Uniform Distribution Varience Exponential Distribution

Question:-

$$f(x) = \frac{1}{15}, \quad 0 \le x \le 15$$

a) Find expected value of writing time X mixtures.

b) Variance and Standard deviation.

Solutions

$$E(x) = \int_a^b x f(x)\,dx$$

$a = 0$, $b = 15$ $\quad f(x) = \frac{1}{15}$

$$= \int_0^{15} x \cdot \frac{1}{15}\,dx$$

$$E(x) = \frac{1}{15}\int_0^{15} x\,dx$$

$$= \frac{1}{15} \cdot \frac{x^2}{2}\Big|_0^{15} \Rightarrow \frac{1}{15}\left[\frac{225}{2}\right]$$

$$E(x) = 7.5$$

b) $\quad Var(x) = E(x^2) - [E(x)]^2$

$$= \int_0^{15} x^2 f(x)\,dx - (7.5)^2$$

$$= \int_0^{15} z^2 \cdot \frac{1}{15}\,dx - (7.5)^2$$

$$= \frac{1}{15}\left.\frac{x^3}{3}\right|_0^{15} - 56.25$$

$$= \frac{1}{15}\left(\frac{3375}{3}\right) - 56.25$$

$$Var(x) = \frac{3375}{45} - 56.25$$

$$= 75 - 56.25$$

$$Var(x) = 18.75$$

$$S.D = \sqrt{18.75}$$

$$\boxed{S.D = 4.33}$$

2) Given that $f(x) = b/x^2$ $1 < x < 3$

a) Determine the value of b that will make $f(x)$ a Pdf.

b) Find prop. of x is exactly equal to 2.

c) Find mean, variance and S.D of this Distribution.

Solution

(a) $f(x) = \frac{b}{x^2}$

$$\int_a^b f(x)\,dx = 1$$

$$\int_a^b b/x^2\,dx = 1$$

$$b\int_a^b x^{-2}\,dx = 1$$

$$b\left[\frac{x^{-1}}{-1}\right]_1^3 = 1$$

$-b[3^{-1} - 1^{-1}] = 1$

$-b[\frac{1}{3} - 1] = 1$

$-b\left[\frac{1-3}{3}\right] = 1$

$-b(-\frac{2}{3}) = 1$

$b = \frac{3}{2}$

$b = 1.5$

(b) $\int_0^2 \frac{1.5}{x^2}\,dx$

$= 1.5 \int_0^2 \frac{1}{x^2}\,dx$

$= 1.5 \int_0^2 x^{-2}\,dx$

$= 1.5 \left[\frac{x^{-1}}{-1}\right]_0^2$

$= 1.5[2^{-1} - 0]$

$= 1.5 \cdot \frac{1}{2}$

$= 0.75$

(c) Find Mean variance and Standard Deviation

(a) $E(x) = \int_a^b x f(x)\,dx$

$= \int_3^1 x \frac{b}{x^2}\,dx$

$$= \int_1^3 x \cdot \frac{1.5}{x^2} dx$$

$$= 1.5 \int_1^3 \frac{1}{x} dx$$

$$= 1.5 [\ln x]_1^3 \Rightarrow 1.5[\ln 3 - \ln 1]$$

$$= 1.5 [1.0986 - 0] \Rightarrow 1.6479$$

Mean = 1.6479

(b) Varience

$$Var(X) = E(x^2) - [E(x)]^2$$

$$= \int_1^3 x^2 f(x) dx - (1.6479)^2$$

$$= \int_1^3 x^2 \cdot \frac{1.5}{x^2} dx - 2.7156$$

$$= 1.5 \int_1^3 1 dx - 2.7156$$

$$= 1.5(3-1) - 2.7156$$

$$= 1.5(2) - 2.7156$$

$$Var(x) = 0.2844$$

$$S.D = \sqrt{\sigma^2} = \sqrt{0.2844}$$

S.D = 0.53329

Question #3

The electricity voltage is determine the prob. density func.

$$f(x) = \frac{1}{2\pi} \; ; \; 0 \le x \le 2\pi$$

b) Find expected value of x and S.D.

