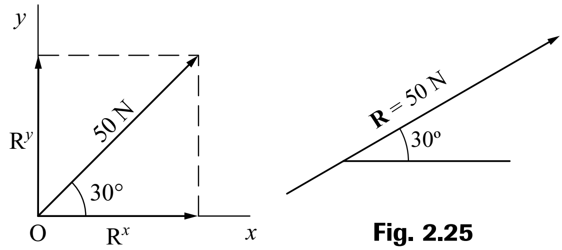


Fig. 2.25

EXAMPLE 2.3

Resolve the given force into components in the directions specified 1 and 2.

$$\frac{R_1}{\sin 75^\circ} = \frac{R_2}{\sin 75^\circ} = \frac{R}{\sin 30^\circ}$$

$$R_1 = R_2 = 80\,\frac{\sin 75^\circ}{\sin 30^\circ} = 154.5\text{ N}$$

Check:

$$R = \sqrt{R_1^2 + R_2^2 - 2R_1R_2\cos 30^\circ}$$

$$R = \sqrt{2\,(154.5)^2 - 2\,(154.5)^2\cos 30^\circ}$$

$$R = 80\text{ N}$$

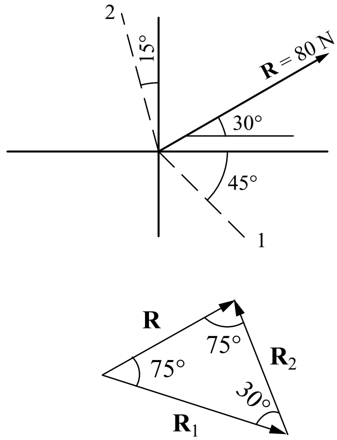


Fig. 2.26

EXAMPLE 2.4

Find the resultant of the forces R_1 and R_2.

(i) Graphical method:

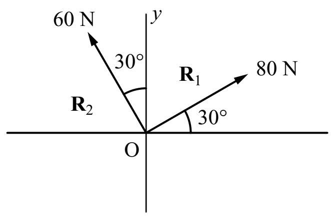


Fig. 2.27

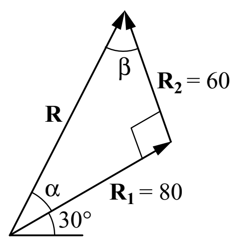
$$\mathbf{R} = \sqrt{R_1^2 + R_2^2}$$

$$\mathbf{R} = 10\sqrt{64 + 36} = 100\text{ N}$$

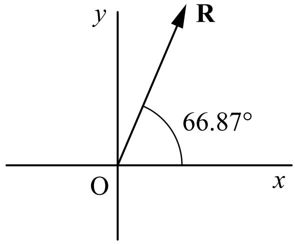
$$\frac{R_1}{\sin\beta} = \frac{R_2}{\sin\alpha} = \frac{R}{\sin 90^\circ}$$

$$\sin\beta = \frac{R_1}{R} = \frac{80}{100} = \frac{4}{5}$$

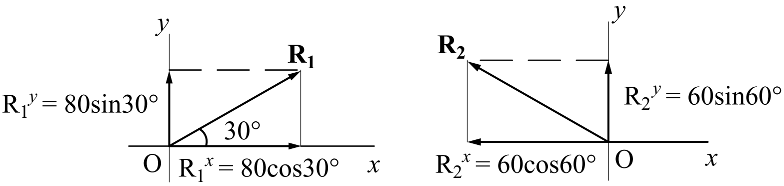
$$\beta = 53.13^\circ$$



$$\sin\alpha = \frac{R_2}{R} = \frac{60}{100} = \frac{3}{5}$$
$$\alpha = 36.87$$



(ii) Resolution of forces:



$$R_1^{x} = 69.28 \rightarrow \qquad R_2^{x} = 30 \leftarrow$$
$$R_1^{y} = 40 \uparrow \qquad R_2^{y} = 51.96 \uparrow$$

Components of resultant R:

$$R^{x} = 39.2 \text{ N} \rightarrow$$
$$R^{y} = 91.96 \text{ N} \uparrow,$$
$$\mathbf{R} = \sqrt{(39.2)^2 + (91.96)^2} = 100 \qquad \tan\theta = \frac{R_y}{R_x} = \frac{91.96}{39.2}$$
$$\theta = 66.87°$$

EXAMPLE 2.5

Determine the resultant of four forces acting on a body as shown below.

