

level of significance. In general 5 per cent and 1 per cent are taken as 'levels of significance' thereby indicating that on an average we may go wrong 5 out of 100 cases and 1 out of 100 cases respectively. To say that 5 per cent level of significance, there is 95 per cent confidence in the result with a margin of error 5 per cent. The 5 per cent level of significance is shown in Fig. 10.1.

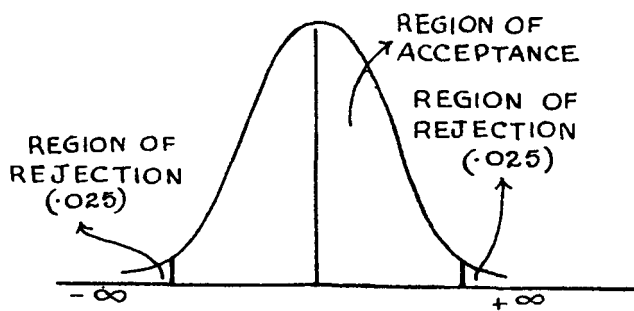


Fig. 10.1. Normal curve.

**10.1.3. Degrees of Freedom:** It is defined as the difference between the total number of items and the total number of constraints.

If  $n$  is the total number of items and  $k$ , the total number of constraints then the degrees of freedom (d.f.) is given by  $d.f. = n - k$

Suppose we want to select 10 values with a restriction that the total of the ten values should be equal to 100. Thus, we can select only 9 items at our choice but the tenth item must be chosen in such a way that the total of them should be equal to 100. Therefore, degrees of freedom of selecting 10 items is only 9 with one constraint.

**10.1.4. Null Hypothesis:** Null hypothesis is the statement about parameters which is likely to be rejected after testing. We start with the hypothesis that the two items are equal.

**10.1.5. Standard Normal Deviate Tests:** If  $X$  follows normal distribution with mean,  $\mu$  and S.D.,  $\sigma$  then  $\bar{X}$  also follows normal distribution with mean  $\mu$  and S.D.,  $\frac{\sigma}{\sqrt{n}}$ . This can be denoted by

$$\bar{X} \rightarrow N \left( \mu, \frac{\sigma}{\sqrt{n}} \right) \text{ i.e., } \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \rightarrow N(0, 1)$$

the expression on the left hand side is called as standard normal variate (or standard normal deviate) which follows normal distribution with mean zero and standard deviation unity. The test of hypothesis based on this deviate is called standard normal deviate test. The confidence limits for the population mean can be obtained as

$$\bar{X} \pm 1.96 \frac{\sigma}{\sqrt{n}}$$

i.e., the population mean, lies between  $\bar{X} - 1.96 \frac{\sigma}{\sqrt{n}}$  and  $\bar{X} + 1.96 \frac{\sigma}{\sqrt{n}}$  where 1.96 is the tabulated value of normal distribution at 5 per cent level of significance.

**10.2. One Sample Test: Case (i)**

- Assumptions:* 1. Population is normal.  
 2. The sample is drawn at random.
- Conditions:* 1. Population S.D.,  $\sigma$  is known.  
 2. Size of the sample may be small or large.
- Null hypothesis:*  $\mu = \mu_0$

$$Z = \frac{|\bar{X} - \mu_0|}{\sigma/\sqrt{n}}$$

we know that 'Z' values follow normal distribution with zero mean and unit S.D. The values of Z on either side of normal curve corresponding to the areas 0.025 and 0.025 are

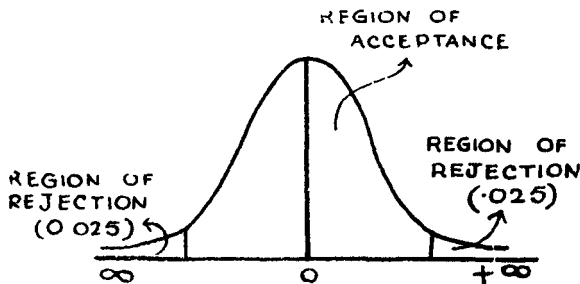


Fig. 10.2. Standard normal curve,

-1.96 and 1.96 respectively. That is the total area of the region of rejection is 0.05 out of the total area 1 sq units which is shown in Fig. 10.2. For 1 per cent level of significance, the regions of rejection on either side of the standard normal curve comprises area of 0.005 and the corresponding Z values are -2.576, and 2.576 which is shown in Fig. 10.3.

