# FUNDAMENTALS OF FLUID MECHANICS

## Chapter 9 External Flow Past Bodies

# MAIN TOPICS

- General Characteristics of External Flow
- Boundary Layer Characteristics
- Drag
- Lift

# Introduction

- Objects are completely surrounded by the fluid and the flows are termed external flows.
- Examples include the flow of air around airplane, automobiles, and falling snowflakes, or the flow of water around submarines and fish.
- External flows involving air are often termed aerodynamics in response to the important external flows produced when an object such as an airplane flies through the atmosphere.

# Application

- Design of cars and trucks – to decrease the fuel consumption and improve the handling characteristics.
- Improve ships, whether they are surface vessels (surrounded by air and water) or submersible vessels.
- Design of building – consider the various wind effects

# Approaches to External Flows 1/2

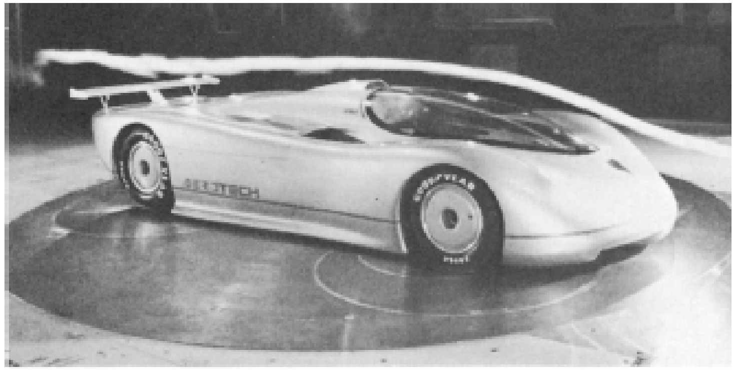
- Two approaches are used to obtain information of external flows:
  - ⇨ Theoretical (analytical and numerical) approaches: Because of the complexities of the governing equations and the complexities of the geometry of the objects involved, the amount of information obtained from purely theoretical methods is limited. With current and anticipated advancements in the area of computational fluid mechanics, computer predication of forces and complicated flow patterns will become more readily available.
  - ⇨ Experimental approaches: Much information is obtained.

# Approaches to External Flows 2/2

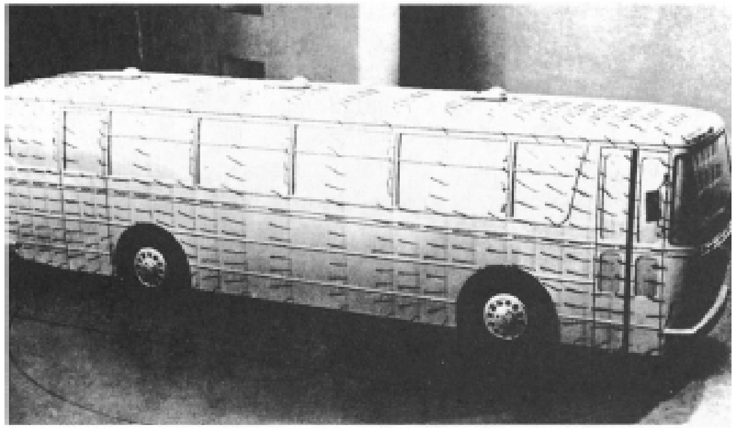
## Flow visualization

(*a*) Flow past a full-sized streamlined vehicle in the GM aerodynamics laboratory wind tunnel, and 18-ft by 34-ft test section facility driven by a 4000-hp, 43-ft-diameter fan.

(*b*) Surface flow on a model vehicle as indicated by tufts attached to the surface.



(*a*)



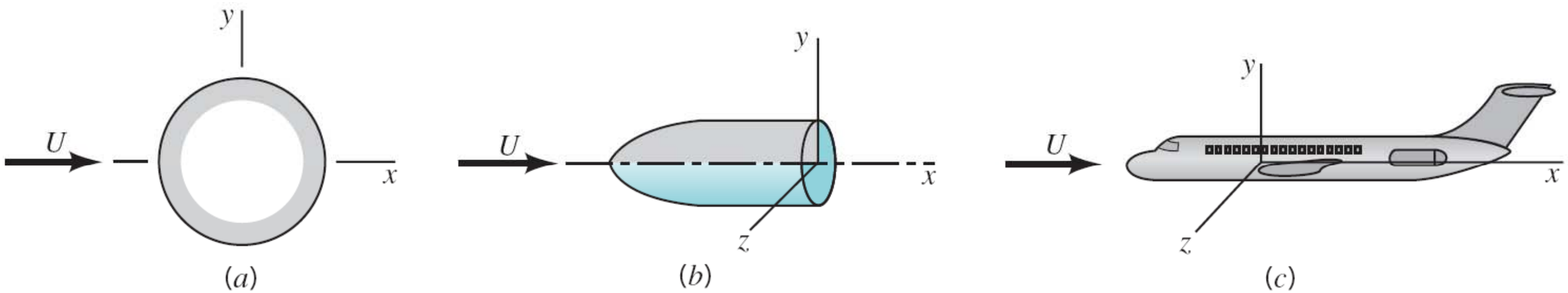
(*b*)

# General Characteristics

- A body immersed in a moving fluid experiences a resultant force due to the interacting between the body and the fluid surrounding:
  - ⇨ The body is stationary and the fluid flows past the body with velocity U.
  - ⇨ The fluid far from the body is stationary and the body moves through the fluid with velocity U.
- For a given-shaped object, the characteristics of the flow depend very strongly on various parameters such as size, orientation, speed, and fluid properties.

# Categories of Bodies

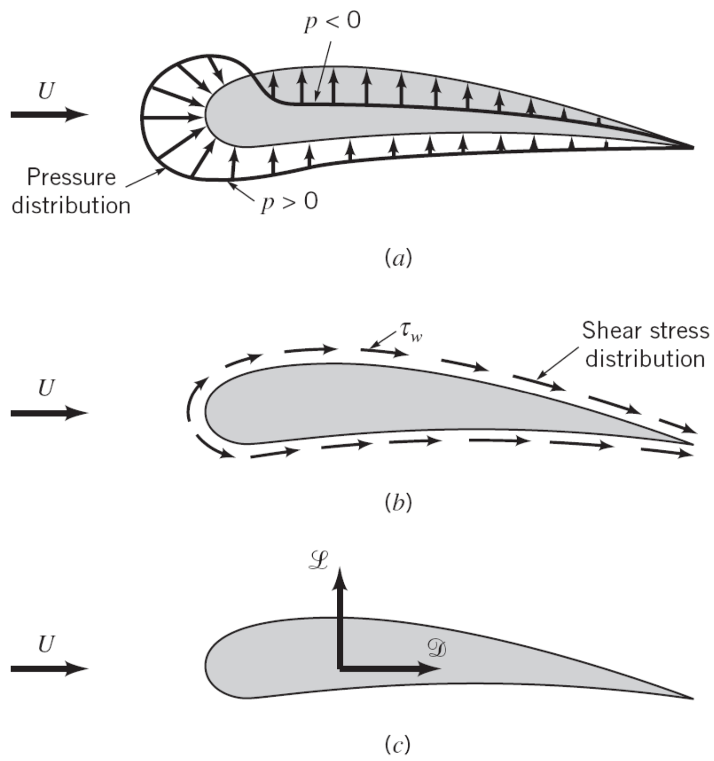
- The structure of an external flow and the ease with which the flow can be described and analyzed often depend on the nature of the body in the flow.
- Three general categories of bodies include (a) two-dimensional objects, (b) axisymmetric bodies, and (c) three-dimensional bodies.




- Another classification of body shape can be made depending on whether the body is **streamlined or blunt**.

# Lift and Drag Concepts 1/3

- The interaction between the body and the fluid:
  - ⇨ Stresses-wall <u>shear stresses</u>, $\tau_w$, due to viscous effects.
  - ⇨ Normal stresses, due to the <u>pressure</u> p.
- Both $\tau_w$ and p vary in magnitude and direction along the surface.
- The detailed distribution of $\tau_w$ and p is **<u>difficult</u>** to obtain.
- However, only the integrated or resultant effects of these distributions are needed.

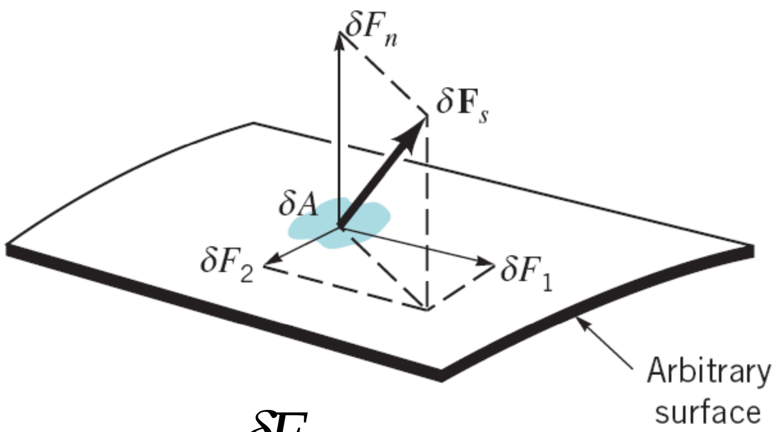


# Forces Acting on Element 1/2

❖ The forces acting on a fluid element may be classified as body forces and surface forces; surface forces include **normal forces and tangential** (shear) **forces**.

$$\delta\vec{F} = \delta\vec{F}_S + \delta\vec{F}_B$$
$$= \delta F_{sx}\vec{i} + \delta F_{sy}\vec{j} + \delta F_{sz}\vec{k}$$
$$+ \delta F_{bx}\vec{i} + \delta F_{by}\vec{j} + \delta F_{bz}\vec{k}$$

Surface forces acting on a fluid element can be described in terms of normal and shear stresses.



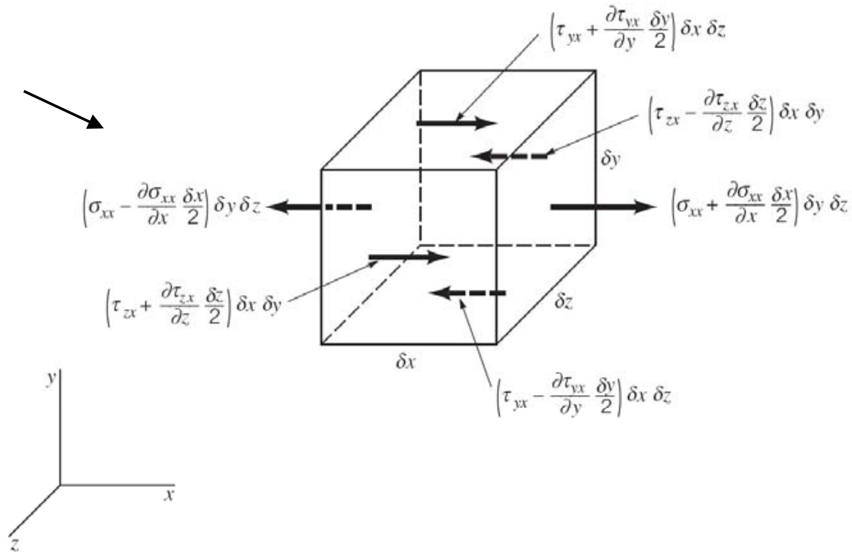

$$\sigma_n = \lim_{\delta t \to 0} \frac{\delta F_n}{\delta A} \quad \tau_1 = \lim_{\delta t \to 0} \frac{\delta F_1}{\delta A} \quad \tau_2 = \lim_{\delta t \to 0} \frac{\delta F_2}{\delta A}$$

# Forces Acting on Element 2/2

$$\delta F_{sx} = \left( \frac{\partial \sigma_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} + \frac{\partial \tau_{zx}}{\partial z} \right) \delta x \delta y \delta z$$

$$\delta F_{sy} = \left( \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \sigma_{yy}}{\partial y} + \frac{\partial \tau_{zy}}{\partial z} \right) \delta x \delta y \delta z$$

$$\delta F_{sz} = \left( \frac{\partial \tau_{xz}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \frac{\partial \sigma_{zz}}{\partial z} \right) \delta x \delta y \delta z$$



$$\delta F_{bx} = \rho g_x \delta x \delta y \delta z$$

$$\delta F_{by} = \rho g_y \delta x \delta y \delta z$$

$$\delta F_{bz} = \rho g_z \delta x \delta y \delta z$$

$$\sigma_{xx} = -p + \tau_{xx}$$

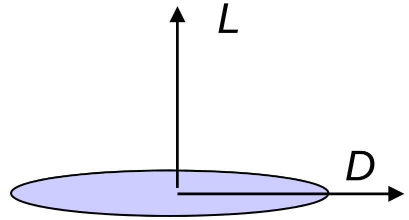
$$\sigma_{yy} = -p + \tau_{yy}$$

$$\sigma_{zz} = -p + \tau_{zz}$$

→ **Equation of Motion**

# Lift and Drag Concepts 2/3

- The resultant force on the object in the downstream direction (by the fluid) is termed the **DRAG**, and the resultant force normal to the upstream velocity is termed the **LIFT**, both of which are surface forces.




$$Drag = D = \underline{e}_x \cdot \int \underline{\sigma}_{(\underline{n})} dA = \underline{e}_x \cdot \int (-p+\tau_{rr})\underline{e}_r + \tau_{r\theta}\underline{e}_\theta dA$$