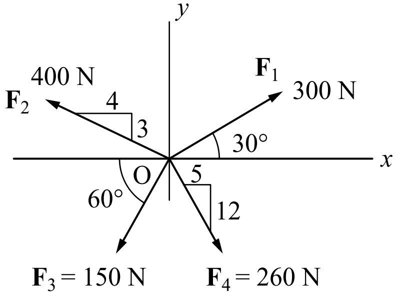
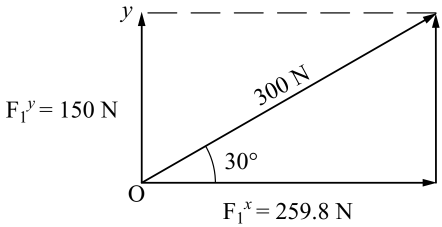


Fig. 2.28

(i) Resolution forces along *x* and *y* axes:



$$F_1^x = 259.8 \text{ N} \rightarrow$$

$$F_2^y = 150 \text{ N} \uparrow$$

$$F_2^x = \frac{400}{5} \times 4 = 320 \text{ N} \leftarrow$$

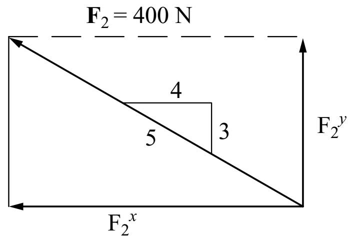
$$F_2^y = \frac{400}{5} \times 3 = 240 \text{ N} \uparrow$$

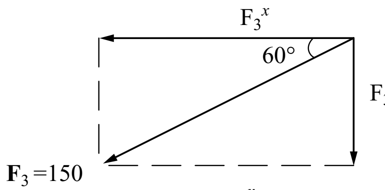
$$F_3^x = 150 \times \frac{1}{2} = 75 \text{ N} \leftarrow$$

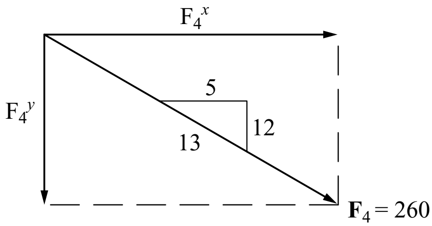
$$F_3^y = 150 \times 0.866 = 130 \text{ N} \downarrow$$

$$F_4^x = \frac{260 \times 5}{13} = 100 \text{ N} \rightarrow$$

$$F_4^y = \frac{260 \times 12}{13} = 240 \text{ N} \downarrow$$







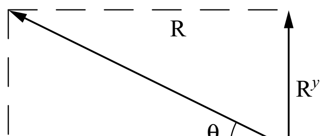
$$R^x = 259.8 - 320 - 75 + 100 = -35.2 \text{ N } (\leftarrow)$$

$$R^y = 150 + 240 - 130 - 240 = 20 \text{ N} \uparrow$$

$$\mathbf{R} = \sqrt{(-35.2)^2 + (20)^2}$$

$$\tan\theta = \frac{R^y}{R^x} = \frac{20}{35.2}$$

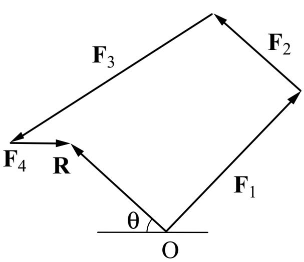
$$\theta = 29.6°$$



(ii) Graphical approach:

$$\mathbf{R} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 + \mathbf{F}_4$$

Closing vector **R** of polygon results in the resultant R.



EXAMPLE 2.6

Resolve the vertical force and horizontal force acting on a body supported by an inclined plane along x and y-axes shown in Fig. 2.29.

