```
        temp*=((xp-table[i][0] /  (table[j][0]-[i][0]));

      ans+=temp*table[j][1];
    }
  cout<<"\nA N S W E R  =          : "<<ans;      //output
}
```

Computer Output

How Many Values of X : 4

Enter the Values of x and f(x)

x	f(x)
1	4
3	7
4	8
6	11

Enter The Value of X : 5

A N S W E R : 9.2

3.6 ITERATIVE INTERPOLATION METHOD

Like Lagrange's method, this formula is also more suitable for computer application and its use is also not limited to only uniformly spaced data. The iterative interpolation process is based on the repeated application of simple (linear) interpolation method. This method is due to **Aitken.**

Consider the following data points (equally or unequally spaced):

x	x_0	x_1	x_2	x_3	...	x_n
f(x)	f_0	f_1	f_2	f_3	...	f_n

In order to estimate the value of the function f corresponding to any value of x, we proceed as follows:

Let $f_0 = f(x_0)$

$f_1 = f(x_1)$

$\vdots$

$$f_k = f(x_k)$$

$$\vdots$$

$$f_n = f(x_n)$$

also let $f(x \mid x_0, x_1, \ldots, x_n)$ denote the unique polynomial of degree n coinciding with f(x) at $x_0, x_1, \ldots, x_n$.

Hence, $f(x \mid x_0) = f(x_0)$

$$f(x \mid x_1) = f(x_1)$$

$$\vdots$$

$$f(x \mid x_n) = f(x_n)$$

Also,
$$f(x \mid x_0, x_1) = \frac{1}{(x_1 - x_0)}\begin{vmatrix} x - x_0 & f(x \mid x_0) \\ x - x_1 & f(x \mid x_1) \end{vmatrix}$$

$$= \frac{1}{x_1 - x_0}\begin{vmatrix} x - x_0 & f_0 \\ x - x_1 & f_1 \end{vmatrix}$$

$$= \frac{1}{(x_1 - x_0)}\{(x - x_0)f_1 - (x - x_1)f_0\}$$

$$f(x \mid x_0, x_2) = \frac{1}{x_2 - x_0}\begin{vmatrix} x - x_0 & f_0 \\ x - x_2 & f_2 \end{vmatrix}$$

$$= \frac{1}{(x_2 - x_0)}\{(x - x_0)f_2 - (x - x_2)f_0\}, \text{ etc.}$$

Similarly, $f(x \mid x_0, x_1, x_2) = \frac{1}{(x_2 - x_1)}\begin{vmatrix} x - x_1 & f(x \mid x_0, x_1) \\ x - x_2 & f(x \mid x_0, x_2) \end{vmatrix}$

$$f(x \mid x_0, x_1, x_3) = \frac{1}{(x_3 - x_1)}\begin{vmatrix} x - x_1 & f(x \mid x_0, x_1) \\ x - x_3 & f(x \mid x_0, x_3) \end{vmatrix}$$

and $f(x \mid x_0, x_1, x_4) = \frac{1}{(x_4 - x_1)}\begin{vmatrix} x - x_1 & f(x \mid x_0, x_1) \\ x - x_4 & f(x \mid x_0, x_4) \end{vmatrix}$

denote polynomials of degree ≤ 2 that pass through the four points (x_0, f_0), (x_1, f_1), (x_2, f_2); (x_0, f_0), (x_1, f_1), (x_3, f_3); and (x_0, f_0), (x_1, f_1), (x_4, f_4), respectively,

whereas $$f(x \mid x_0, x_1, x_2, x_3) = \frac{1}{x_3 - x_2}\begin{vmatrix} x - x_2 & f(x \mid x_0, x_1, x_2) \\ x - x_3 & f(x \mid x_0, x_1, x_3) \end{vmatrix}$$

denotes polynomial of degree ≤ 3 and so on.

Continuing the above process, we can develop the interpolating polynomials to any degree we want:

$$f(x \mid x_0, x_1, x_2, x_3, x_4) = \frac{1}{(x_4 - x_3)}\begin{vmatrix} x - x_3 & f(x \mid x_0, x_1, x_2, x_3) \\ x - x_4 & f(x \mid x_0, x_1, x_2, x_4) \end{vmatrix}$$

$$f(x \mid x_0, x_1, x_2, x_3, x_5) = \frac{1}{(x_5 - x_3)}\begin{vmatrix} x - x_3 & f(x \mid x_0, x_1, x_2, x_3) \\ x - x_5 & f(x \mid x_0, x_1, x_2, x_5) \end{vmatrix}$$