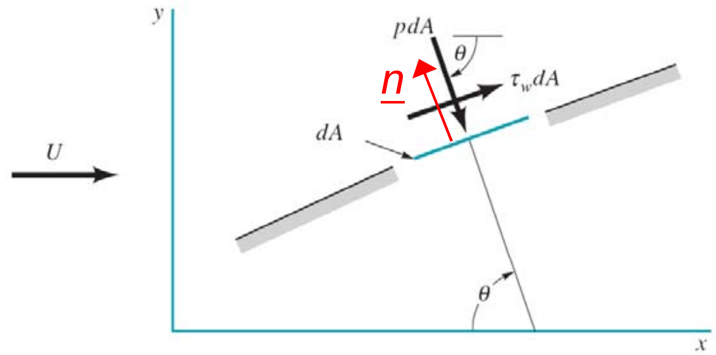
$$= \int (-p+\tau_{rr})(-\cos\theta) + \tau_{r\theta}\sin\theta dA$$

$$Lift = L = \underline{e}_y \cdot \int \underline{\sigma}_{(\underline{n})} dA = \underline{e}_y \cdot \int (-p+\tau_{rr})\underline{e}_r + \tau_{r\theta}\underline{e}_\theta dA$$

$$= \int (-p+\tau_{rr})(\sin\theta) + \tau_{r\theta}\cos\theta dA$$

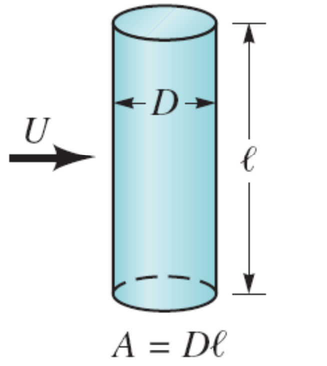
$$\underline{e}_x \cdot \underline{e}_r = \cos(\pi-\theta) = -\cos\theta, \underline{e}_x \cdot \underline{e}_\theta = \cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta$$

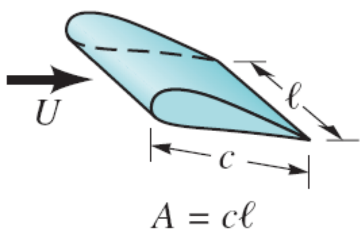
$$\underline{e}_y \cdot \underline{e}_r = \cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta, \underline{e}_y \cdot \underline{e}_r = \cos\theta$$



# Lift and Drag Concepts 3/3

- Without detailed information concerning the shear stress and pressure distributions on a body, the drag and the lift are difficult to obtain by integration.
- A widely used alternative is to define dimensionless lift and drag coefficients and determine their approximate values by means of either a simplified analysis, some numerical technique, or an appropriate experiment.




Lift coefficient $C_L = \dfrac{L}{\frac{1}{2}\rho U^2 A}$ Drag coefficient $C_D = \dfrac{D}{\frac{1}{2}\rho U^2 A}$

*Characteristic pressure*

# Characteristics of Flow Past an Object 1/2

- For a given-shaped object, the characteristics of the flow depend very strongly on various parameters such as size, orientation, speed, and fluid properties.
- According to dimensional analysis arguments, the character of flow should depend on the various dimensionless parameters. For typical external flows the most important of these parameters are the Reynolds number, the Mach number, and for the flow with a free surface, the Froude number.

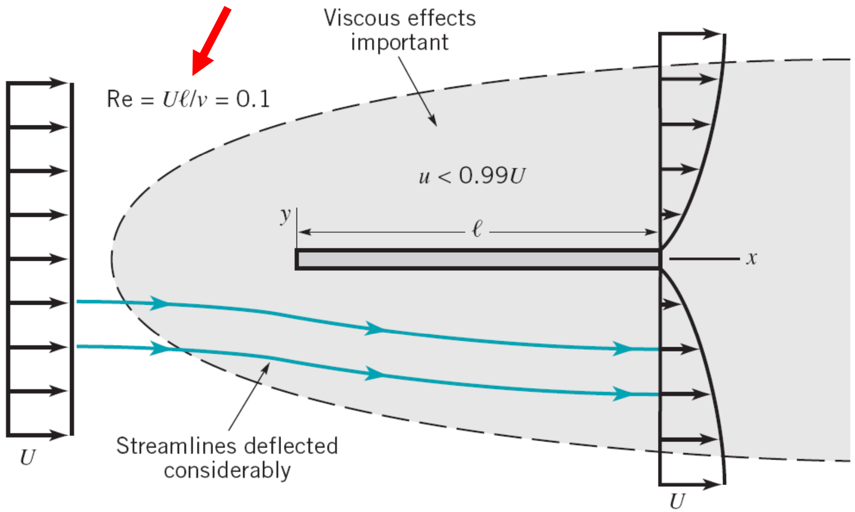
# Characteristics of Flow Past an Object 2/2

- For the present, we consider how the external flow and its associated lift and drag vary as a function of Reynolds number.
- For most external flows, the characteristic length of objects are on the order of 0.10m~10m. Typical upstream velocities are on the order of 0.01m/s~100m/s. The resulting Reynolds number range is approximately $10 \sim 10^9$.
  - ⇨ Re>100. The flows are dominated by inertial effects.
  - ⇨ Re<1. The flows are dominated by viscous effects.

# Flow Past an Flat Plate 1/4

- With Re ≒ 0.1, the viscous effects are relatively strong and the plate affects the uniform upstream flow far ahead, above, below, and behind the plate. In low Reynolds number flows, the viscous effects are felt far from the object in all directions.

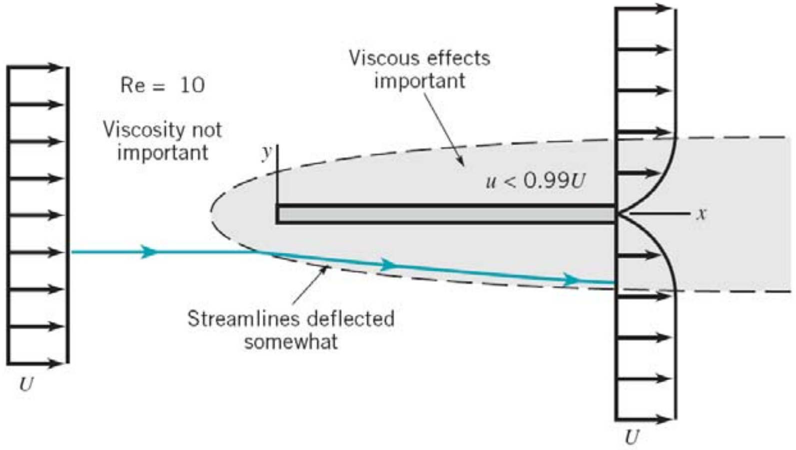
**Within the boundary layer, the viscous force is comparable to the inertial forces.**



(a)

# Flow Past an Flat Plate 2/4

❖ With Re = 10, the region in which viscous effects are important become smaller in all directions except downstream. One does not need to travel very far ahead, above, or below the plate to reach areas in which the viscous effects of the plate are not felt. The streamlines are displaced from their original uniform upstream conditions, but the displacement is not as great as for the Re=0.1 situation.



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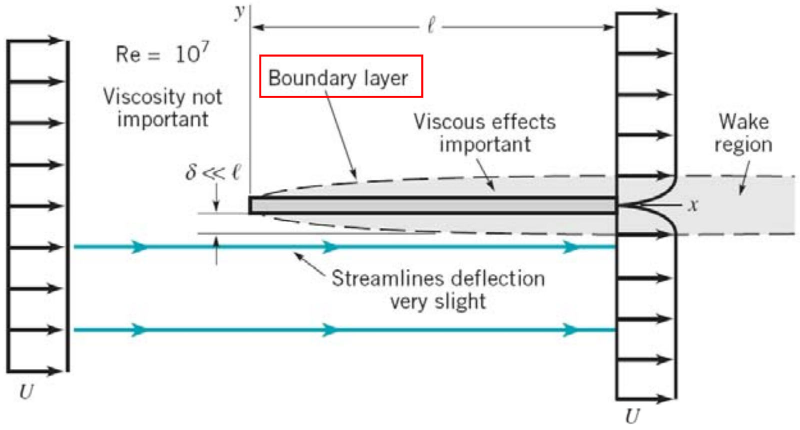
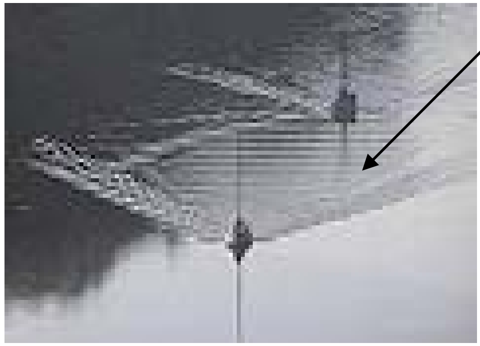


(c)

# Flow Past an Flat Plate 3/4

- With Re = $10^7$ , the flow is dominated by inertial effects and the **viscous effects are negligible everywhere "except in a region very close to the plate and in the relatively thin *wake region* behind the plate."**



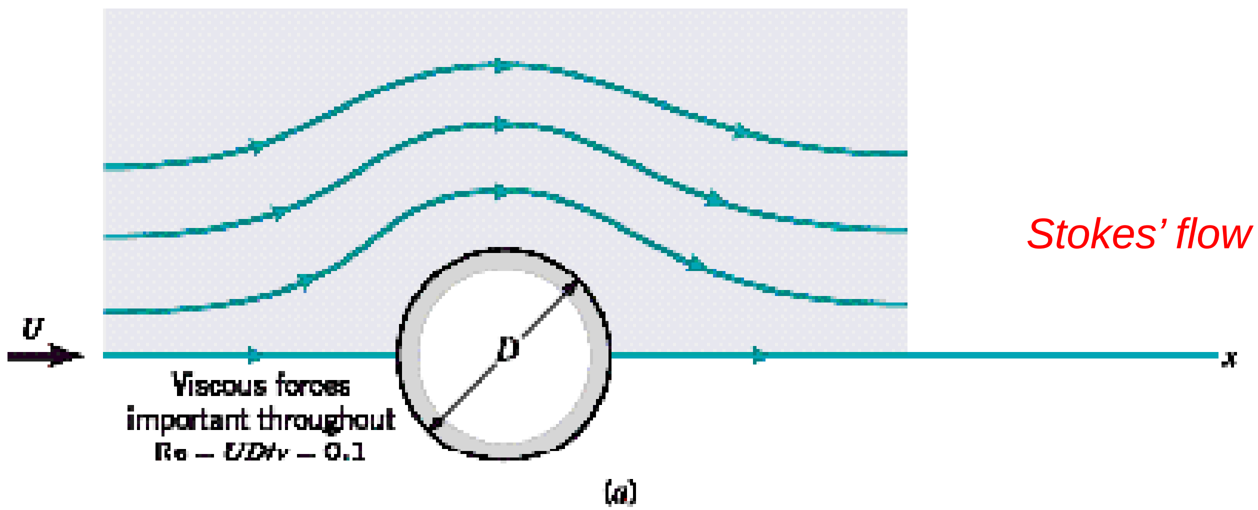
- Since the fluid must stick to the solid surface, there is a thin boundary layer region of thickness $\delta << \ell$ next to the plate in which the fluid velocity changes from U to zero on the plate.
- The thickness of boundary layer increases in the direction of flow, starting from zero at the forward or leading edge of the plate.

# Flow Past an Flat Plate 4/4

- The flow within the boundary layer may be laminar or turbulent depending on various parameters involved.
- The streamline of the flow outside of the boundary layer is nearly parallel to the plate. -> no viscous effects.
- The existence of the plate has very little effect on the streamline outside of the boundary layer – either ahead, above, and below the plate.

# Flow Past an Circular Cylinder 1/4

- When Re ≒ 0.1, the viscous effects are important several diameters in any direction from the cylinder. A somewhat surprising characteristic of this flow is that the streamlines are essentially symmetric about the center of the cylinder-the streamline pattern is the same in front of the cylinder as it is behind the cylinder.