Resolution of horizontal force F:

$$F^x = F\cos\theta\,(\nearrow)$$

$$F^y = +F\sin\theta\,(\searrow)$$

$$F = F^x\mathbf{i} - F^y\mathbf{j}$$

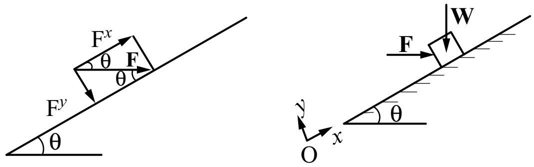
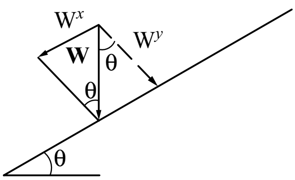


Fig. 2.29



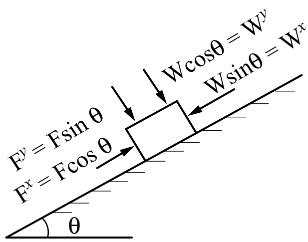
Resolution of vertical for W:

$$W^x = +W\sin\theta\,(\swarrow)$$

$$W^y = +W\cos\theta\,(\searrow)$$

$$\mathbf{W} = -W^x\mathbf{i} - W^y\mathbf{j}$$

The resolution of vertical and horizontal forces on an inclined plane will provide force components useful in many applications.



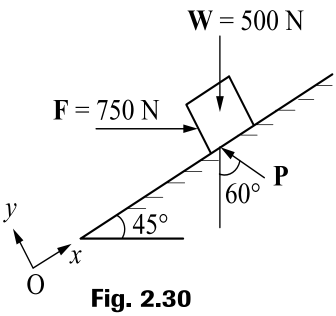
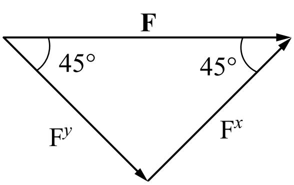


Fig. 2.30

EXAMPLE 2.7

A block is acted upon by its weight **W** = 500 N and a horizontal force **F** = 750 N and force exerted by the inclined plane is **P**. Determine the resultant **R** of these forces which is parallel to the incline and the magnitude of P.

Let us define the coordinate axes x and y parallel and perpendicular to the inclined plane respectively.



(i) Resolution of force **F** parallel and perpendicular to the plane

$$F^x = +F\cos 45^\circ = +750 \times 0.707 = +530.25\text{ N }(\rightarrow)$$

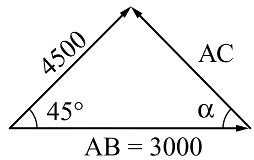
$$F^y = +F\sin 45^\circ = +750 \times 0.707 = +530.25\text{ N }(\downarrow)$$

$$F = F^x\mathbf{i} - F^y\mathbf{j} = 530.25\mathbf{i} - 530.25\mathbf{j}$$

$$AC = \sqrt{(3000)^2 + (4500)^2 - 2(3000)(4500)\cos 45}$$

$$AC = 3187.6$$

$$\frac{AC}{\sin 45} = \frac{4500}{\sin \alpha}$$



$$\sin\alpha = \frac{4500}{3187.6} \times 0.707$$

$$\alpha = 86.45^\circ$$

$$AC = 3187.6 \text{ N}$$

EXAMPLE 2.10

Three forces shown in Fig. 2.33 produce a horizontal resultant force through the point A. Find the magnitude and sense of **P** and **F**.

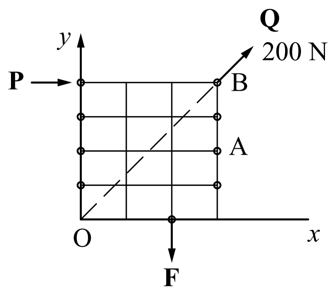
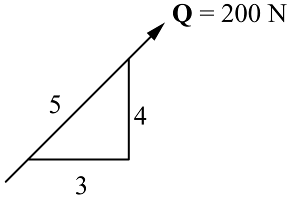


Fig. 2.33

Resolving the force Q = 200 N along x and y axes.