The following table illustrates the arrangement of the work needed to construct $f(x \mid x_0, x_1, \ldots, x_n)$:

x_0	$x - x_0$	$f(x \mid x_0)$				
x_1	$x - x_1$	$f(x \mid x_1)$	$f(x \mid x_0, x_1)$			
x_2	$x - x_2$	$f(x \mid x_2)$	$f(x \mid x_0, x_2)$	$f(x \mid x_0, x_1, x_2)$		
x_3	$x - x_3$	$f(x \mid x_3)$	$f(x \mid x_0, x_3)$	$f(x \mid x_0, x_1, x_3)$	$f(x \mid x_0, x_1, x_2, x_3)$	
x_4	$x - x_4$	$f(x \mid x_4)$	$f(x \mid x_0, x_4)$	$f(x \mid x_0, x_1, x_4)$	$f(x \mid x_0, x_1, x_2, x_4)$	$\cdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	

The tabular values are generated row-wise (or column-wise). Since the current value are generated from the previous values that is why this method is often called the **iterative interpolation method** and also named as **Neville's formula**. The rightmost value in the table is the required value of interpolation.

Example 8 (a) Using Aitken's iterative scheme, find the value of log 4.5 from the following values:

x	4.0	4.2	4.4	4.6
f(x)	0.60206	0.62325	0.64345	0.66276

(b) Write a computer program to implement Aitken's method.

Solution (a) $x = 4.5$

Aitken's table is as follows:

$x_0 = 4.0$	$x - x_0 = .5$	0.60206			
$x_1 = 4.2$	$x - x_1 = .3$	0.62325	0.65504		
$x_2 = 4.4$	$x - x_2 = .1$	0.64345	0.65380	0.65318	
$x_3 = 4.6$	$x - x_3 = -.1$	0.66276	0.65264	0.65324	0.65321

$$f(x \mid x_0, x_1) = \frac{1}{(x_1 - x_0)}\begin{vmatrix} x - x_0 & f(x \mid x_0) \\ x - x_1 & f(x \mid x_1) \end{vmatrix}$$

$$= \frac{1}{(4.2 - 4.0)}\begin{vmatrix} .5 & .60206 \\ .3 & .62325 \end{vmatrix}$$

$$= \frac{(.5 \times .62325 - .3 \times .60206)}{0.2}$$

$$= \frac{(.311625 - .180618)}{0.2} = 0.65504$$

$$f(x \mid x_0, x_2) = \frac{1}{(x_2 - x_0)}\begin{vmatrix} x - x_0 & f(x \mid x_0) \\ x - x_2 & f(x \mid x_2) \end{vmatrix}$$

$$= \frac{1}{(4.4 - 4.0)}\begin{vmatrix} .5 & .60206 \\ .1 & .64345 \end{vmatrix}$$

$$= \frac{(.321725 - .060206)}{0.4} = 0.65380$$

$$f(x \mid x_0, x_3) = \frac{1}{(x_3 - x_0)}\begin{vmatrix} x - x_0 & f(x \mid x_0) \\ x - x_3 & f(x \mid x_3) \end{vmatrix}$$

$$= \frac{1}{(4.6 - 4.0)}\begin{vmatrix} .5 & .60206 \\ -.1 & .66276 \end{vmatrix}$$

$$= \frac{(.5 \times .66276 + .1 \times .60206)}{0.6}$$

$$= \frac{(.33138 - .060206)}{0.6} = 0.65264$$

$$f(x \mid x_0, x_1, x_2) = \frac{1}{(x_2 - x_1)} \begin{vmatrix} x - x_1 & f(x \mid x_0, x_1) \\ x - x_2 & f(x \mid x_0, x_2) \end{vmatrix}$$

$$= \frac{1}{(4.4 - 4.2)} \begin{vmatrix} .3 & .65504 \\ -.1 & .65380 \end{vmatrix}$$

$$= \frac{(.3 \times .65380 - .1 \times .65504)}{0.2}$$

$$= \frac{(.19614 - .065504)}{0.2} = 0.65318$$

$$f(x \mid x_0, x_1, x_3) = \frac{1}{(x_3 - x_1)} \begin{vmatrix} x - x_1 & f(x \mid x_0, x_1) \\ x - x_3 & f(x \mid x_0, x_3) \end{vmatrix}$$

$$= \frac{1}{(0.4)} \begin{vmatrix} .3 & .65504 \\ -.1 & .65264 \end{vmatrix}$$

$$= \frac{(.195792 - .065504)}{0.4} = 0.65324$$

$$f(x \mid x_0, x_1, x_2, x_3) = \frac{1}{(x_3 - x_2)} \begin{vmatrix} x - x_2 & f(x \mid x_0, x_1, x_2) \\ x - x_3 & f(x \mid x_0, x_1, x_3) \end{vmatrix}$$

$$= \frac{1}{(0.2)} \begin{vmatrix} .1 & .65318 \\ -.1 & .65324 \end{vmatrix}$$

$$= \frac{(.195324 - .065318)}{0.2} = 0.65321$$

The rightmost entry in each row in the table gives,

$f(4.5 \mid x_0, x_1) = 0.65504$

$f(4.5 \mid x_0, x_1, x_2) = 0.65318$

$f(4.5 \mid x_0, x_1, x_2, x_3) = 0.65321$

It is seen that log 4.5 = 0.65321, which is the anticipated answer.