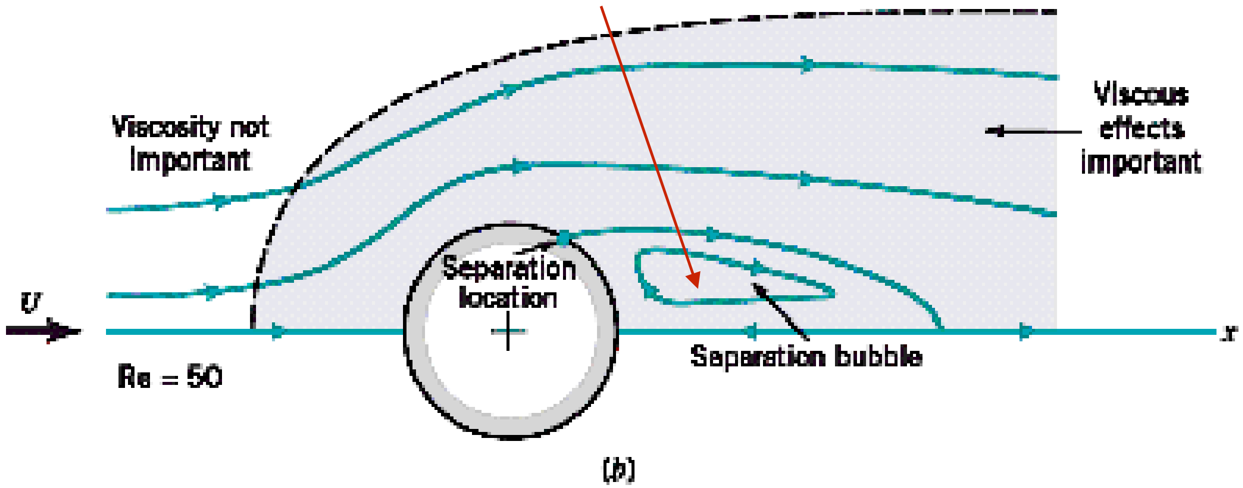



Stokes' flow

# Flow Past an Circular Cylinder 2/4

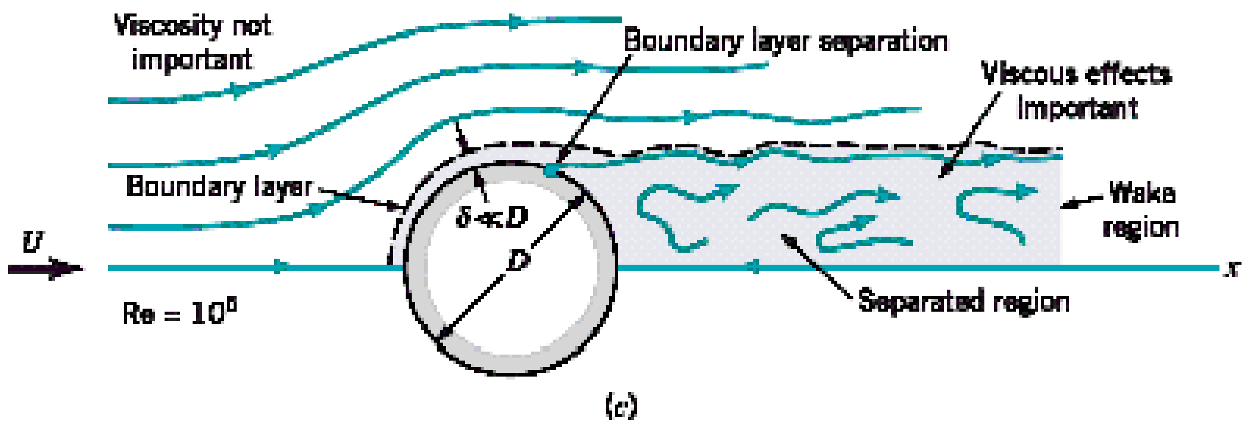
- As Reynolds number is increased (Re =50), the region ahead of the cylinder in which viscous effect are important becomes smaller, with the viscous region extending only a short distance ahead of the cylinder.
- The flow separates from the body at the separation point.
- With the increase in Reynolds number, the fluid inertia becomes more important and at the some on the body, denoted the separation location, the fluid's inertia is such that it cannot follow the curved path around to the rear of the body.

Some of the fluid is actually flowing upstream, against the direction of the upstream flow.



# Flow Past an Circular Cylinder 4/4

- With larger Reynolds numbers ($Re=10^5$), the area affected by the viscous forces is forced farther downstream until it involve only a boundary layer ($\delta<<D$) on the front portion of the cylinder and an irregular, unsteady wake region that extends far downstream of the cylinder.
- The velocity gradients within the boundary layer and wake regions are much larger than those in the remainder of the flow field. -> Shear stress is a product of fluid viscosity and the velocity gradient, so **viscosity effects are confined to the boundary layer and wake regions**.



# Prior to Prandtl

- Theoretical hydrodynamics evolved from Euler's equation of motion for a inviscid (nonviscous) fluid. (published by Leonhard Euler in 1755)
  - ⇨Contradicted many experimental observations. (NO DRAG in the equation) Practicing engineers developed their own empirical art of hydraulics.

$$\rho\vec{g} - \nabla p = \rho\left(\frac{\partial \vec{V}}{\partial t} + (\vec{V}\cdot\nabla)\vec{V}\right) \qquad \rho g_z - \frac{\partial p}{\partial z} = \rho\left(\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial x} + v\frac{\partial w}{\partial y} + w\frac{\partial w}{\partial z}\right)$$

- Mathematical description of a viscous fluid by Navier-Stokes equations, developed by Navier,1827, and *independently (extended)* by Stokes, 1845.
  - ⇨Mathematical difficulties in solving these equations.

# Boundary Layer Concepts

- Introduced by Ludwig Prandtl, a German aerodynamicist, in 1904.
  - ⇨ Many viscous flows can be analyzed by dividing the flow into two regions, one close to solid boundaries, the other covering the rest of flow.
  - ⇨ Only in the thin region adjacent to a solid boundary (the boundary layer) is the effect of viscosity important.
  - ⇨ In the region outside of the boundary layer, the effect of viscosity is negligible and the fluid may be treated as inviscid.
- The boundary layer concept permitted the solution of viscous flow problems that would have been impossible through application of the Navier-Stokes to the complete flow field.

# Boundary Layer Characteristics

- The flow past an object can be treated as a combination of viscous flow in the boundary layer and inviscid flow elsewhere.

⇨ Inside the boundary layer the friction is significant and across the width of which the velocity increases rapidly from zero (at the surface) to the value inviscid flow theory predicts.

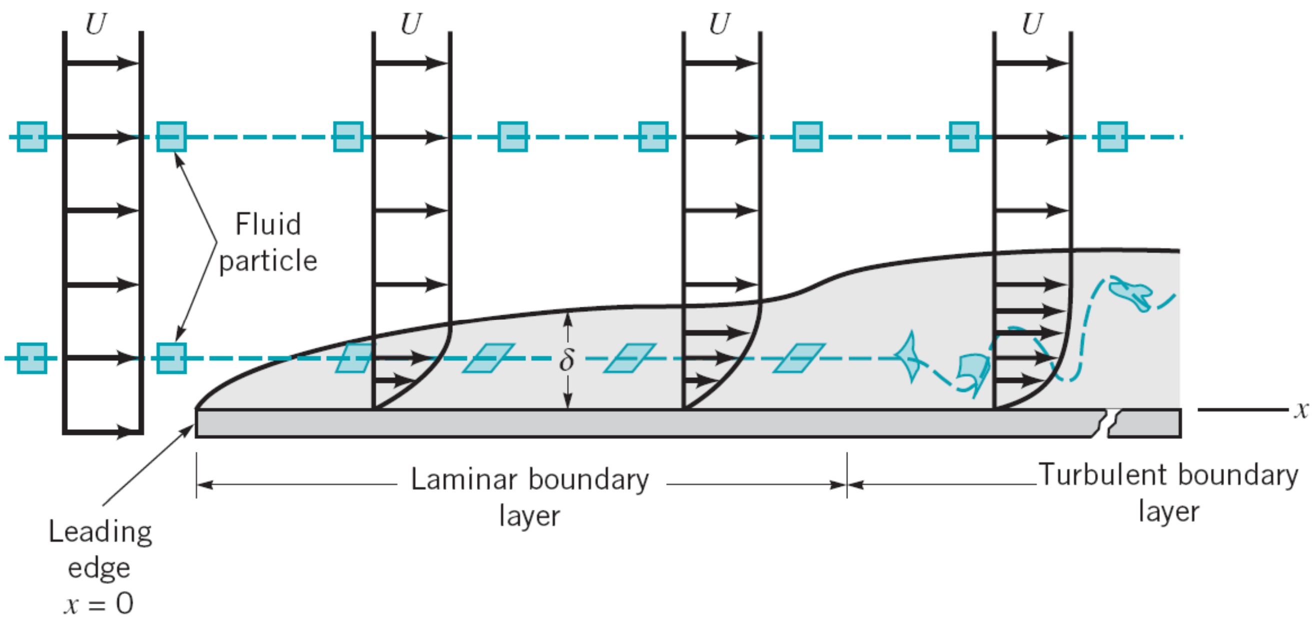
⇨ Outside the boundary layer the velocity gradients normal to the flow are relatively small, and the fluids acts as if it were inviscid, even though the viscosity is not zero.

# Boundary Layer on Solid Surface

- **Inviscid flow (1st order eq.) -> No drag -> Unrealistic**
- By Pradtl in 1904:
  - ⇨ The no-slip condition requires that the velocity everywhere on the surface of the object be zero.
  - ⇨ There will always be a thin boundary layer, in which friction is significant and across the width of the layer the velocity increases rapidly from zero (at the surface) to the value inviscid flow theory predicts.
  - ⇨ Outside of the boundary layer, the velocity gradients normal to the flow are relative small, and the fluid acts as if it were inviscid, even though the viscosity is not zero.

# Boundary Layer on Solid Surface

❖ Consider the flow over a flat plate as shown, the boundary layer is laminar for a short distance downstream from the leading edge; transition occurs over a region of the plate rather than at a single line across the plate.

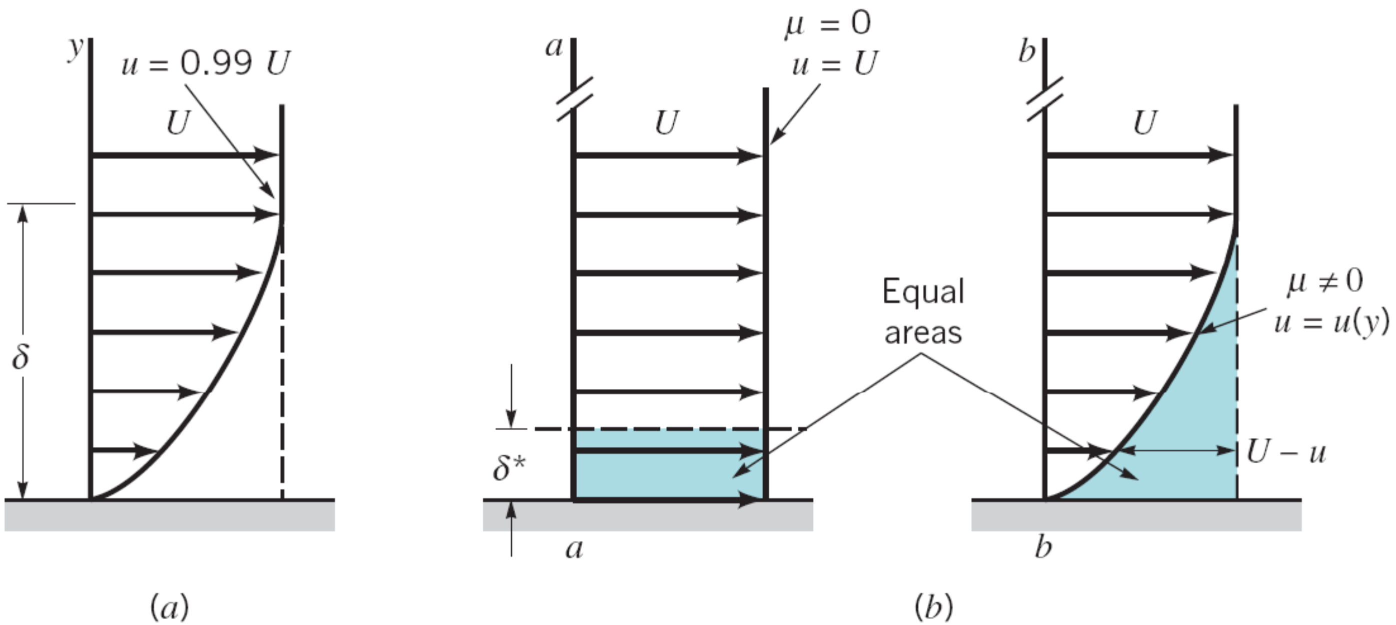


# Boundary Layer on Solid Surface

- The transition region extends downstream to the location where the boundary layer flow becomes completely turbulent.
- For a finite length plate, it is clear that the plate length, $\ell$, can be used as the characteristic length, with the Reynolds number as $Re=U\ell/\nu$.
- For the infinitely long flat plate we use x, the coordinate distance along the plate from the leading edge, as the characteristic length and define the Reynolds number as $Re_x=Ux/\nu$.
- For any fluid or upstream velocity the Reynolds number will be sufficiently large for boundary layer type flow if the plate is long enough.

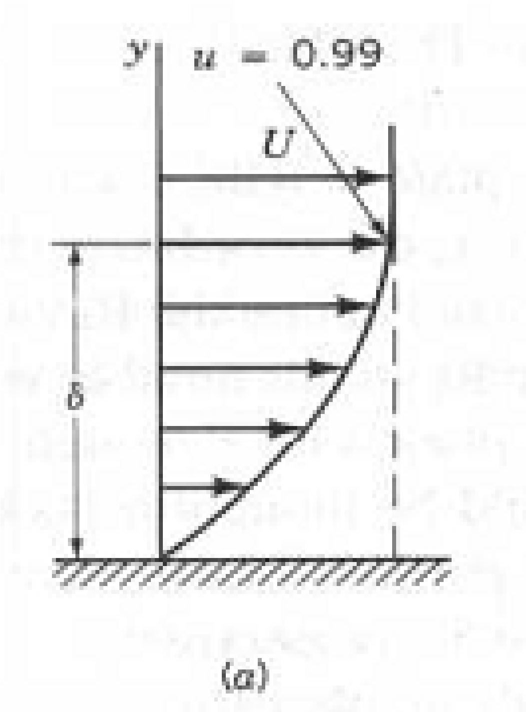
# Boundary Layer Thickness

- ❖ Standard Boundary layer thickness
- ❖ Boundary layer displacement thickness
- ❖ Boundary layer momentum thickness



# Standard Boundary Layer Thickness

❖ The standard boundary layer thickness is the distance from the plate at which the fluid velocity is within some arbitrary value of the upstream velocity.