$$Q^x = \frac{3}{5}(200) = 120 \text{ N} \rightarrow$$

$$Q^y = \frac{4}{5}(200) = 160 \text{ N} \uparrow$$

Let the resultant force at A be R^x and there is no component of the resultant force in the y-direction,

i.e., $R^y = 0$

Equations: $Q^x + P = R^x$

$Q^y - F = 0$, $\quad$ $\mathbf{F = 160}$ N $\downarrow$

Taking moments about B, forces P and Q will not cause moments.

$$(2)R^x + (1)F = 0$$

$$\therefore \quad R^x = -\frac{F}{2} = -80 \text{ N}$$

$$R^x = 80 \text{ N} \leftarrow$$

$$Q^x + P = R^x$$

$$120 + P = -80$$

$$\mathbf{P} = -200 \text{ N}$$

$$\mathbf{P} = 200 \text{ N} \leftarrow; \qquad \mathbf{F} = 160 \text{ N} \downarrow; \qquad \mathbf{R} = 80 \text{ N} \leftarrow$$

EXAMPLE 2.11

Compute the moment of 1500 N force shown in Fig. 2.34 about points A and B

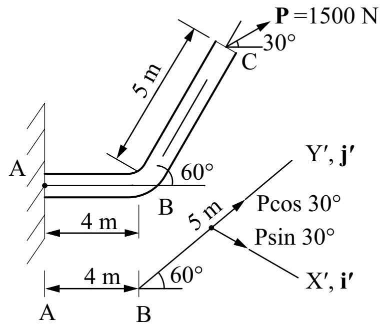


Fig. 2.34

(i) (a) Resolving the force **P** along BC and perpendicular BC

Moment of force P about B

$$M_B^P = (5)(P\sin 30°) = (5)(750) = 3750 \text{ N}\cdot\text{m} \qquad \text{(clockwise moment)}$$

(b) *Vector method:*

Position Vector $\mathbf{r}_{BC} = 5\mathbf{j}'$ in x′cy′ coordinate system

Force Vector $\mathbf{P} = P\sin 30\mathbf{i}' + P\cos 30°\mathbf{j}'$

$$\mathbf{M}_B^P = \mathbf{r}_{BC} \times \mathbf{P} = \begin{vmatrix} \mathbf{i}' & \mathbf{j}' & \mathbf{k}' \\ 0 & 5 & 0 \\ P\sin 30° & P\cos 30° & 0 \end{vmatrix}$$

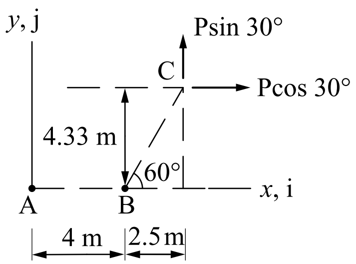
$$\mathbf{M}_B^P = -5(P\sin 30°)\mathbf{k}' = +3750(-\mathbf{k}') \text{ N·m}$$

($-\mathbf{k}'$) indicates clockwise moment.

$$M_B^P = 3750 \text{ N·m.}$$

(ii) (a) Resolving the force **P** parallel to AB and perpendicular AB.

Moment of force P about A



$$M_A^P = (6.5)P\sin 30° - (4.33)P\cos 30°$$

$$M_A^P = (6.5)(750) - (4.33)1299$$

$$M_A^P = -749.67 \text{ N·m}$$

$$M_A^P = 749.67 \text{ N·m} \qquad \text{(clockwise moment)}$$

EXAMPLE 2.14

What are the force components of 1000 N shown in Fig. 2.37. What are the direction cosines associated with the 1000 N force.

