Example:→ $L(s) = \dfrac{s+5}{s(s+2)}$

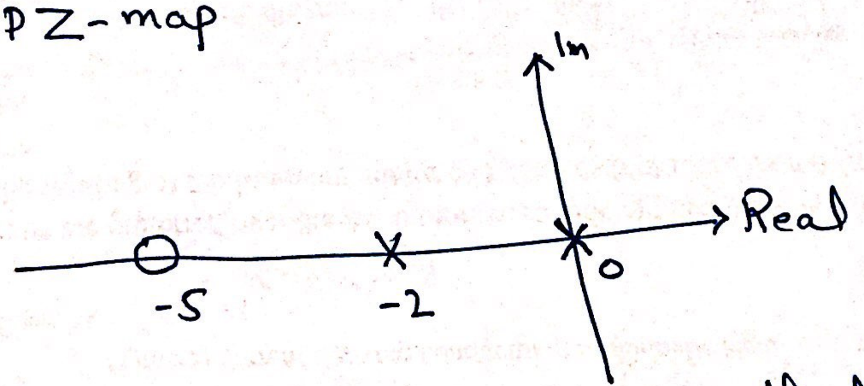
$n = 2$, $p_1 = 0$, $p_2 = -2$

$m = 1$, $z_1 = -5$

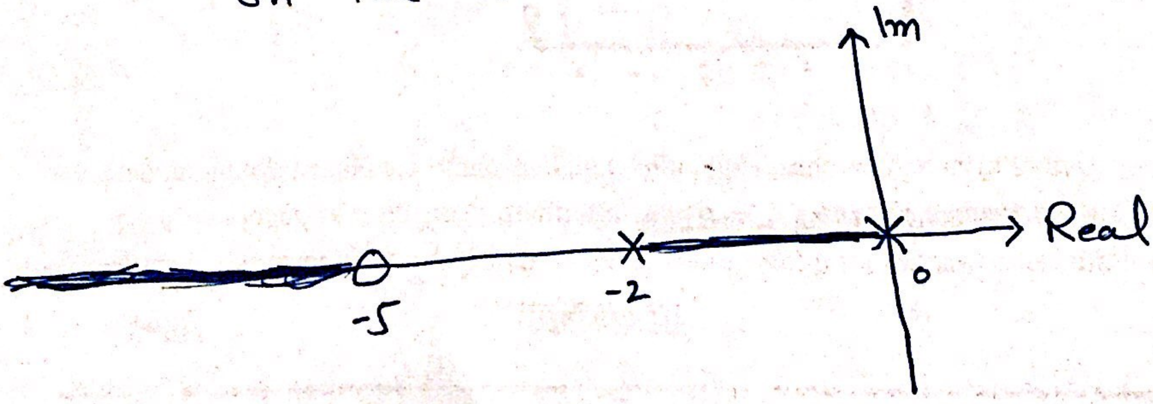
no. of branches of the R|L = $n = 2$

no. of assymptotes = No. of branches that go to infinity = $n - m = 1$

① PZ-map



② Identify part of Real axis that is on the Locus.