$y = \delta$ where $u = 0.99\,U$

↑ boundary layer thickness

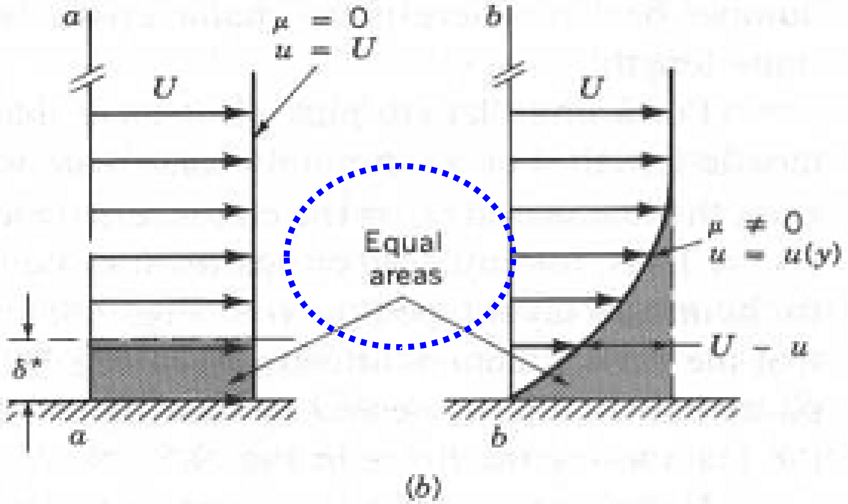
# Boundary Layer Displacement Thickness

- The boundary layer retards the fluid, so that the **mass flux** and **momentum flux** are both less than they would be in the absence of the boundary layer.
- The displacement distance is the distance the plate would be moved so that the loss of mass flux (due to reduction in uniform flow area) is equivalent to the loss the boundary layer causes.

**The loss of mass flow rate due to the boundary layer**

$$\rho\delta^* U w = \int_0^\infty \rho(U-u)w\,dy$$

$$\Rightarrow \delta^* = \int_0^\infty \left(1-\frac{u}{U}\right)dy$$



# Boundary Layer Momentum Thickness

- The momentum thickness is the distance the plate would be moved so that the loss of momentum flux is equivalent to the loss the boundary layer actually causes.

**The loss of momentum due to the boundary layer**

$$\rho w U^2 \Theta = \int_0^\infty \rho w u (U - u) dy$$

$$\Rightarrow \Theta = \int_0^\infty \frac{u}{U}\left(1 - \frac{u}{U}\right) dy$$

# How to Solve Boundary Layer

## How To Solve Boundary Layer

- By Blasius (called Blasius solution)

  Limited to **laminar** boundary layer only, and for a flat plate only (**without a pressure gradient**).

- Momentum integral equation

  Used to obtain approximate information on boundary layer growth for the general case ( **laminar or turbulent boundary layers, with or without a pressure gradient**).

# Prandtl/Blasius Solution

Prandtle used boundary layer concept and imposed approximation (valid for large Reynolds number flows) to simplify the governing Navier-Stokes equations. H. Blasius (1883-1970), one of Prandtl's students, solved these simplified equations.

# Prandtl/Blasius Solution 1/10

- The details of viscous incompressible flow past any object can be obtained by solving the governing Navier-Stokes equation.
- For steady, two dimensional laminar flow with negligible gravitational effects, these equations reduce to the following

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial x}+\nu\left(\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}\right)$$

$$u\frac{\partial v}{\partial x}+v\frac{\partial v}{\partial y}=-\frac{1}{\rho}\frac{\partial p}{\partial y}+\nu\left(\frac{\partial^2 v}{\partial x^2}+\frac{\partial^2 v}{\partial y^2}\right)$$

- In addition, the conservation of mass

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

**No analytical solution**

# Prandtl/Blasius Solution 2/10

- Assumptions for simplification
  1. Since the boundary layer is thin, it is expected that the component of velocity (v) normal to the plate is much smaller than stream-wise velocity component and that the rate of change of any parameter across the boundary layer should be much greater than that along the flow direction. $u >> v, \quad \frac{\partial}{\partial y} >> \frac{\partial}{\partial x}$
  2. Zero pressure gradient. (Note potential flow and constant velocity)
  3. Within the boundary layer, the viscous force is comparable to the inertial forces.

## Nondimensionalization

$$\tilde{u} = \frac{u}{U}, \tilde{v} = \frac{v}{V}, \tilde{x} = \frac{x}{L}, \tilde{y} = \frac{y}{\delta}$$

$$\left(\frac{U\delta}{VL}\right)\frac{\partial \tilde{u}}{\partial \tilde{x}} + \frac{\partial \tilde{v}}{\partial \tilde{y}} = 0 \text{ -> } \left(\frac{U\delta}{VL}\right) = 1, \text{ -> } V = \frac{\delta}{L}U$$

$$\left(\frac{\rho U\delta^2}{\mu L}\right)\left[\tilde{u}\frac{\partial \tilde{u}}{\partial \tilde{x}} + \tilde{v}\frac{\partial \tilde{u}}{\partial \tilde{y}}\right] = -\left(\frac{\Pi\delta^2}{\mu UL}\right)\frac{\partial \tilde{p}}{\partial \tilde{x}} + \left(\left(\frac{\delta}{L}\right)^2\frac{\partial^2 \tilde{u}}{\partial \tilde{x}^2} + \frac{\partial^2 \tilde{u}}{\partial \tilde{y}^2}\right), \text{ -> } \left(\frac{\rho U\delta^2}{\mu L}\right) = 1 \text{ -> } \left(\frac{\delta}{L}\right) = \frac{1}{\sqrt{\mathrm{Re}}}$$

$$\left(\frac{\delta}{L}\right)^2\left[\tilde{u}\frac{\partial \tilde{v}}{\partial \tilde{x}} + \tilde{v}\frac{\partial \tilde{v}}{\partial \tilde{y}}\right] = -\frac{\partial \tilde{p}}{\partial \tilde{y}} + \left(\frac{\delta}{L}\right)^2\left(\left(\frac{\delta}{L}\right)^2\frac{\partial^2 \tilde{v}}{\partial \tilde{x}^2} + \frac{\partial^2 \tilde{v}}{\partial \tilde{y}^2}\right)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$0 = -\frac{\partial p}{\partial y}$$

# Prandtl/Blasius Solution 3/10

Governing equations

**Second order partial differential equations**

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\upsilon\frac{\partial^2 u}{\partial y^2} \qquad \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

Boundary conditions

$$y=0, \quad u=0, \quad v=0$$

$$y=\infty, \quad u=U, \quad \frac{\partial u}{\partial y}=0$$

Solution ? …… ***are extremely difficult to obtain.***

# Prandtl/Blasius Solution 4/10

- Blasius reduced the partial differential equations to an ordinary differential equation…

The velocity profile, u/U, should be similar for all values of x. Thus the velocity profile is of the dimensionless form

**Is an unknown function to be determined.**

$$\frac{u}{U} = g(\eta)$$ where $\eta = \frac{y}{\delta} = \frac{y}{\sqrt{\frac{\nu x}{U}}}$

The boundary layer thickness grows as the square root of x and inversely proportional to the square root of U. That is

$$\delta \sim \left(\frac{\nu x}{U}\right)^{1/2}$$

# Prandtl/Blasius Solution 5/10

Set a dimensionless similarity variable

$$\eta = \frac{y}{\delta} = \frac{y}{\sqrt{\frac{\nu x}{U}}}$$

The velocity component

$$f'(\eta) = \frac{u}{U}, u = Uf'(\eta), dy = \sqrt{\frac{\nu x}{U}} d\eta, d\eta = -\frac{\eta}{2x} dx$$

$$\frac{\partial u}{\partial x} = Uf''(\eta)\frac{\partial \eta}{\partial x} = -Uf''(\eta)\frac{\eta}{2x}$$

$$v = \int_0^y -\frac{\partial u}{\partial x} dy = \int_0^\eta Uf''(\eta)\frac{\eta}{2x}\sqrt{\frac{\nu x}{U}} d\eta = \frac{U}{2x}\sqrt{\frac{\nu x}{U}}\int_0^\eta f''(\eta)\eta d\eta = \frac{U}{2x}\sqrt{\frac{\nu x}{U}}\left[f'(\eta)\eta - f(\eta)\right]_0^\eta$$

$$= \frac{U}{2x}\sqrt{\frac{\nu x}{U}}\left[f'(\eta)\eta - f(\eta)\right]_0^\eta = \frac{1}{2}\sqrt{\frac{U\nu}{x}}\left[f'(\eta)\eta - f(\eta)\right] \text{ where } f(0) = 0$$

# Prandtl/Blasius Solution 6/10

$$u = Uf'(\eta), \quad dy = \sqrt{\frac{\upsilon x}{U}} d\eta, \quad d\eta = -\frac{\eta}{2x} dx,$$

$$\frac{\partial u}{\partial x} = Uf''(\eta)\frac{\partial \eta}{\partial x} = -Uf''(\eta)\frac{\eta}{2x}, \quad \frac{\partial u}{\partial y} = Uf''(\eta)\frac{\partial \eta}{\partial y} = Uf''(\eta)\sqrt{\frac{U}{\upsilon x}}$$

$$\frac{\partial^2 u}{\partial y^2} = Uf'''(\eta)\frac{U}{\upsilon x}, \quad v = \frac{1}{2}\sqrt{\frac{U\upsilon}{x}}\left[f'(\eta)\eta - f(\eta)\right] \text{ where } f(0) = 0$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \upsilon\frac{\partial^2 u}{\partial y^2}$$

$$Uf'(\eta)\left[-Uf''(\eta)\frac{\eta}{2x}\right] + \frac{1}{2}\sqrt{\frac{U\upsilon}{x}}\left[f'(\eta)\eta - f(\eta)\right]Uf''(\eta)\sqrt{\frac{U}{\upsilon x}} = \upsilon Uf'''(\eta)\frac{U}{\upsilon x}$$

$$-f'(\eta)f''(\eta)\eta + \left[f'(\eta)\eta - f(\eta)\right]f''(\eta) = 2f'''(\eta)$$

$$2f'''(\eta) + f(\eta)f''(\eta) = 0$$

# Prandtl/Blasius Solution 7/10

$$2\frac{d^3 f}{d\eta^3} + f\,\frac{d^2 f}{d\eta^2} = 2f''' + ff'' = 0$$

Nonlinear, third-order ordinary differential equation

With boundary conditions

**f = f' = 0 at η = 0**

**f' →1 at η →∞**

$$f'(\eta) = \frac{u}{U}$$

Solution ? No analytical solution !

Easy to integrate to obtain numerical solution

Blasius solved it using a power series expansion about **η = 0 …Blasius solution**

# Prandtl/Blasius Solution 8/10

**TABLE 9.1**

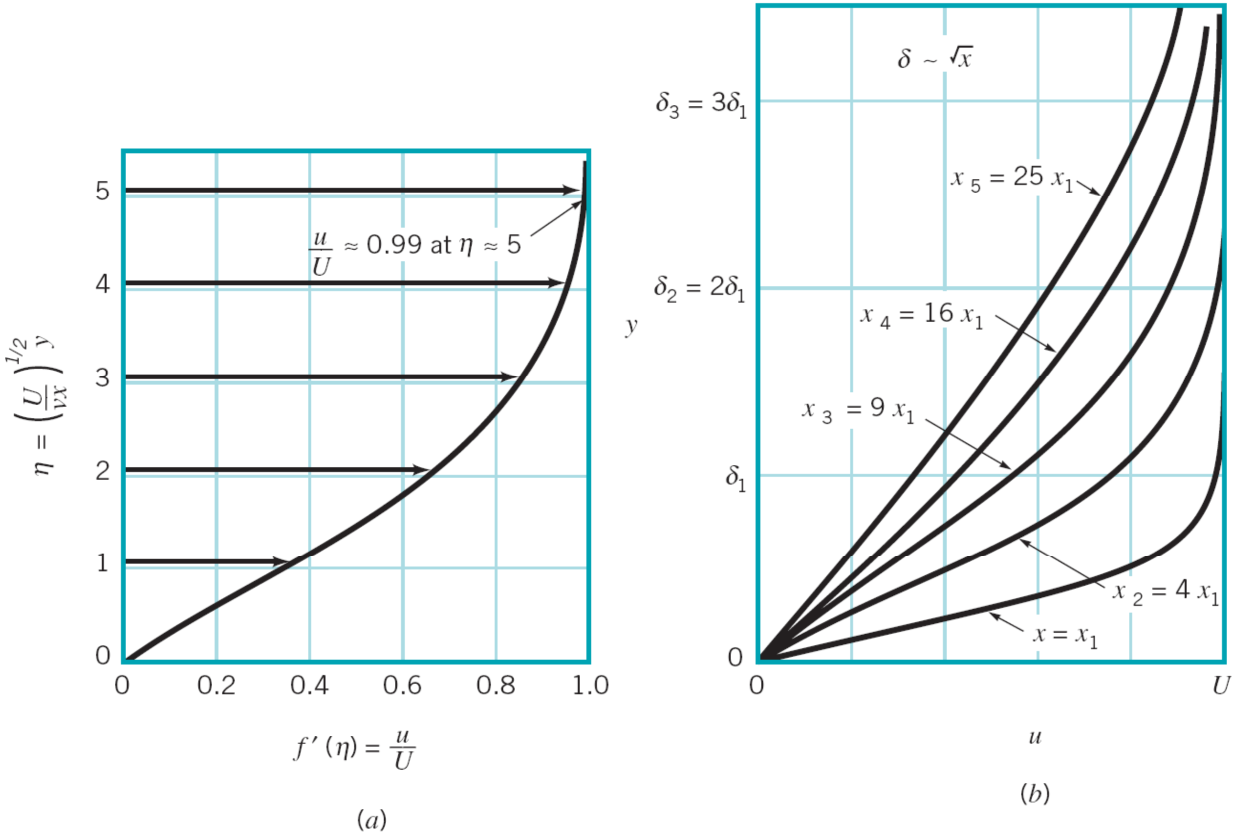
**Laminar Flow along a Flat Plate (the Blasius Solution)**

← Numerical solution of

$$2\frac{d^3 f}{d\eta^3} + f\frac{d^2 f}{d\eta^2} = 2f''' + ff'' = 0$$

| $\eta = y(U/\nu x)^{1/2}$ | $f'(\eta) = u/U$ | $\eta$ | $f'(\eta)$ |
|---|---|---|---|
| 0 | 0 | 3.6 | 0.9233 |
| 0.4 | 0.1328 | 4.0 | 0.9555 |
| 0.8 | 0.2647 | 4.4 | 0.9759 |
| 1.2 | 0.3938 | 4.8 | 0.9878 |
| 1.6 | 0.5168 | 5.0 | 0.9916 |
| 2.0 | 0.6298 | 5.2 | 0.9943 |
| 2.4 | 0.7290 | 5.6 | 0.9975 |
| 2.8 | 0.8115 | 6.0 | 0.9990 |
| 3.2 | 0.8761 | $\infty$ | 1.0000 |

# Prandtl/Blasius Solution 9/10




Numerical solution of

$$2\frac{d^3 f}{d\eta^3} + f\,\frac{d^2 f}{d\eta^2} = 2f''' + ff'' = 0$$

Blasius boundary layer profile: (*a*) boundary layer profile in dimensionless form using the similarity variable η. (*b*) similar boundary layer profiles at different locations along the flat plate.