Length of force Vector L

$$L = \sqrt{(15)^2 + (5)^2 + (3)^2}$$
$$L = 16.1 \text{ m}$$

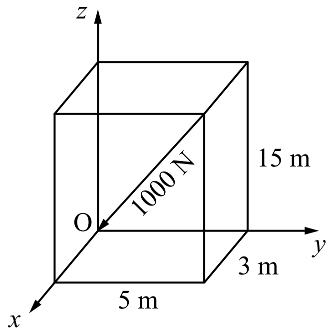


Fig. 2.37

Direction cosines:

$$l = -\frac{x}{L} = -\frac{3}{16.1} = -0.19$$
$$m = -\frac{y}{L} = -\frac{5}{16.1} = -0.31$$
$$n = -\frac{z}{L} = -\frac{15}{16.1} = -0.93$$

$$F^x = Fl = -(0.19)\ (1000) = -190 \text{ N}$$
$$F^y = Fm = -(0.31)\ (1000) = -310 \text{ N}$$
$$F^z = Fn = -(0.93)\ (1000) = -930 \text{ N.}$$

EXAMPLE 2.15

Determine the resultant of the system of concurrent forces having the following magnitudes passing through the origin and the indicated points: P = 300 N (+12, +6, –4), T = 500 N (–3, –4, +12) F = 250 N (+6, –3, –6)

Direction cosines: $l = \frac{x}{L}, \quad m = \frac{y}{L}, \quad n = \frac{z}{L}$

Force (N)	**Coordinates**			**L**	**Direction cosines**		
	x	***y***	***z***		***l***	***m***	***n***
P = 300	12	+6	–4	14	0.857	0.43	–0.29
T = 500	–3	–4	12	13	–0.23	–0.3	0.92
F = 250	6	–3	–6	9	0.66	–0.33	–0.66

Force component in x-direction = Force × l

y-direction = Force × m

z-direction = Force × n

Force components:

Force (N)	Force component (N)		
	x	*y*	*z*
P = 300	257	129	–87
T = 500	–115	–150	460
F = 250	165	–82.5	–165
Total	307	–103.5	208

$$\text{Resultant} = \sqrt{(307)^2 + (-103.5)^2 + (208)^2}$$
$$= 385 \text{ N}$$

Direction cosines of resultant:

$$l = 0.793,\ m = -0.268,\ n = 0.54$$

EXAMPLE 2.16

Find the resultant of the force system shown in Fig. 2.38 in which **P** = 280 N, **Q** = 260 N and **R** = 210 N.

Coordinates of points on the line of action of forces:

Point	*x*	*y*	*z*
A	0	12	0
B	–4	0	–3
C	–4	0	6
D	6	0	4

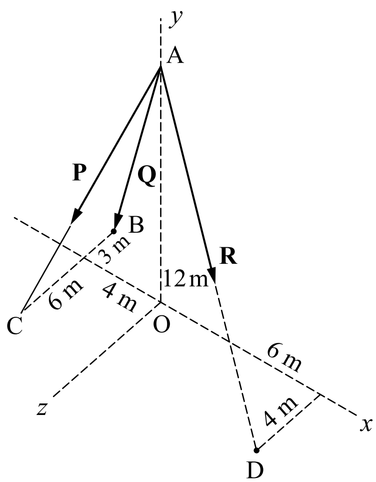


Fig. 2.38

We use the following equations for calculating components of force:

$$\frac{F_x}{\bar{x}} = \frac{F_y}{\bar{y}} = \frac{F_z}{\bar{z}} = \frac{F}{L}$$

Force multiplier $F_m = \frac{F}{L}$, $L = \sqrt{(\bar{x})^2 + (\bar{y})^2 + (\bar{z})^2}$

$$\text{X component of force } F_x = F_m \cdot \bar{x}$$
$$\text{Y component of force } F_y = F_m \cdot \bar{y}$$
$$\text{Z component of force } F_z = F_m \cdot \bar{z}$$

Length of force vector L:

$$\overline{AC} = \sqrt{(-4-0)^2 + (0-12)^2 + (6-0)^2} = 14$$

$$\overline{AB} = \sqrt{(-4-0)^2 + (0-12)^2 + (-3-0)^2} = 13$$

$$\overline{AD} = \sqrt{(6-0)^2 + (0-12)^2 + (4-0)^2} = 14$$

Force (N)	**Components of distance**			**L**	**F_m (N/m)**	**Force components**		
	$\bar{x}$	$\bar{y}$	$\bar{z}$			F_x	F_y	F_z
P = 280	–4	–12	6	14	20	–80	–240	120
Q = 260	–4	–12	–3	13	20	–80	–240	–60
R = 210	6	–12	–4	14	15	90	–180	60
TOTAL						–70	–660	120

$$\text{Resultant force} = 10\sqrt{(7)^2 + (66)^2 + (12)^2} = 674.45 \text{ N}$$

EXAMPLE 2.17

A vertical boom A_E is supported by guy wires from A to B, C, D. If the tensile load in AD = 252 N, find the forces in AC and AB so that the resultant force on A will be vertical.