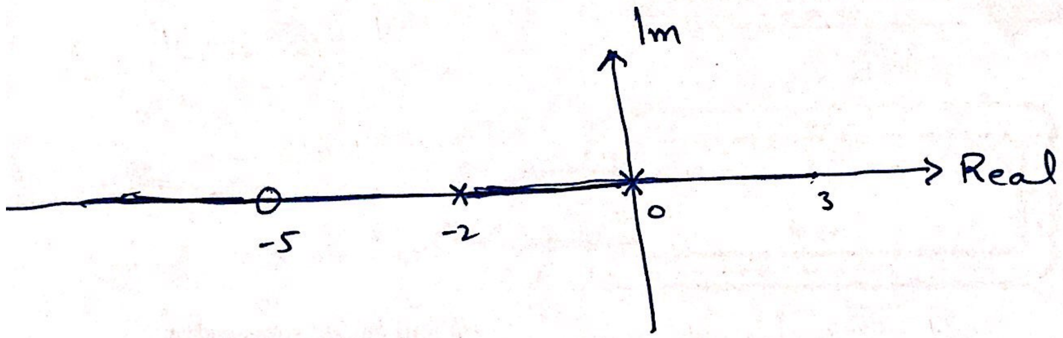


3 Centre & angle of assymptotes

$$\alpha = \frac{\Sigma P_i - \Sigma z_i}{n-m} = \frac{0-2+5}{2-1} = 3$$

$$\phi_i = \frac{180^\circ + 360^\circ(i-1)}{n-m}$$

$$\phi_1 = \frac{180^\circ}{2-1} = 180^\circ$$



4 Breakaway

$$L(s) = \frac{s+5}{s^2+2s} = \frac{a(s)}{b(s)}$$

$$b(s)\frac{d\,a(s)}{ds} - a(s)\frac{d\,b(s)}{ds} = 0$$

$$(s^2+2s)(1) - (s+5)(2s+2) = 0$$

$$s^2 + \cancel{2s} - 2s^2 - \cancel{2s} - 10s - 10 = 0$$

$$-s^2 - 10s - 10 = 0$$

$$s^2 + 10s + 10 = 0$$

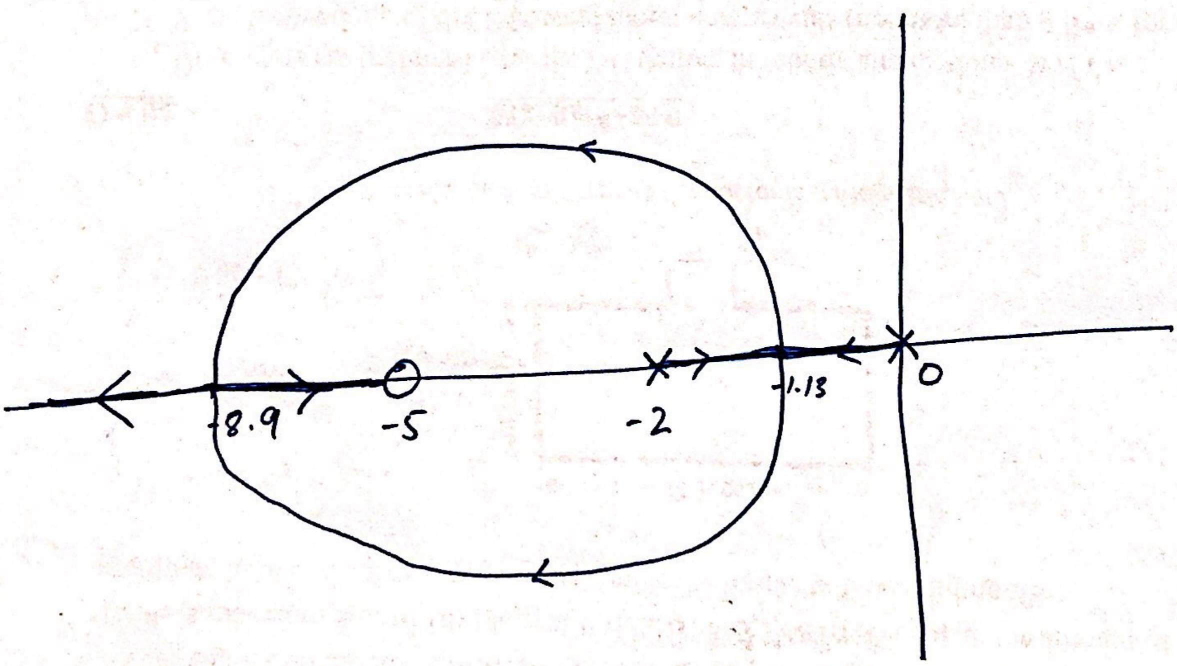
$$s_1 = -1.13 \; , \; s_2 = -8.9$$

Both of the points are valid
(as both are on the locus)

[Remember: Breakaway can't be imaginary
→ it will always be real.]

⇒ In such case one is "breakaway" pt. and
the other is "breakin" pt.

Breakin Point :- The point on real
axis where multiple branches of the
root locus enters the real axis.



