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Assignment

Mathematical Economics

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Cramer's Rule For Matrix Solutions:

Cramer's Rule provide a simplified method of solving a system of linear equations through the use of determinents.

Crame's rule states

$$\bar{x}_i = \frac{|A_i|}{|A|}$$

where x_i is the ith unknown variable in a series of equations, $|A|$ is the determinent of the co-efficient matrix and $|A_i|$ is the determinent of a special matrix formed from the original co-efficient matrix by replacing the column of co-efficients of x_i with the column vector of constants

Example#9

Solve the following system of linear equations using cramer's rule.

$$6x_1 + 5x_2 = 49$$

$$3x_1 + 4x_2 = 32$$

Solutions:

Express the equations in matrix form

$$\begin{bmatrix} 6 & 5 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 49 \\ 32 \end{bmatrix}$$

det of $A = |A| = \begin{vmatrix} 6 & 5 \\ 3 & 4 \end{vmatrix}$

$= (6)(4) - (5)(3)$

$= 24 - 15$

$= 9$

To solve for x_1, replace column 1, the co-efficients of x_1, with the vector of constants B, forming a new matrix A_1;

$$A_1 = \begin{bmatrix} 49 & 5 \\ 32 & 4 \end{bmatrix}$$

$$|A_1| = \begin{vmatrix} 49 & 5 \\ 32 & 4 \end{vmatrix}$$

$= (49)(4) - (5)(32)$

$= 196 - 160 = 36$

Similarly get A_2 by replacing 2nd column of co-efficient matrix with ~~vec~~matrix B.

$$A_2 = \begin{bmatrix} 6 & 49 \\ 3 & 32 \end{bmatrix}$$

$$|A_2| = \begin{vmatrix} 6 & 49 \\ 3 & 32 \end{vmatrix}$$

$= (6)(32) - (49)(3)$

$= 192 - 147 = 45$

Now;

By using cramer's Rule, we have

$\bar{x}_1 = \frac{|A_1|}{|A|}$ and $\bar{x}_2 = \frac{|A_2|}{|A|}$

$= \frac{36}{9}$ $\qquad = \frac{45}{9}$

$\bar{x}_1 = 4$ $\qquad \bar{x}_2 = 5$

$\boxed{\bar{x}_1 = 4}$, $\boxed{\bar{x}_2 = 5}$

Question No. 8

Use cramer's Rule to solve the following system of Eqvations

$$11P_1 - P_2 - P_3 = 31$$
$$-P_1 + 6P_2 - 2P_3 = 26$$
$$-P_1 - 2P_2 + 7P_3 = 24$$

Solution:

In Matrix form, we have

$$\underset{A}{\begin{bmatrix} 11 & -1 & -1 \\ -1 & 6 & -2 \\ -1 & -2 & 7 \end{bmatrix}} \underset{X}{\begin{bmatrix} P_1 \\ P_2 \\ P_3 \end{bmatrix}} = \underset{B}{\begin{bmatrix} 31 \\ 26 \\ 24 \end{bmatrix}}$$

$$|A| = \begin{vmatrix} 11 & -1 & -1 \\ -1 & 6 & -2 \\ -1 & -2 & 7 \end{vmatrix}$$

$$= 11(38) + 1(-9) - 1(8) = 401$$

$$|A_1| = \begin{vmatrix} 31 & -1 & -1 \\ 26 & 6 & -2 \\ 24 & -2 & 7 \end{vmatrix}$$

$$= 31(38) + 1(230) - 1(-196) = 1604$$

$$|A_2| = \begin{vmatrix} 11 & 31 & -1 \\ -1 & 26 & -2 \\ -1 & 24 & 7 \end{vmatrix}$$

$$= 11(196) + 1(2) + 31(8)$$

$$= 2807$$

$$|A_3| = \begin{vmatrix} 11 & -1 & 31 \\ -1 & 6 & 26 \\ -1 & -2 & 24 \end{vmatrix}$$

$$= 11(196) + 1(2) + 31(8) = 2406$$

Thus, By cramer's Rule, we have

$$\bar{P}_1 = \frac{|A_1|}{|A|} = \frac{1604^4}{401} = 4$$

$$\bar{P}_2 = \frac{|A_2|}{|A|} = \frac{2807}{401} = 7$$

$$\bar{P}_3 = \frac{|A_3|}{|A|} = \frac{2406}{401} = 6$$

Question No.9

Use cramer's Rule to find the critical values of Q_1 and Q_2, given the first order conditions for constrained utility maximization.

$$Q_2 - 10\lambda = 0$$

$$Q_1 - 2\lambda = 0$$

$$-10Q - 2Q_2 = -240$$

Solution:

$$\begin{bmatrix} 0 & 1 & -10 \\ 1 & 0 & -2 \\ -10 & -2 & 0 \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \\ Q_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 240 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 0 & 1 & -10 \\ 1 & 0 & -2 \\ -10 & -2 & 0 \end{vmatrix}$$

$$= 0 - 1(-20) + (-10)(-2)$$
$$= 20 + 20 = 40$$

$$|A_1| = \begin{vmatrix} 0 & 1 & -10 \\ 0. & 0 & -2 \\ -240 & -2 & 0 \end{vmatrix}$$

$$= 0 - 1(0 - 480)$$
$$= 480$$

$$|A_2| = \begin{vmatrix} 0 & 0 & -10 \\ 0 & 0 & -2 \\ -10 & -240 & 0 \end{vmatrix}$$

$$= 0 - 0 + (-240)(-10)$$
$$= 2400$$

$$|A_3| = \begin{vmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ -10 & -2 & -240 \end{vmatrix}$$

$$= 0 - 1(-240) + 0$$
$$= 240$$

Thus By cramer's Rule, we have

$$\bar{Q}_1 = \frac{|A_1|}{|A|} = \frac{480}{40} = 12$$

$$\bar{Q}_2 = \frac{|A_2|}{|A|} = \frac{2400}{40} = 60$$

$$\bar{\lambda} = \frac{|A_3|}{|A|} = \frac{240}{40} = 6$$