Coordinates of points.

Point	***x***	***y***	***z***
A	0	12	0
B	0	0	–9
C	–4	0	3
D	6	0	4
E	0	0	0

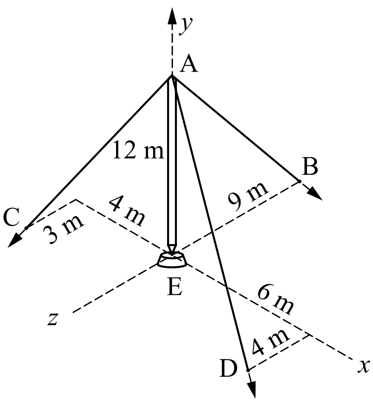


Fig. 2.39

Components of force P: along x-direction $\quad P_x = P_m\overline{X} = 10(-4) = -40$ N

along y-direction $\quad P_y = P_m\overline{Y} = 10(8) = 80$ N

along z-direction $\quad P_z = P_m\overline{Z} = 10(6) = 60$ N

$$\mathbf{P} = -40\mathbf{i} + 80\mathbf{j} + 60\mathbf{k}$$

$$\text{Position vector } \mathbf{r}_{CB} = 4\mathbf{i} + 5\mathbf{j}$$

$$\text{(i)}\quad \mathbf{M}_C^P = \mathbf{r}_{CB} \times \mathbf{P} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & 5 & 0 \\ -40 & 80 & 60 \end{vmatrix} = 300\,\mathbf{i} - 240\,\mathbf{j} + 520\,\mathbf{k}$$

Moment of force P about $C = \mathbf{M}_C^P = 300\mathbf{i} - 240\mathbf{j} - 520\mathbf{k}$

(ii) Position vector $\mathbf{r}_{DB} = 4\mathbf{i} - 5\mathbf{j} + 3\mathbf{k}$

Moment of force P about D = $\mathbf{M}_D^P = \mathbf{r}_{DB} \times \mathbf{P} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 4 & -5 & 3 \\ -40 & 80 & 60 \end{vmatrix}$

$$\mathbf{M}_D^P = -540\mathbf{i} - 360\mathbf{j} + 120\mathbf{k}$$

(iii) Position vector $\mathbf{r}_{CD} = 10\mathbf{j} - 3\mathbf{k}$

Unit vector along CD = $CD = \mathbf{n}_{CD} = \dfrac{10\,\mathbf{j} - 3\,\mathbf{k}}{10.44}$

Moment about a line directed from C to D of $P = \mathbf{M}_C^P \cdot \mathbf{n}_{CD} = -379.3$ N·m

EXAMPLE 2.19

Replace the force and couple shown in Fig. 2.41 by a wrench passing through E in xz plane.

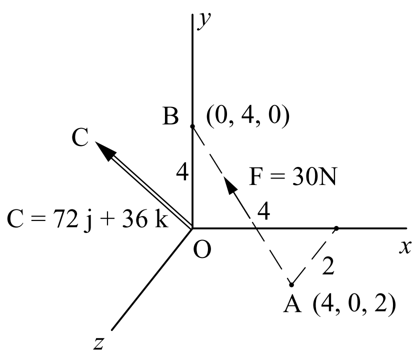


Fig. 2.41

The direction cosines of force vector are

$$l = \frac{x}{L} = -\frac{4}{6} = -\frac{2}{3}$$

$$m = \frac{y}{L} = \frac{4}{6} = \frac{2}{3}$$

$$n = \frac{z}{L} = -\frac{2}{6} = -\frac{1}{3}$$

where L is length of AB, $\quad L = \sqrt{(0-4)^2 + (4-0)^2 + (0-2)^2} = 6$

∴ The force vector components when F is equal to 30 N

$$F = F^x\mathbf{i} + F^y\mathbf{j} + F^z\mathbf{k} = (Fl)\mathbf{i} + (Fm)\mathbf{j} + (Fn)\mathbf{k}$$

