Measure of Central tendency:-

Trend of obs. in a data set, that is known as measure of location or position. The measures of central tendency or location are generally known as averages.

Different type of average:-

(i) Arithmetic Mean (ii) Geometric Mean

(iii) Harmonic Mean (iv) Median

(v) Mode (vi) Quartile

(vii) Decile (viii) Percentile

Mean → A.M, G.M, H.M

Median → Quartile, Decile, Percentile

Mode → Uni-mode, Bi mode, Tri mode

(i) Arithmetic Mean:- (A.M)

The arithmetic mean of "n" obs. (Sum) are defined as the total number of all obs. divided by the number of obs.

- The A.M of sample data is denoted by "$\bar{x}$" & the symbol "μ" is used for mean of pop^n data

For ungrouped data:-

$$\bar{X} = \frac{x_1 + x_2 + x_3 + \cdots + x_n}{n}$$

Grouped data:- $\bar{X} = \Sigma f_i x_i / n$

$$\bar{X} = \sum_{i=1}^{n} x_i / n$$

$\bar{X} = \frac{\Sigma X}{n}$ for sample

$M = \frac{\Sigma X}{N}$ for Population

Example :-

Find the A.M of the set of numbers. 84, 91, 72, 68, 87, & 78.

So,

$$n = 6$$

$$\bar{X} = \frac{84+91+72+68+87+78}{6} = \frac{480}{6} = 80$$

It means that most values of data lies around 80.

Geometric Mean :-

$$G.M = \text{antilog}\left(\frac{\sum_{i=1}^{n} \log x_i}{n}\right)$$

Harmonic Mean :- Reciprocal of the A.M of the reciprocal of the values

$$H.M = \frac{\sum_{i=1}^{n} 1/x_i}{n} = \frac{n}{\sum 1/x_i}$$

$$H.M = \frac{\frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{1}{x_n}}{n} = \frac{\sum_{i=1}^{n} 1/x_i}{n}$$

eolian:-

"The median of a data is the middle item when the item (the obs.) are arranged in ascending or decending order."

Median Symbol.. $\tilde{X}$ (X tilda).

When n is ~~odd~~ The value of :- $\left(\frac{n+1}{2}\right)^{th}$ obs.

When n is even :- $\left(\frac{n}{2}\right)^{th}$ obs. OR $\left(\frac{n+2}{2}\right)^{th}$

Example:-

The wages of 5 workers in rupees are 1800, 1900, 1700, 2000, & 2200. Find the median.

: odd no. usually divided data into 2 equal parts

Ascending order:-

1700, 1800, 1900 (↓Median), 2000, 2200

$\tilde{X} = \frac{5+1}{2} = \frac{6}{2} = 3^{rd}$ obs. = 1900 median

1700, 1800, 1900, 2000, 2200, 2500.

$\tilde{X} = \left(\frac{n}{2}\right)^{th}$ obs. - $\frac{1900+2000}{2} = 1950$

$= \left(\frac{6}{2}\right)^{th} = 3^{rd}$ obs. = 1900

Example:-(2):- The minimum Temp. in Murree for the 1st 10 days of March was.

-1°c, -2, 1, 0, 3, 3, 4, 3, 2, 6. Find median.

-2, -1, 0, 1, 2, 3, 3, 3, 4, 6

$\tilde{X} = \left(\frac{n}{2}\right)^{th}$ obs. $= \frac{10}{2} = 5^{th}$ obs. = 2

For even add central 2 values & divided by 2.

$\tilde{X} = \frac{2+3}{2} = \frac{5}{2} = 2.5 \Rightarrow$ Median

Quartiles:- ≈ Quantiles:-

Quartiles are the values of variates that divide a set of data into 4 equal parts after arranging the obs ~~into~~ in ascending order of magnitude.

OR

The 3 values which divide the data set into 4 equal parts.