# Prandtl/Blasius Solution 10/10

❖ From Table 9.1. We see that at $\eta = 5.0$ , $u/U=0.992$, $f=3.283$.

$$\frac{\delta}{x} = \frac{5}{\sqrt{U/\nu x}} \qquad \delta = \frac{5.0}{\sqrt{U/\nu x}} = \frac{5.0}{\sqrt{Re_x}} \qquad \frac{\delta^*}{x} = \frac{1.721}{\sqrt{Re_x}}$$

$$\frac{\Theta}{x} = \frac{0.664}{\sqrt{Re_x}} \qquad Re_x = Ux/\nu$$

$$\delta^* = \int_0^\infty \left(1 - \frac{u}{U}\right) dy = \int_0^\delta (1 - f')\sqrt{\frac{\upsilon x}{U}}\, d\eta = \sqrt{\frac{\upsilon x}{U}}\left[\eta - f\right]_0^\delta$$

$$= \sqrt{\frac{\upsilon x}{U}}\left[5 - 3.283 = 1.72\right], \quad \frac{\delta^*}{x} = 1.72\sqrt{\frac{\upsilon}{Ux}}$$

Shear stress $$\tau_w = \mu \frac{\partial u}{\partial y}\Big|_{y=0} = 0.332 U^{3/2} \sqrt{\frac{\rho\mu}{x}} = \frac{0.332 \rho U^2}{\sqrt{Re_x}}$$

HW: derive momentum thickness and shear stress formulas.

# HW Solution

- Revised Table 9.1.

| η | f'' | f' | f |
|---|---|---|---|
| 0 | 0.3321 | 0 | 0 |
| 1 | 0.3230 | 0.3298 | 0.1656 |
| 2 | 0.2668 | 0.6298 | 0.6500 |
| 3 | 0.1614 | 0.8460 | 1.3968 |
| 4 | 0.0642 | 0.9555 | 2.3057 |
| 5 | 0.0159 | 0.9915 | 3.2833 |
| 6 | 0.0024 | 0.9990 | 4.2796 |
| Infinity | 0 | 1.000 | Infinity |

$$\Theta = \int_0^\infty \frac{u}{U}\left(1-\frac{u}{U}\right)dy = \int_0^5 f'(\eta)(1-f'(\eta))\sqrt{\frac{\upsilon x}{U}}d\eta = \sqrt{\frac{\upsilon x}{U}}\int_0^5 f'(\eta)(1-f'(\eta))d\eta$$

$$= \sqrt{\frac{\upsilon x}{U}}\int_0^5 f'(\eta)(1-f'(\eta))d\eta = \sqrt{\frac{\upsilon x}{U}}\left[f(\eta)(1-f'(\eta))\Big|_0^5 + \int_0^5 ff''d\eta\right]$$

$$= \sqrt{\frac{\upsilon x}{U}}\left[f(\eta)(1-f'(\eta))\Big|_0^5 - \int_0^5 2f'''d\eta\right] = \sqrt{\frac{\upsilon x}{U}}\left[f(\eta)(1-f'(\eta))\Big|_0^5 - 2f''\Big|_0^5\right]$$

$$= \sqrt{\frac{\upsilon x}{U}}\left[f(5)(1-f'(5)) - 2(f''(5)-f''(0))\right] = \sqrt{\frac{\upsilon x}{U}}\left[3.2833(1-0.9915) - 2(0.0159-0.3321)\right]$$

$$= \sqrt{\frac{\upsilon x}{U}}\,0.660$$

Shear stress

$$\tau_w = \mu\frac{\partial u}{\partial y}\Big|_{y=0} = 0.332U^{3/2}\sqrt{\frac{\rho\mu}{x}} = \frac{0.332\rho U^2}{\sqrt{Re_x}}$$

$$\tau = \mu\frac{\partial u}{\partial y}\Big|_{y=0} = \mu U f''(\eta)\frac{\partial \eta}{\partial y}\Big|_{\eta=0} = \mu U f''(\eta)\sqrt{\frac{U}{\upsilon x}}\Big|_{\eta=0} = \mu U f''(0)\sqrt{\frac{U}{\upsilon x}} = 0.332\mu U\sqrt{\frac{U}{\upsilon x}}$$

# Momentum Integral Equation

## Used to obtain approximate information on boundary layer growth

How To Solve Boundary Layer

1. By Blasius (called Blasius solution)

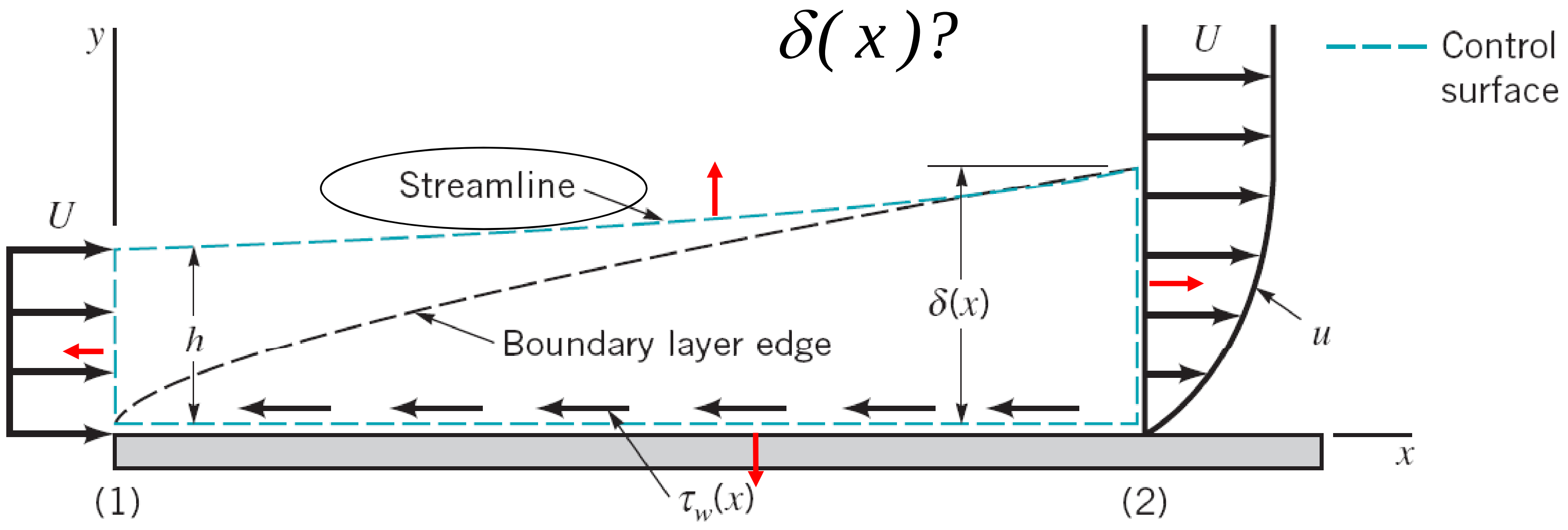
   Limited to **laminar** boundary layer only, and for a flat plate only (**without a pressure gradient**).

2. Momentum integral equation

   Used to obtain approximate information on boundary layer growth for the general case ( **laminar or turbulent boundary layers, with or without a pressure gradient**).

# Momentum Integral Equation 1/12

- Consider incompressible, steady, two-dimensional flow over a solid surface.



# Momentum Integral Equation 2/12

- Assume that the pressure is constant throughout the flow field.
- X-component of the momentum equation to the steady flow of fluid within this control volume

$$\sum \vec{F} = \frac{\partial}{\partial t}\int_V \rho \vec{V} dV + \int_A \vec{n}\cdot\left(\vec{V}\rho\vec{V}\right)dA$$

$$\sum F_x = \rho\int_{(1)} \vec{V}\vec{V}\cdot\vec{n}dA + \rho\int_{(2)} \vec{V}\vec{V}\cdot\vec{n}dA$$

For a plate of width b

$$\sum F_x = -D = -\int_{plate} \tau_w dA = -b\int_{plate} \tau_w dy$$

where $D$ is the drag that the fluid exerts on the object.

# Momentum Integral Equation 3/12

❖ Since the plate is solid and the upper surface of the control volume is a streamline, there is no flow through these area. Thus

$\vec{V} \cdot \vec{n} = 0$

$$-D = \rho \int_{(1)} U(-U)dA + \int_{(2)} u^2 dA$$

$$\Rightarrow D = \rho U^2 bh - \rho b \int_0^\delta u^2 dy$$

***h ???***

❖ The conservation of mass

Drag on a flat plate is related to momentum deficit within the boundary layer

$$Uh = \int_0^\delta u dy$$

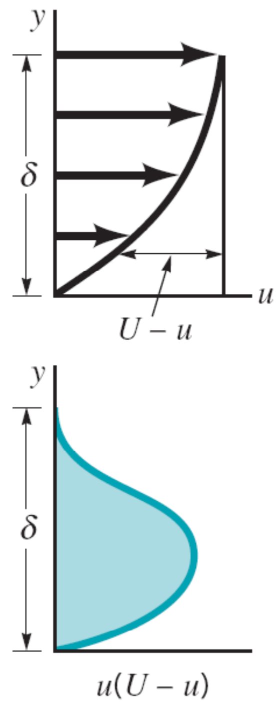
$$Uh\rho Ub = \rho b \int_0^\delta U u dy$$

# Momentum Integral Equation 4/12

$$D = \rho b \int_0^{\delta} u(U - u)dy$$

A balance between shear drag and a decrease in the momentum of the fluid

- As x increases, $\delta$ increases and the drag increases.
- The thickening of the boundary layer is necessary to overcome the drag of the viscous shear stress on the plate. (This is contrary to horizontal fully developed pipe flow in which the momentum of the fluid remains constant and the shear force is overcome by the pressure gradient along the pipe.)



# Momentum Integral Equation 5/12

- By T. von Karman (1881-1963)

$$D = \rho b \int_0^{\delta} u(U - u)dy$$

$$\Theta = \int_0^{\infty} \frac{u}{U}\left(1 - \frac{u}{U}\right)dy$$

$$\Rightarrow D = \rho b U^2 \Theta$$

*Valid for laminar or turbulent flows*

$$\frac{dD}{dx} = \rho b U^2 \frac{d\Theta}{dx} \qquad dD = \tau_w b dx \Rightarrow \frac{dD}{dx} = b\tau_w$$

$$\Rightarrow \tau_w = \rho U^2 \frac{d\Theta}{dx}$$

Momentum integral equation for the boundary layer flow on a flat plate

# Momentum Integral Equation 6/12

If we knew the detailed velocity profile in the boundary layer (i.e., the Blasius solution), we could evaluate either the drag force or the shear stress.