Example:→ $L(s) = \dfrac{1}{s^2+2s+2}$

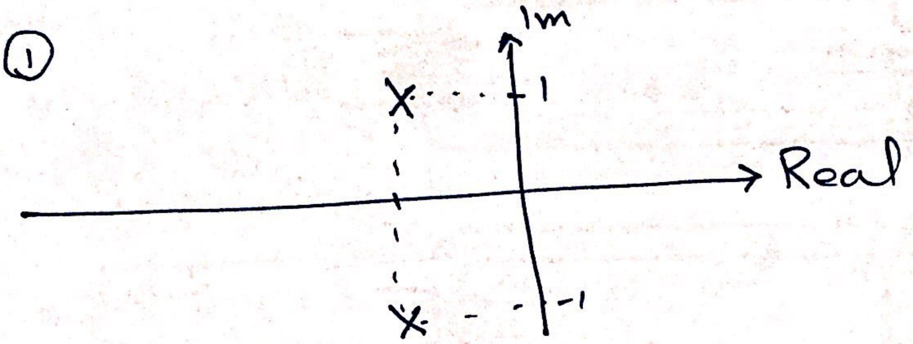
$n = 2, \quad m = 0$

$P_1 = -1+j, \quad P_2 = -1-j$

no. of branches $= n = 2$

no. of assymptotes $= n-m = 2$

1



2 No part of real axis is on the locus.

3 $\alpha = \dfrac{\sum P_i - \sum z_i}{n-m} = \dfrac{-1+j-1-j-0}{2}$

Centre of assymptotes

$= \dfrac{-2}{2} = -1$

Angle of assymptotes $\phi_i = \dfrac{180^\circ + 360^\circ(i-1)}{n-m}$

$i = 1$

$\phi_1 = 90^\circ$

$i = 2$

$\phi_2 = -90^\circ$

④ ~~Angle of~~ Break away

$$L(s) = \frac{1}{s^2+2s+2} = \frac{a(s)}{b(s)}$$

$$b(s)\frac{da(s)}{ds} - a(s)\frac{db(s)}{ds} = 0$$

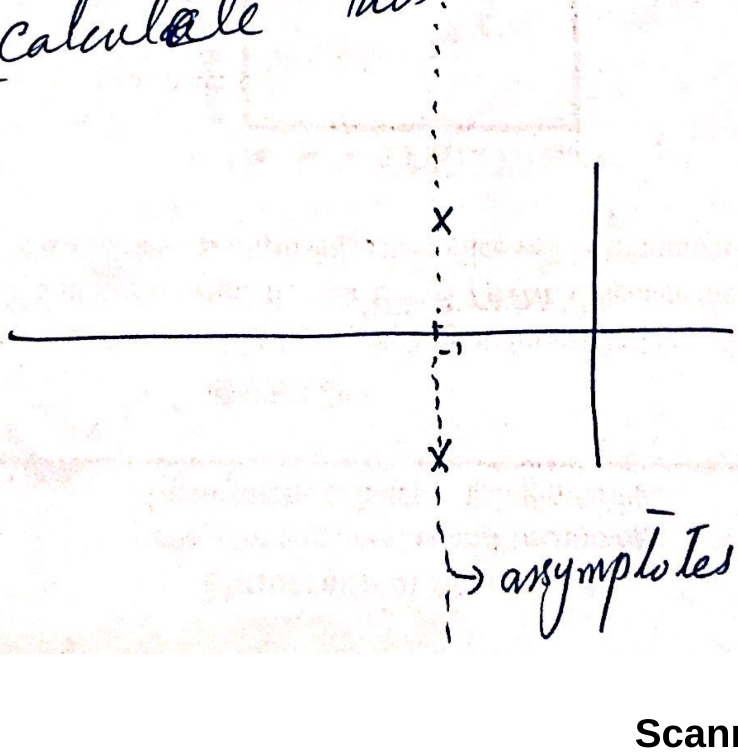
$$s^2+2s+2\,(0) - 1\cdot(2s+2) = 0$$

$$2s = -2$$

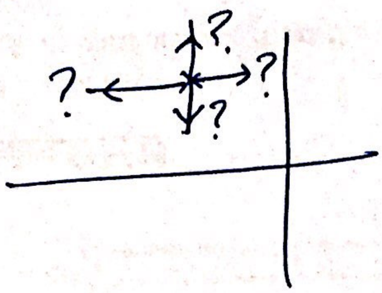
$$\boxed{s = -1}$$ → This is not valid

∵ This is not on the locus.

[whenever there are no "real" poles or zeros then we don't need to calculate this.]



Now "HOW will the branches depart from the poles?"



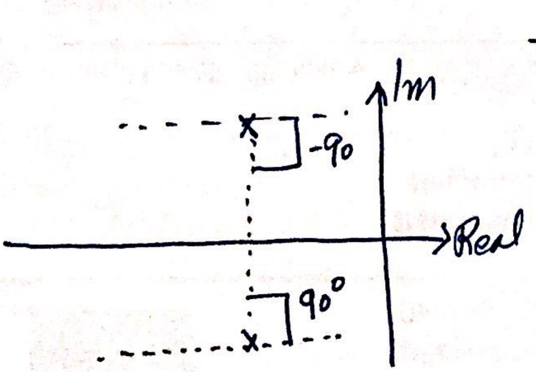
5 Angle of departure

→ Necessary only for complex poles.