$$F^x = -\frac{2}{3}\times 30 = -20$$

$$F^y = \frac{2}{3}\times 30 = 20$$

$$F^z = -\frac{1}{3}\times 30 = -10$$

$$\mathbf{F} = -20\mathbf{i} + 20\mathbf{j} - 10\mathbf{k}$$

unit vector in the direction of force $\mathbf{n}_F = \frac{1}{3}(-2\mathbf{i}+2\mathbf{j}-\mathbf{k})$

Couple vector $\mathbf{C} = 72\mathbf{j} + 36\mathbf{k}$

$$C\cdot\mathbf{n}_F = (72\mathbf{j}+36\mathbf{k})\cdot\frac{(-2\mathbf{i}+2\mathbf{j}-\mathbf{k})}{3} = 36$$

Couple vector collinear with the force **F** is

$$\mathbf{C}_F = (\mathbf{C}\cdot\mathbf{n}_F)\mathbf{n}_F = \frac{36}{3}(-2\mathbf{i}+2\mathbf{j}-\mathbf{k}) = -24\mathbf{i}+24\mathbf{j}-12\mathbf{k}$$

Couple vector perpendicular to the force **F** is

$$\mathbf{C}_p = (\mathbf{C}-\mathbf{C}_F) = (72\mathbf{j} + 36\mathbf{k} + 24\mathbf{i} - 24\mathbf{j} + 12\mathbf{k})$$

$$\mathbf{C}_p = (24\mathbf{i} + 48\mathbf{j} + 48\mathbf{k})$$

The coordinates of point A (4, 0, 2) and E (x, o, z)

Position vector $\mathbf{r}_{AE} = (x-4)\mathbf{i} + (z-2)\mathbf{k}$

$$\mathbf{r}_{AE}\times\mathbf{F} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ (x-4) & 0 & (z-2) \\ -20 & 20 & -10 \end{vmatrix} = -20(z-2)\mathbf{i} + [10(x-4) - 20(z-2)]\mathbf{j} + 20(x-4)\mathbf{k}$$

$\mathbf{r}_{AE}\times\mathbf{F} = \mathbf{C}_p$ and equating the coefficients of **i** and **k**

$$-20z + 40 = 24, \qquad z = 0.8$$

$$20x - 80 = 48, \qquad x = 6.4$$

Point E is (6.4, 0, 0.8)

2.13. Find the resultant force of the applied loads to the body at the points A and B.

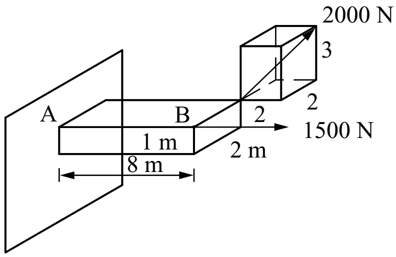


Fig. P. 2.13

2.14. The moment of a force F is 400 N·m clockwise about O and 1500 N·m counter-clockwise about B. If the moment of the force about A is zero, determine the force.

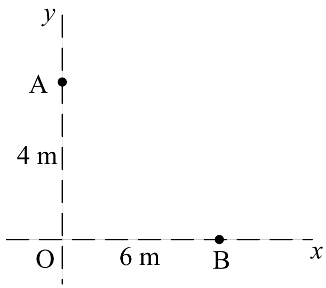


Fig. P. 2.14

2.15. Force and couple–moment acting on a body are given below.

$$\mathbf{F} = 15\mathbf{i} - 5\mathbf{j} + \mathbf{6k}$$

$$\mathbf{C} = 3\mathbf{i} + 12\mathbf{j}$$

If the force goes through the point (3, 5, 8) replace the system by wrench.