Q_1: lower quartile (it cover 25% area of data set)

Q_2: Median (it cover 50% area of data set).

Q_3: Upper quartile (it cover (75%) area of data set).

Formulas for ungrouped data:- General $Q_j = \left[\frac{j(n+1)}{4}\right]^{th}$ obs

Q_1 = The value of $\left(\frac{n+1}{4}\right)^{th}$ obs.

Q_2 = The value of $\left[\frac{2(n+1)}{4}\right]^{th}$ obs.

Q_3 = The value of $\left[\frac{3(n+1)}{4}\right]^{th}$ obs.

Example:- Find the Q_1, Q_2, Q_3 from the following data.

26, 22, 14, 30, 18, 11, 35, 41, 12, 32.

11, 12, 14, 18, 22, 26, 30, 32, 35, 41.

Q_1 Q_2 Q_3

$$Q_1 = \left(\frac{n+1}{4}\right)^{th} = \frac{10+1}{4}$$

Q_1 = The value of $(2.75)^{th}$ obs.

$= 2^{nd}$ obs. $+ 0.75(3rd\ obs - 2^{nd}\ obs)$

$= 12 + 0.75(14-12)$

$= 12 + 0.75(2)$

$Q_1 = 13.5$

$Q_2 = \left[\frac{2(n+1)}{4}\right]^{th}$

$= \left[\frac{2(10+1)}{4}\right]^{th}$

= The value of 5.5^{th} obs.

$= 5^{th}$ obs $+ 0.5(6^{th}$ obs $- 5^{th}$ obs$)$

$= 22 + 0.5(26 - 22)$

$Q_2 = 24$

$Q_3 = \left[\frac{3(n+1)}{4}\right]^{th}$

$= \frac{3(10+1)}{4}$

= The value of 8.25^{th} obs.

$= 8^{th}$ obs. $+ 0.25(9^{th}$ obs $- 8^{th}$ obs$)$

$= 32 + 0.25(35 - 32)$

$= 32.75$

Percentile :-

The set of values which divide the group of data into 100 equal parts.

$1/100^{th}$ of data set.

P_x = the value of $x\frac{(n+1)^{th}}{100}$ obs

Example :-

Percentile Question.

53, 74, 89, 42, 29, 81, 68, 58, 28, 67, 54, 93, 70, 30, 55, 36, 37, 29, 51

Asending order :-

20, 28, 29, 30, 36, 37, 39, 42, 53, 54, 55, 61, 67, 68, 70, 74, 81, 89, 93.

P_{15} = The value of $15\left(\frac{n+1}{100}\right)$ obs

= 3.15 obs.

3.15 obs

= 3^{rd} obs + 0.15 (4^{th} obs - 3^{rd} obs)

= 29 + 0.15 (30 - 29)

P_{15} = 29.15

Decile:-

The value of variable that divide an ordered data set into Ten equal parts. So, that each part represents $1/10^{th}$ of Sample | pop^n.

OR

Decile are the q^{th} value that divide data set into Ten equal parts.

Formula:-

$$\frac{j(n+1)^{th}}{10} \text{ obs.}$$

Example:-

No. of defective items are:

45, 30, 36, 26, 16, 21, 33, 40, 32, 14, 10, 29, 23, 39, 17, 11, 15, 34, 19, 24, 21, 35, 42, 37.

10, 11, 14, 16, 17, 18, 19, 21, 21, 23, 24, 26, 29, 30, 32, 33, 34, 35, 36, 37, 39, 40, 42, 45.

Find 5^{th} decile.

D_5 = The value of $\frac{5(24+1)}{10}$

$= 12.5^{th}$ obs.

$= 12^{th}$ obs $+ 0.5(13^{th}$ obs $- 12^{th}$ obs$)$

$= 26 + 0.5(29 - 26)$

$= 27.5$

Similarly with D_7.

Show that:-

$Q_2 = D_5 = P_{50} = \tilde{X}$