→ denoted by ϕ_d

Formula:→ ϕ_d for a complex pole is given by

$$-\phi_d - \underbrace{\sum_{i=1}^{n} p_i}_{\text{Angles from all the poles}} + \underbrace{\sum_{i=1}^{m} z_i}_{\text{Angles from all the zeros.}} = -180^0$$



In the given example only two poles

→ ϕ_d for pole at $-1+j$

$$-\phi_d - 90^\circ = -180^\circ$$

$$-\phi_d = -90^\circ \Rightarrow \phi_d = 90^\circ$$

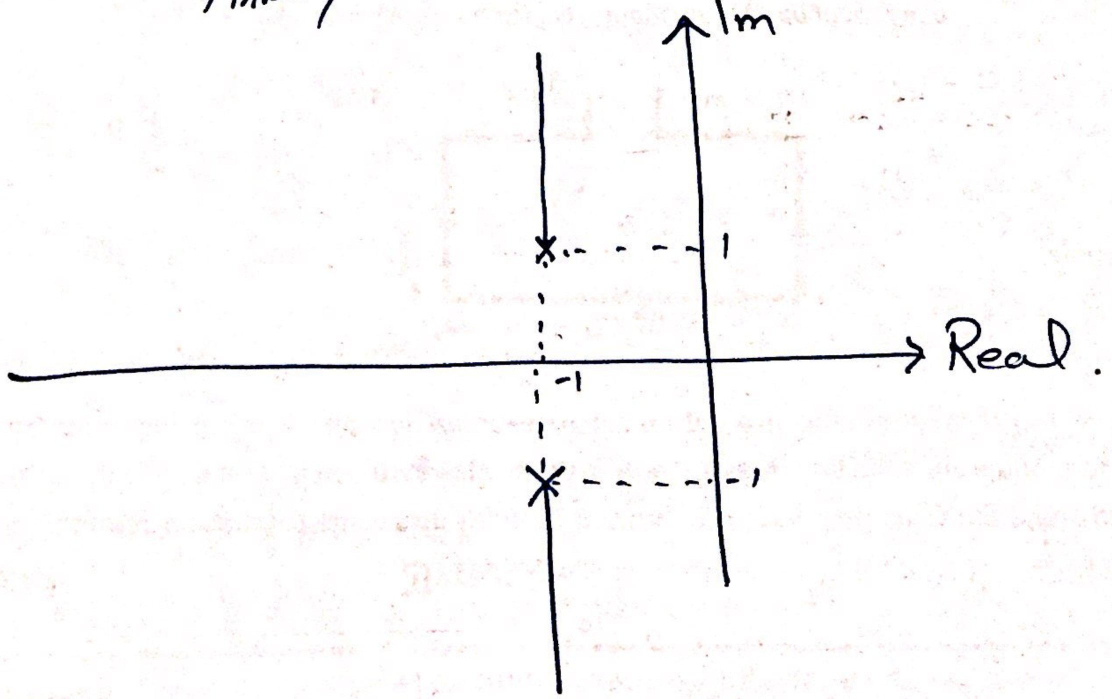
→ ϕ_d for pole at $-1-j$

$$-\phi_d - (-90^\circ) = -180^\circ$$

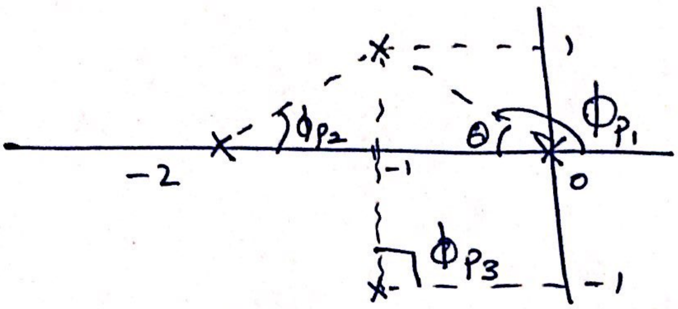
$$-\phi_d = -180^\circ - 90^\circ = -270^\circ$$

$$\phi_d = 270^\circ = -90^\circ$$

Finally the R/L is



An other example of ϕ_d



ϕ_d for pole at $-1+j$?

$$-\phi_d - \sum_{i=1}^{n} \angle p_i + \sum \angle z_i = -180^\circ$$

$\sum \angle z_i = 0$ as no zeros.

$$\sum \angle p_i = \phi_{p_1} + \phi_{p_2} + \phi_{p_3}$$

$$\phi_{p_1} = 180^\circ - \theta = 180^\circ - \tan^{-1}\left(\tfrac{1}{1}\right) = 135^\circ$$

$$\phi_{p_2} = \tan^{-1}\left(\tfrac{1}{1}\right) = 45^\circ$$

$$\phi_{p_3} = 90^\circ$$

So

$$-\phi_d - 135^\circ - 45^\circ - 90^\circ = -180^\circ$$

$$-\phi_d = 90^\circ \Rightarrow \phi_d = -90^\circ$$

Remember that for conjugate pair of poles if one angle is say ϕ_d then the ϕ_d at its conjugate pair will be $-\phi_d$ i.e for pole at $-1-j$ we don't need to calculate ϕ_d its $+90^\circ$