(b) **Program No. 4** **Aitken's Method**

```
# include<conio.h>
# include<iostream.h>
# include<complex.h>
# include<stdio.h>

void main ( )
  {
    clrscr( );
    float x[10],f[10],r[10][10],diff[10],xp;
    int i,j,l,m,n,p,k,y,z;
    double term1,term2,term3;
    cout<<"n\t\t\t Aitken Method\n\n";
    cout<<"Enter the number of  X data   :     ";
    cin>>n;
    cout<<"Enter value of  xp   :     ";
    cin>>xp;
    for(i=0;i<n;i++)
      {
        cout<<"Enter value of  X["<<i<<"]\t";
        cin>>x[i];
        diff[i] = xp – x[i];
      }

  cout<<"\n\n\nGiven the values of function\n\n\n";
  for(i=0;i<=n-1;i++)
  {
     cout<<"Enter value of  F("<<i<<")\t";
     cin>>f[i];
  }
  for(i=0;i<=n-1;i++)
     r[i][0] = f[i];
  for(i=0;i<=n-1;i++)
  {
     for(j=0;j<n-1;j++)
     {
       term1 = diff[i]*r[j+1][i];
       term2 = diff[j+1]*r[i][i];
       term3 = x[j+1] – x[i];
```

```
        if(term3 !=0)
                r[j+1][i+1] = (term1 – term2)/term3;
    }
}
y = 13;
clscr( );
gotoxy(3,5);
cout<<"                    Implementation of Aitken's Method\n"
for(i=0;i<=n-1;i++)     //loop to print the value of x differences
  {
    y = i + 13;
    gotoxy(5,y);

    cout<<setiosflags(ios::fixed)<<setiosflags(ios::showpoint)<<setprecision(5)<<x[1];
    gotoxy(15,y);
    cout<<"\t"<<setiosflags(ios::fixed)<<setprecision(5)<<diff[i];
  }
  p=0;
  m=25;
  k=13;
  for(i=0;i<=n-1;i++)
    {
      k = i + 13;
      for(j=p;j<=n-1;j++)
        {
          gotoxy(m,k);
          cout<<setiosflags(ios::fixed)<<setprecision(5)<<setw(10)<<r[j][i];
          k = k + 1;
        }
      p = p + 1;
      m = m + 11;
    }
    z = 0;

    for(y=0;y<n-1;y++)
          z = z + 1;
    cout<<"\n\n\n\tAtXp="<<setw(15)<<setiosflags(ios::fixed)<<setprecision(3)<<xp"
    function value is\t"<<setiosflags(ios::fixed)<<setprecision(5)<<setw(15)<<r[y][z];
    getch( );
}
```

Computer Output

```
        Aitken Method
Enter the number of X data  : 4
Enter value of xp : 4.5
Enter value of X[0]      4.0
Enter value of X[1]      4.2
Enter value of X[2]      4.4
Enter value of X[3]      4.6

Given the values of function

Enter value of F[0]      .60206
Enter value of F[1]      .62325
Enter value of F[2]      .64345
Enter value of F[3]      .66276

              Implementation of Aitken's Method
   4.00000     0.50000     0.60206
   4.20000     0.30000     0.62325    0.65504
   4.40000     0.10000     0.64345    0.65380    0.65318
   4.60000    -0.10000     0.66276    0.65264    0.65324     0.65321

        At Xp = 4.500          Function value is  0.65321
```

3.7 ERROR ESTIMATION IN INTERPOLATION

So far, we have studied several formulas for interpolation. The basic principle in all these formulas is the approximation of a polynomial so that this polynomial passes through the set of points in a given table.

The error in an interpolation process is introduced by several sources:

(a) The truncation error due to terminating the series at the term in, say, the nth differences.

(b) The round-off errors in the function values and resulting errors in the differences causing oscillation in the differences.

(c) The round-off errors in the individual terms of the formula and their sum.

(d) Inaccuracy, usually due to rounding-off, in the given value of p.