Solution:-

$$E(x) = \int_a^b x f(x) dx$$

$a = 0$, $b = 2\pi$, $f(x) = \frac{1}{2\pi}$

$$E(x) = \int_0^{2\pi} x \cdot \frac{1}{2\pi} dx$$

$$= \frac{1}{2\pi} \int_0^{2\pi} x dx$$

$$= \frac{1}{2\pi} \cdot \frac{x^2}{2} \Big|_0^{2\pi}$$

$$= \frac{1}{2\pi} \left[\frac{(2\pi)^2}{2} - 0 \right]$$

$$= \frac{1}{2\pi} \left[\frac{4\pi^2}{2} - 0 \right]$$

$$= \frac{1}{2\pi} (19.7192)$$

$$= \frac{19.7192}{6.28}$$

$$\boxed{E(x) = 3.14}$$

$$Var(x) = E(x^2) - [E(x)]^2$$

$$= \int_0^{2\pi} x^2 \cdot \frac{1}{2\pi}\, dx - (3.14)^2$$

$$= \frac{1}{2\pi} \int_0^{2\pi} x^2 dx - (3.14)^2$$

$$= \frac{1}{2\pi} \cdot \frac{x^3}{3}\Big|_0^{2\pi} - (3.14)^2$$

$$= \frac{1}{2\pi}\left[\frac{(2\pi)^3}{3} - 0\right] - (3.14)^2$$

$$= \frac{1}{2(3.1417)}(82.5577) - (3.14)^2$$

$$Var(x) = 13.146 - 9.8596$$

$$\boxed{Var(x) = 3.2864}$$

$$S.D = \sqrt{3.2864}$$

$$\boxed{S.D = 1.8129}$$

Question #4

The daily amount of Coffee...?

a) at most 8.8 litres

b) More from 7.4 Litres but less than 9.5 litres.

c) at least 85 litres.

$$f(x) = \frac{1}{b-a} = 1/3$$

$$P(X \leq 8.8) = \int_{7}^{8.8} 1/3 \, dx$$

$$= 1/3 \int_{7}^{8.8} 1 \, dx$$

$$= 1/3 \; x \Big|_{7}^{8.8} \Rightarrow 1/3 [8.8 - 7]$$

$$= 0.6$$

b) $P(7.4 \leq x \leq 9.5) = \int_{7.4}^{9.5} 1/3 \, dx$

$$= 1/3 \; x \Big|_{7.4}^{9.5}$$

$$= 1/3 [9.5 - 7.4]$$

$$= 0.7$$

c) $P(X \geq 8.2) = \int_{8.2}^{10} 1/3 \, dx$

$$= 1/3 \int_{8.2}^{10} 1 \, dx$$

$$= 1/3 [10 - 8.2]$$

$$= 0.6$$

Question # 5:-

Bus arrives every 10 mints at a bus Stop...?

a) what is the probability that the individual waits more than 7 min.

b) What is the probability that individual waits b/w 2 & 7 min.

Solution:

$$f(x) = \frac{1}{b-a}$$

$$f(x) = \frac{1}{10}$$

(a) $P(x \geq 7) = \int_7^{20} \frac{1}{10} dx$

$$= \frac{1}{10} \int_7^{20} 1 dx$$

$$= \frac{1}{10} x \Big|_7^{20} \Rightarrow \frac{1}{10}(20-7)$$

$$= \frac{3}{10} = 0.3$$

$$\boxed{P(x \geq 7) = 0.3}$$

b) $P(2 \leq x \leq 7) = \int_2^7 \frac{1}{10} dx$

$$= \frac{1}{10} \int_2^7 1 dx$$

$$= \frac{1}{10} [x]_2^7 \Rightarrow \frac{1}{10}[7-2]$$

$$= \frac{5}{10} = \frac{1}{2} = 0.5$$

$$\boxed{P(2 \leq x \leq 7) = 0.5}$$