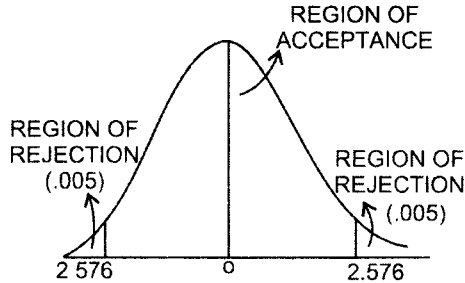


Fig. 10.3.

**CONCLUSION :** If  $Z$  (calculated)  $\leq Z$  (tabulated), at 5 per cent (or 1 per cent) level of significance, the null hypothesis is accepted i.e., there is no significant difference between sample mean and population mean.

Otherwise, the null hypothesis is rejected. That is, there is significant difference between sample and population means. In other words, the sample may not belong to the population having given mean with respect to character under consideration.

Here, we are comparing with positive values of tabulated  $Z$  since we are taking modulus in S.N.D. test as the  $Z$  values on either side of mean are identical.

**EXAMPLE :** The average number of mango fruits per tree in a particular region was known from a considerable experience as 520 with a standard deviation 4.0. A sample of 20 trees gives an average number of fruits 450 per tree. Test whether the average number of fruits per tree selected in the sample is in agreement with the average production in that region ?

*Null hypothesis :*  $\mu = \mu_0 = 520$

$$Z = \frac{|450 - 520|}{4/\sqrt{20}} = 78.26$$

**Conclusion :**  $Z$  (calculated)  $> Z$  (tabulated), 1.96 at 5 per cent level significance. Therefore, it can be concluded that there is

significant difference between sample mean and population mean with respect to average performance.

**10.2. Case (ii):** If the S.D. in the population is not known still we can use the standard normal deviated test.

*Assumptions:* 1. Population is normal  
2. Sample is drawn at random

*Conditions:* 1.  $\sigma$  is not known  
2. Size of the sample is large (say  $> 30$ )

*Null hypothesis:*  $\mu = \mu_0$

$$Z = \frac{|\bar{X} - \mu_0|}{S/\sqrt{n}}$$

$$\text{where } S = \sqrt{\frac{1}{n} \sum_{i=1}^n (X_i - \bar{X})^2}$$

where  $\bar{X}$  = mean of sample  
n = Size of the sample

**CONCLUSION:** Same as in the case (i).

**EXAMPLE:** The average daily milk production of a particular variety of buffalo was given as 12 kgs. The distribution of daily milk yield in a dairy farm was given as follows:

TABLE 10.1

Daily milk yield (kgs)	6-8	8-10	10-12	12-14	14-16	16-18
No. of buffaloes	9	20	35	42	17	7

Test whether the performance of dairy farm was in agreement with the record.

*Null hypothesis:*  $\mu = \mu_0 = 12$

Using the formula given in Section 4.1.3 (b), we have

$$\bar{X} = 11.91$$

Using the formula given in Section 5.4.2, we have

$$S = 2.49$$

$$\text{Therefore, } Z = \frac{|11.91 - 12|}{\frac{2.49}{\sqrt{130}}} = 0.41$$

**CONCLUSION:** The  $Z$  (calculated)  $<$   $Z$  (tabulated), 1.96 at 5 per cent level of significance. Therefore, the null hypothesis is