**What if we don't know u=u(x)??? With an assumed velocity profile** in the boundary layer **to obtain reasonable, approximate boundary layer result. The accuracy of the result depends on how closely the shape of the assumed velocity profile approximates the actual profile.**

$$\frac{u}{U} = g(Y) \qquad 0 \le Y \le 1 \quad Y = \frac{y}{\delta}$$

$$\frac{u}{U} = 1 \qquad Y > 1$$

**B.C.**

$$\begin{cases} at & y = 0, & u = 0 \\ at & y = \delta, & u = U \\ at & y = \delta, & \partial u / \partial y = 0 \end{cases}$$

$$g(0) = 0 \text{ and } g(1) = 1$$

# Momentum Integral Equation 7/12

g(Y) ? g(Y)=Y (Example 9.4)

For a given g(Y), the drag can be determined

$$D = \rho b \int_0^{\delta} u(U-u)dy = \rho b U^2 \delta \int_0^1 g(Y)[1-g(Y)]dY = \rho b U^2 \delta \ C_1$$

$$\text{If } g(Y) = Y, D = \rho b U^2 \delta \int_0^1 Y[1-Y]dY = \rho b U^2 \delta \left[\frac{1}{2} - \frac{1}{3}\right]$$

$$C_1 = \int_0^1 g(Y)[1-g(Y)]dY$$

$$\tau_w = \mu \frac{du}{dy}\Big|_{y=0} = \frac{\mu U}{\delta}\frac{dg}{dY}\Big|_{Y=0} = \frac{\mu U}{\delta} C_2$$

$$C_2 = \frac{dg(Y)}{dY}\Big|_{Y=0}$$

# Momentum Integral Equation 8/12

$$\frac{dD}{dx} = \rho b U^2 \frac{d\delta}{dx} C_1 = b\tau_{wall} = b\frac{\mu U}{\delta}C_2 -> \rho U \frac{d\delta}{dx} C_1 = \frac{\mu}{\delta}C_2$$

$$\delta d\delta = \frac{\mu C_2}{\rho U C_1} dx$$

Integrating… from δ=0 at x=0 to give

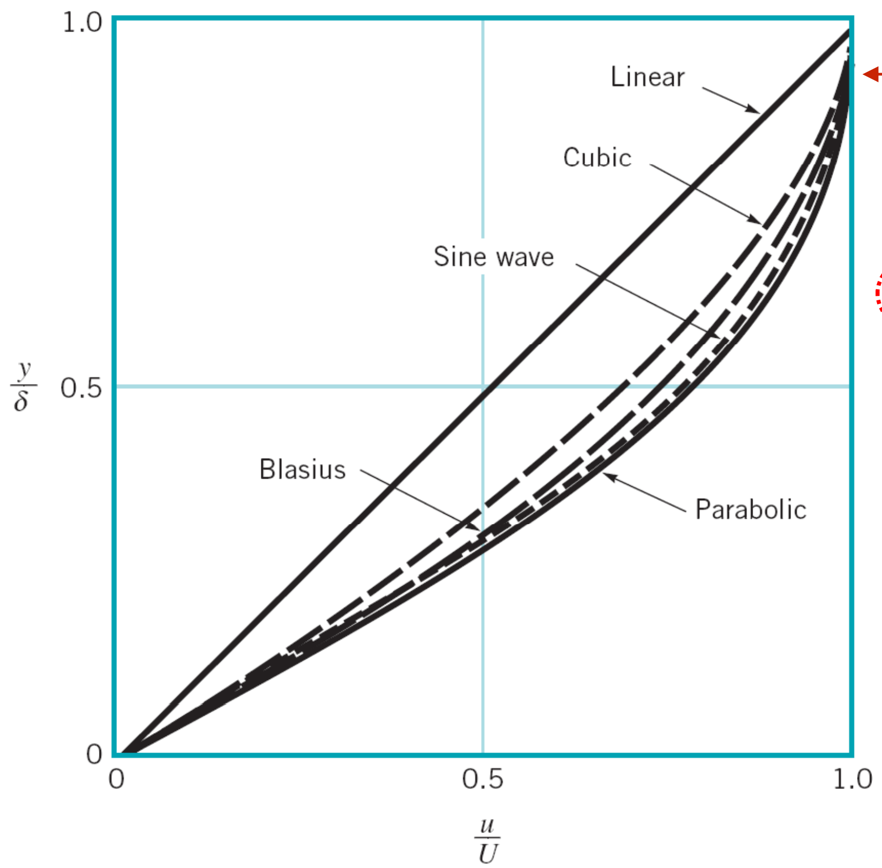
$$\delta = \sqrt{\frac{2\nu C_2 x}{UC_1}} \Rightarrow \frac{\delta}{x} = \frac{\sqrt{2C_2 / C_1}}{\sqrt{Re_x}}$$

$$\tau_w = \sqrt{\frac{C_1 C_2}{2}} U^{3/2} \sqrt{\frac{\rho\mu}{x}}$$

$C_1$ and $C_2$ must be determined

# Momentum Integral Equation 9/12

❖ Several assumed velocity profiles and the resulting value of δ




**Typical approximate boundary layer profiles** used in the momentum integral equation.

■ **TABLE 9.2**

**Flat Plate Momentum Integral Results for Various Assumed Laminar Flow Velocity Profiles**

| Profile Character | $\delta Re_x^{1/2}/x$ | $c_f Re_x^{1/2}$ | $C_{Df} Re_\ell^{1/2}$ |
|---|---|---|---|
| a. Blasius solution | 5.00 | 0.664 | 1.328 |
| b. Linear $u/U = y/\delta$ | 3.46 | 0.578 | 1.156 |
| c. Parabolic $u/U = 2y/\delta - (y/\delta)^2$ | 5.48 | 0.730 | 1.460 |
| d. Cubic $u/U = 3(y/\delta)/2 - (y/\delta)^3/2$ | 4.64 | 0.646 | 1.292 |
| e. Sine wave $u/U = \sin[\pi(y/\delta)/2]$ | 4.79 | 0.655 | 1.310 |

# Momentum Integral Equation 10/12

- The more closely the assumed shape approximates the actual (i.e., Blasius) profile, the more accurate the final results.
- For any assumed profile shape, the functional dependence of δ and $\tau_w$ on the physical parameters ρ,μ, U, and x is the same. Only the constants are different. That is,

$$\delta \sim \left(\frac{\mu x}{\rho U}\right)^{1/2} \qquad \tau_w \sim \left(\frac{\rho \mu U^3}{x}\right)^{1/2} \qquad \frac{\delta \, \mathrm{Re}_x^{1/2}}{x} = \text{constant}$$

# Momentum Integral Equation 11/12

❖ Defining dimensionless local friction coefficient

$$c_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} \Rightarrow c_f = \frac{\sqrt{2C_1C_2}}{\sqrt{Re_x}}$$

$$\tau_w = \sqrt{\frac{C_1C_2}{2}}U^{3/2}\sqrt{\frac{\rho\mu}{x}}$$

$$c_f = \frac{\tau_w}{\frac{1}{2}\rho U^2} = \frac{0.644}{\sqrt{Re_x}}$$ (Blasius solution)

# Momentum Integral Equation 12/12

❖ For a flat plate of length $\ell$ and width b, the net friction drag $D_f$ and frictional drag coefficient $C_{Df}$ are defined as

$$D_f = C_{Df} \cdot \frac{1}{2}\rho U^2 b\ell = b\int_0^{\ell} \tau_w dx \qquad C_{Df} = \frac{D_f}{\frac{1}{2}\rho U^2 b\ell} = \frac{1}{\ell}\int_0^{L} c_f dx = \frac{\sqrt{8C_1C_2}}{\sqrt{Re_\ell}}$$

$$c_f = \sqrt{2C_1C_2\mu/\rho U x} \qquad Re_\ell = U\ell/\nu$$

$$C_{Df} = \frac{D_f}{\frac{1}{2}\rho U^2 b\ell} = \frac{1}{\ell}\int_0^{L} c_f dx = \frac{1.328}{\sqrt{Re_\ell}}$$

$$c_f = \frac{0.644}{\sqrt{Re_x}}$$ (Blasius solution)

# Transition from Laminar to Turbulent 1/5

- Above analytical results agree quite well with experimental results up to a point where the boundary layer flow becomes turbulent, which will occur for any free stream velocity and any fluid provided the plate is long enough.
- The parameter that governs the transition to turbulent flow is the Reynolds numbers – in this case, the Reynolds number based on the distance from the leading edge of the plate, $Re_x = Ux/\nu$.
- The value of the Reynolds number at the transition location is a rather complex function of various parameters involved, including the roughness of the surface, the curvature of the surface, and some measure of the disturbances in the flow outside the boundary layer.

# Transition from Laminar to Turbulent 2/5

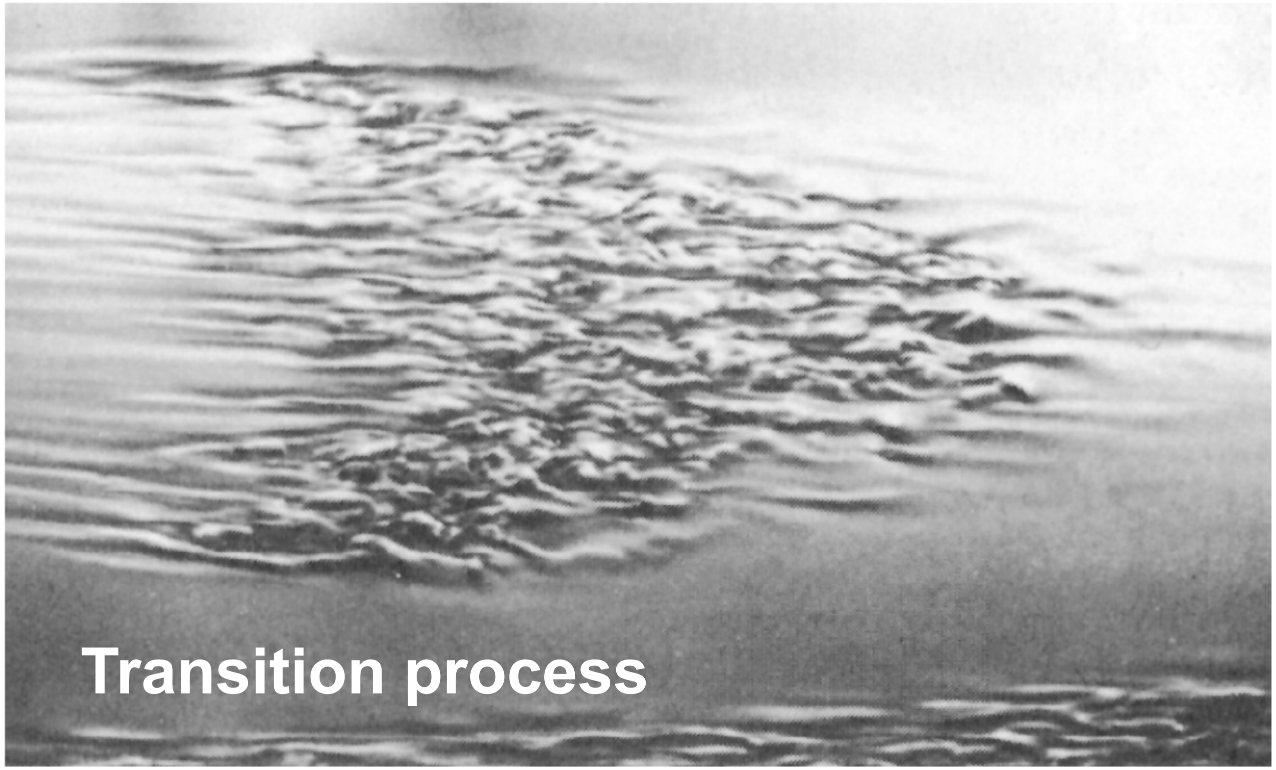
- On a flat plate with a sharp leading edge in a typical air-stream, the transition takes place at a distance x from the leading edge given by $Re_{xcr}=2\times10^5$ to $3\times10^6$. **$Re_{xcr}=5\times10^5$ is used.**
- The actual transition from laminar to turbulent boundary layer flow may occur over a region of the plate, not a specific single location.
- Typical, the transition begins at random location on the plate in the vicinity of $Re_x=$ **$Re_{xcr}$**
- The complex process of transition from laminar to turbulent flow involves the instability of the flow filed.

# Transition from Laminar to Turbulent 3/5

- Small disturbances imposed on the boundary layer will either grow or decay, depending on where the disturbance is introduced into the flow.
- If the disturbances occur at a location with $Re_x < Re_{xcr}$, they will die out, and the boundary layer will return to laminar flow at that location.
- If the disturbances occur at a location with $Re_x > Re_{xcr}$, they will grow and transform the boundary layer flow downstream of this location into turbulence.

**The boundary layer on a flat plate will become turbulent if the plate is long enough -> large Reynolds number .**

# Transition from Laminar to Turbulent 4/5

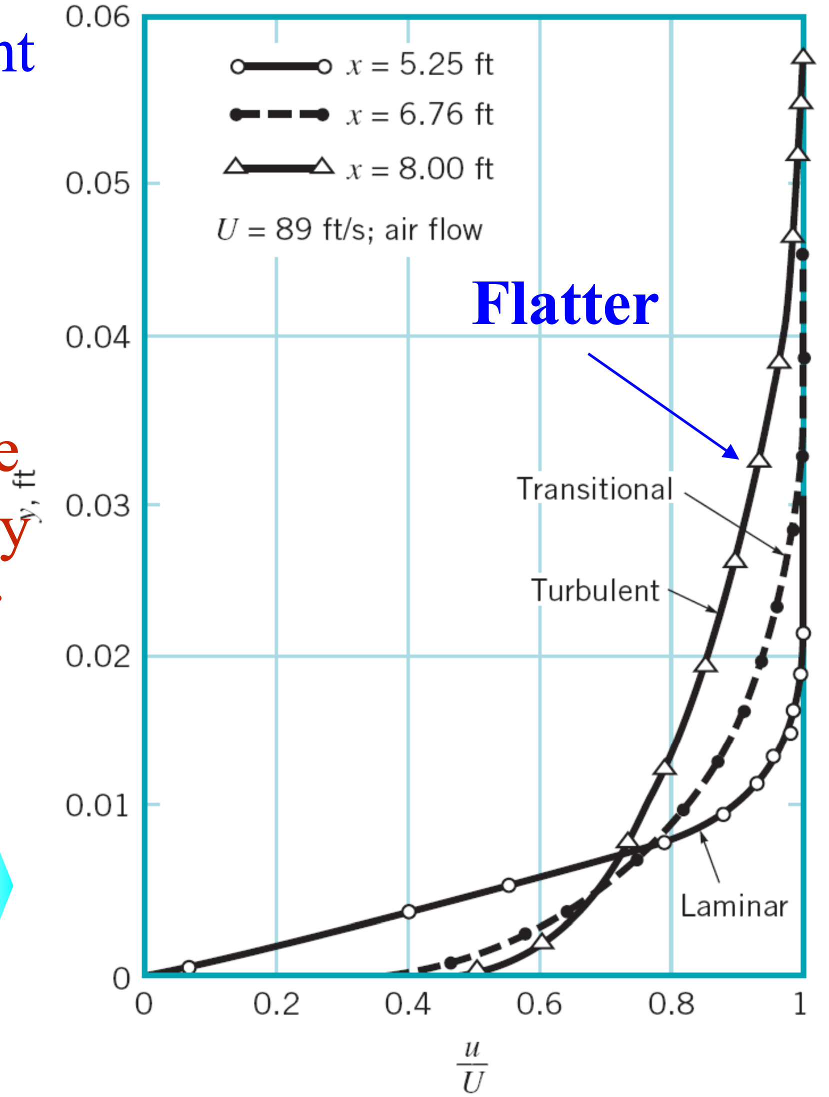



Turbulent spots and the transition from laminar to turbulent boundary layer flow on a flat plate. Flow from left to right

# Transition from Laminar to Turbulent 5/5

- Transition from laminar to turbulent flow involves a noticeable change in the shape of the boundary layer velocity profiles.
- The turbulent profiles are flatter, have a large velocity gradient at the wall, and produce a larger boundary layer thickness than do the laminar profiles.

boundary layer velocity profiles on a flat plate for laminar, transitional, and turbulent flow.



# Laminar and Turbulent

- For laminar flow

$$\frac{\delta}{x} = \frac{5.48}{\sqrt{Re_x}} \qquad \tau_w = \frac{1}{2}\rho U^2 \frac{0.730}{\sqrt{Re_x}}$$

- For turbulent flow

$$\frac{\delta}{x} = \frac{0.382}{Re_x^{1/5}} \qquad \tau_w = \frac{1}{2}\rho U^2 \frac{0.0594}{Re_x^{1/5}}$$

⇨ The turbulent boundary layer develops more rapidly than the laminar boundary layer.

⇨ Wall shear stress is much higher in the turbulent boundary layer than in the laminar layer.

# Effects of Pressure Gradient

**Fox**

# Effect of Pressure Gradients 1/4

❖ Use the boundary conditions at the wall u=v=0 at y=0

$$\rho\left(\frac{\partial u}{\partial t}+u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial x}\right)=-\frac{\partial p}{\partial x}+\mu\frac{\partial^2 u}{\partial y^2} \quad \Rightarrow \quad \mu\left.\frac{\partial^2 u}{\partial y^2}\right|_{y=0}=\frac{\partial p}{\partial x}$$

which (momentum equation) relates the curvature of the velocity profile at the surface to the pressure gradient.

# Effect of Pressure Gradients 1/4

- Favorable pressure gradient: the pressure decreases in the flow direction

  To counteract the slowing the fluid particles in the boundary layer.

$$\partial p / \partial x < 0$$

- Adverse pressure gradient: the pressure increases in the flow direction

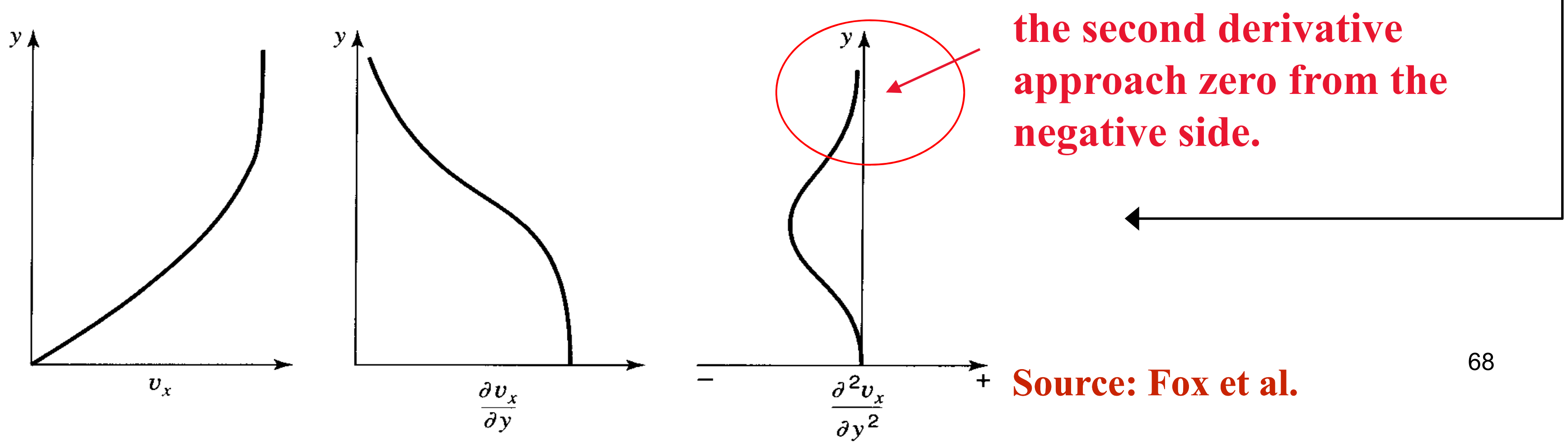
  To contribute to the slowing of the fluid particles.

$$\partial p / \partial x > 0$$

# $\partial p / \partial x = 0$

$$\mu \left.\frac{\partial^2 u}{\partial y^2}\right|_{y=0} = \frac{\partial p}{\partial x}$$

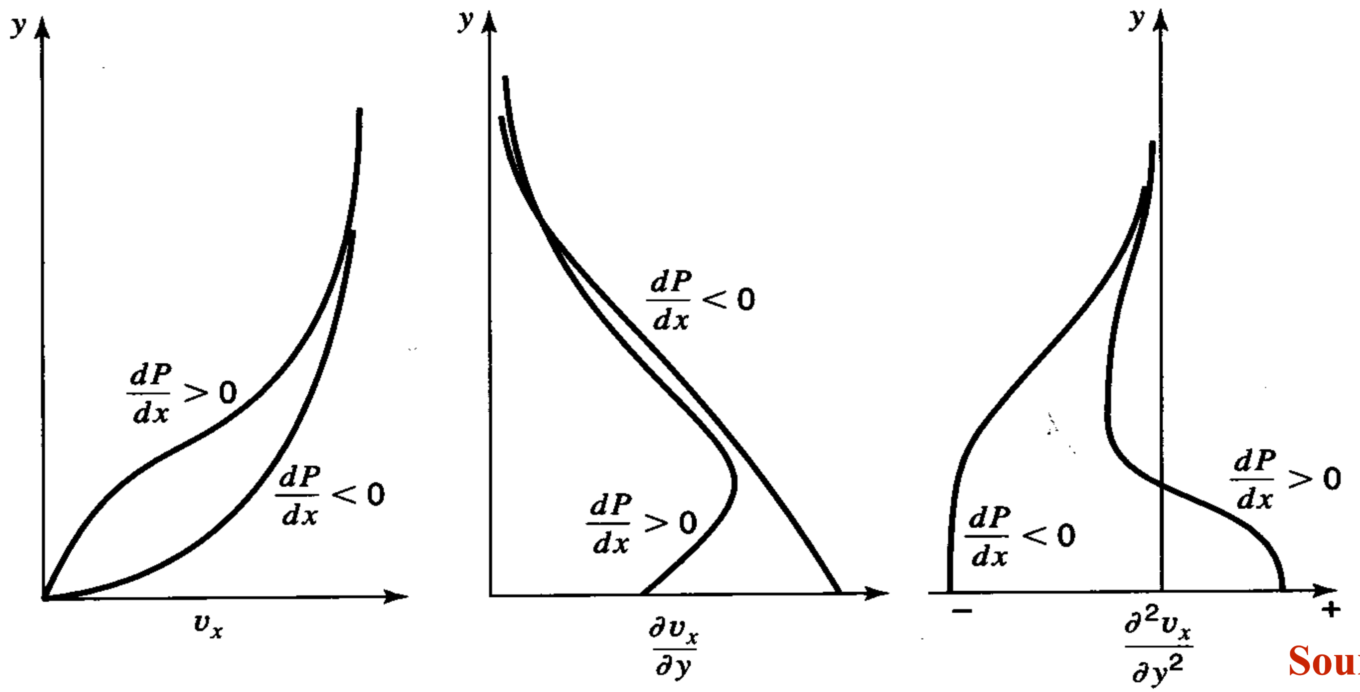
- The velocity profile is **linear** near the wall.
- In the boundary layer, the velocity gradient becomes smaller and gradually approaches zero. The decrease in the velocity gradient means that the second derivative of the velocity must be negative.
- The second derivative  is shown as being zero at the wall, negative within the boundary layer, and approaching zero at the outer edge of the boundary layer.




Source: Fox et al.

# $\partial p / \partial x < 0$

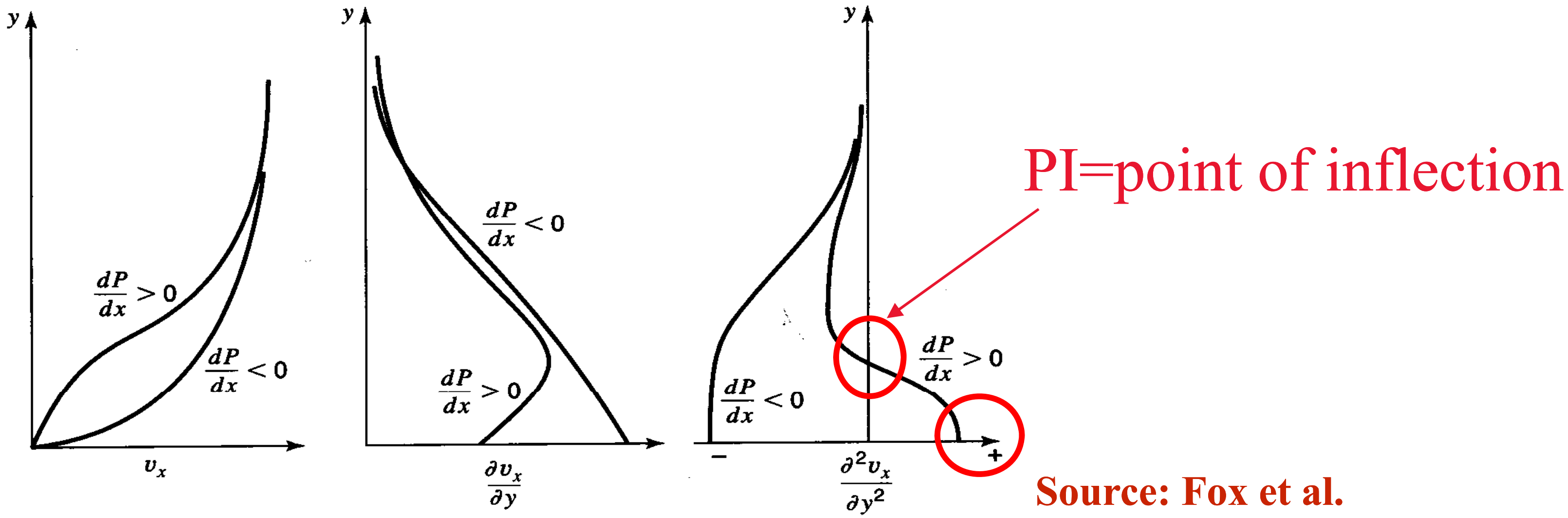
- A negative pressure gradient is seen to produce a velocity variation somewhat similar to that of the zero pressure gradient case.




Source: Fox et al.

# $\partial p / \partial x > 0$

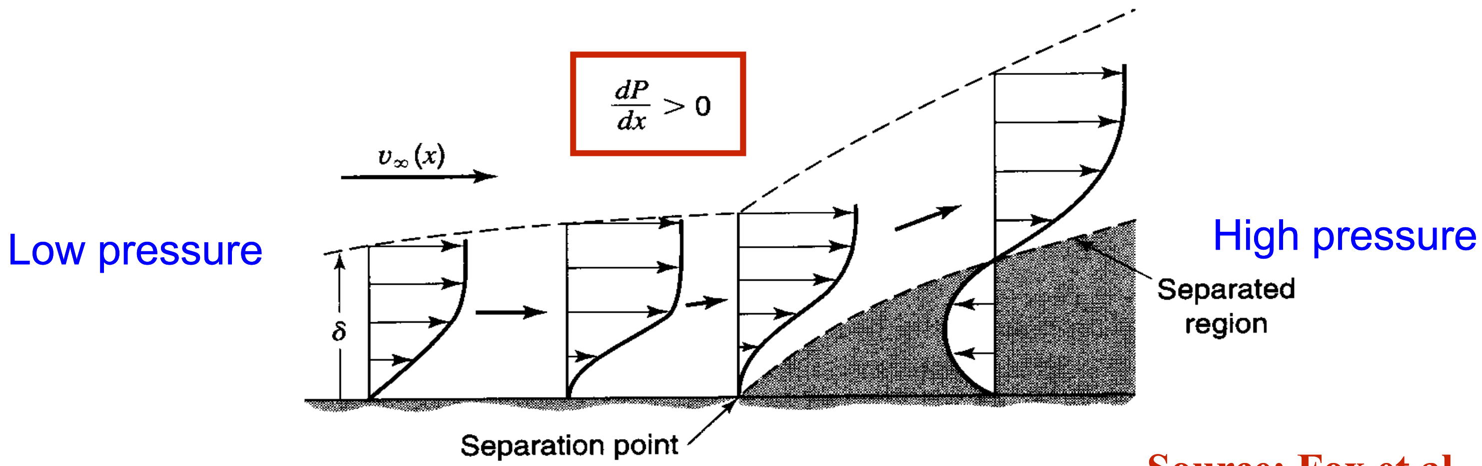
- A <u>positive pressure gradient</u> requires a positive derivative value of the second derivative of the velocity at the wall.
- Since this derivative must approach zero from the negative side, at some point within the boundary layer the second derivative must equal to zero. A zero second derivative, it will be recalled, is associated with an inflection point.




**Source: Fox et al.**

# $\partial p / \partial x > 0$

- If the adverse pressure gradient is severe enough, the fluid particle in the boundary layer will actually be brought to rest (the velocity in the layer of fluid adjacent to the wall must be zero or negative ).
- If the adverse pressure gradient is severe enough, the particle will be forced away from the body surface *(called flow separation)* as they make room for following particles, ultimately leading to a *wake* in which flow is turbulent.




**Source: Fox et al.**

# Effect of Pressure Gradients 2/4

- For **uniform flow over a flat plate** the flow **never separates**, and we never develop a wake region, whether the boundary layer is laminar or turbulent, regardless of plate length.
  - ⇨ $\partial p/\partial x=0$ : no flow separation.
  - ⇨ $\partial p/\partial x<0$ : no flow separation.
  - ⇨ $\partial p/\partial x>0$ : could have flow separation, not always leads to flow separation and a wake. $\partial p/\partial x>0$ is a "necessary condition" for flow separation to occur.

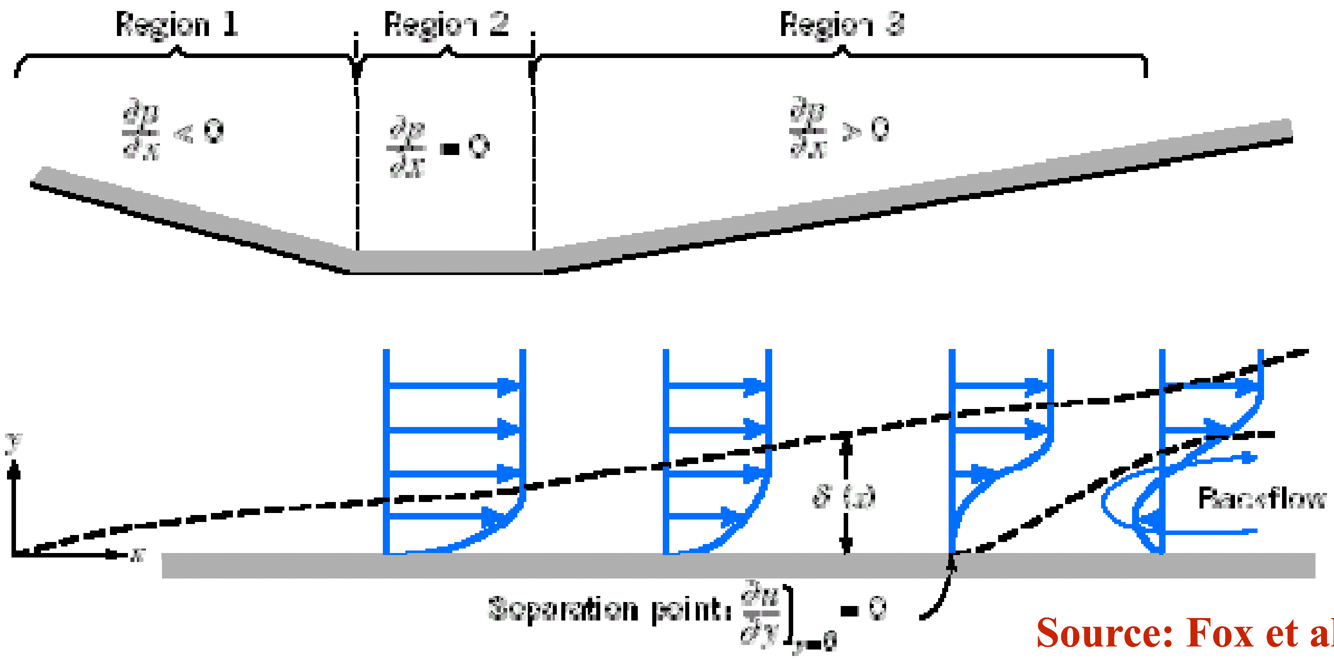
# Effect of Pressure Gradients 3/4

Outside the boundary layer.
The flow accelerates.

Constant velocity

Separation occurs.
The flow decelerates.

? How small the adverse pressure gradient needs to eliminate flow separation?

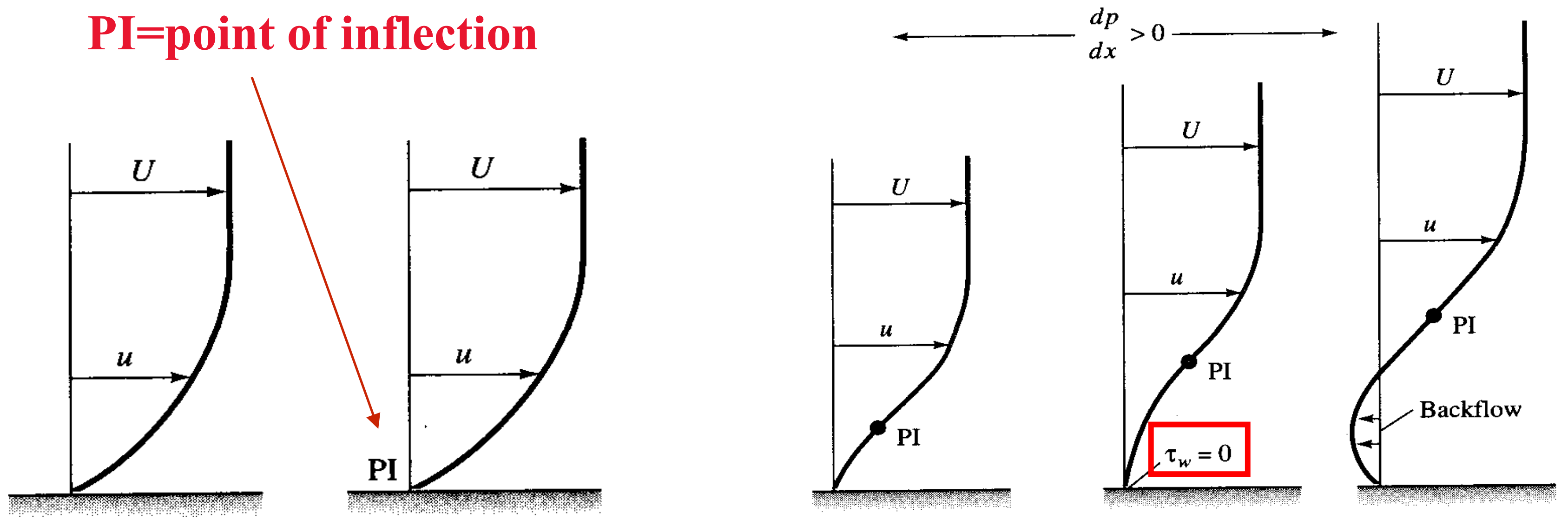



**Source: Fox et al.**

**Fig. 9.6** Boundary-layer flow with pressure gradient (boundary-layer thickness exaggerated for clarity).

# Effect of Pressure Gradients 4/4

**PI=point of inflection**




(a) Favorable gradient: $\frac{dU}{dx} > 0$, $\frac{dp}{dx} < 0$. No separation, PI inside wall

(b) Zero gradient: $\frac{dU}{dx} = 0$, $\frac{dp}{dx} = 0$. No separation, PI at wall

(c) Weak adverse gradient: $\frac{dU}{dx} < 0$, $\frac{dp}{dx} > 0$. No separation, PI in the flow

(d) Critical adverse gradient: Zero slope at the wall: *Separation*

(e) Excessive adverse gradient: Backflow at the wall: Separated flow region

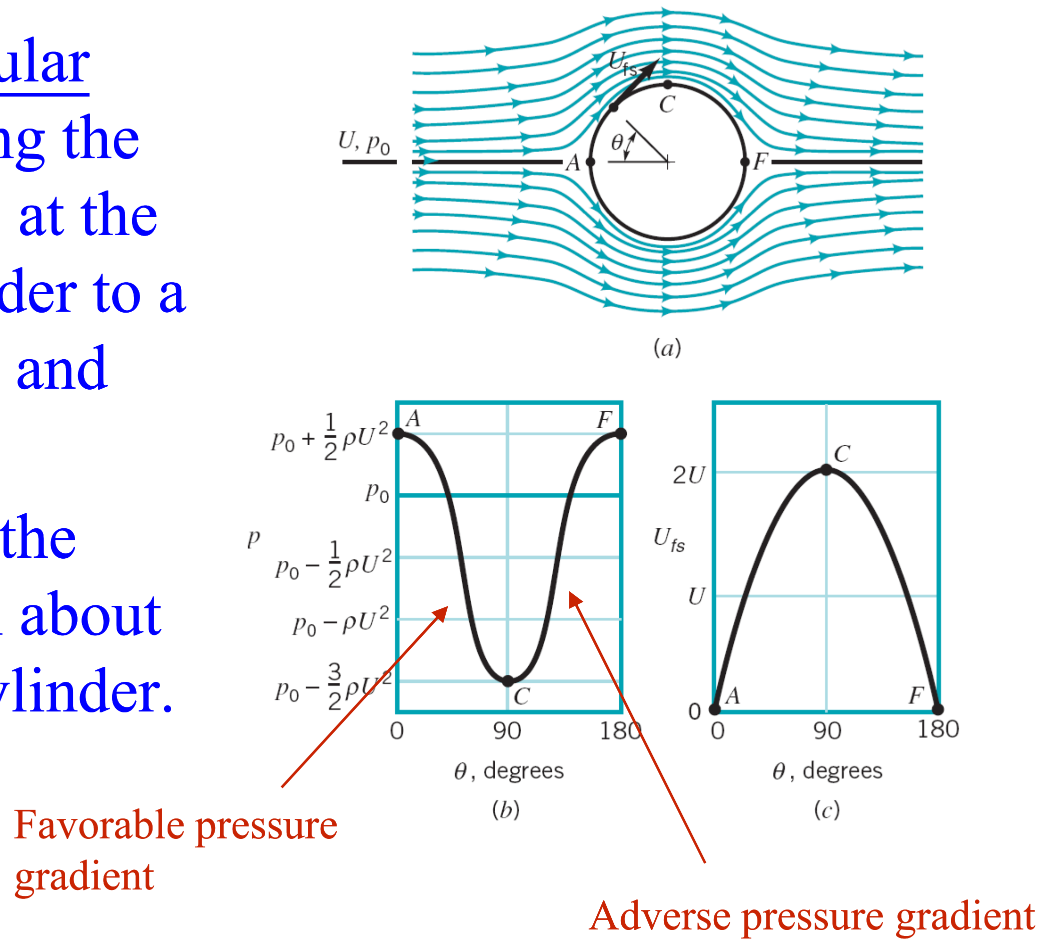
**Source: Fox et al.**

# Effects of Pressure Gradient

## Munson

# Flows Past Circular Cylinder
## Inviscid Flow 1/2

- For "**inviscid**" flow past a circular cylinder, the fluid velocity along the surface would vary from $U_{fs}=0$ at the very front and rear of the cylinder to a maximum of $U_{fs}=2U$ at the top and bottom of the cylinder.
- The pressure on the surface of the cylinder would be symmetrical about the vertical mid-plane of the cylinder.



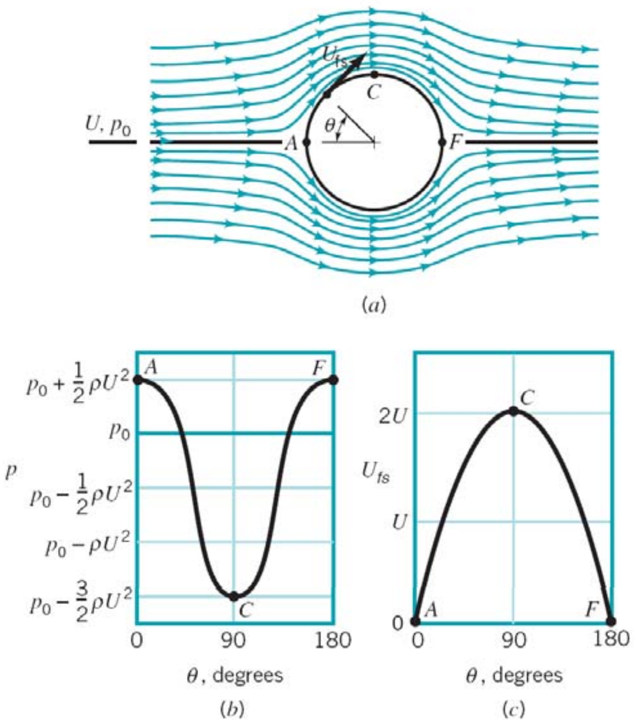
# Flows Past Circular Cylinder

## Inviscid Flow 2/2

❖ The drag on the cylinder is zero.

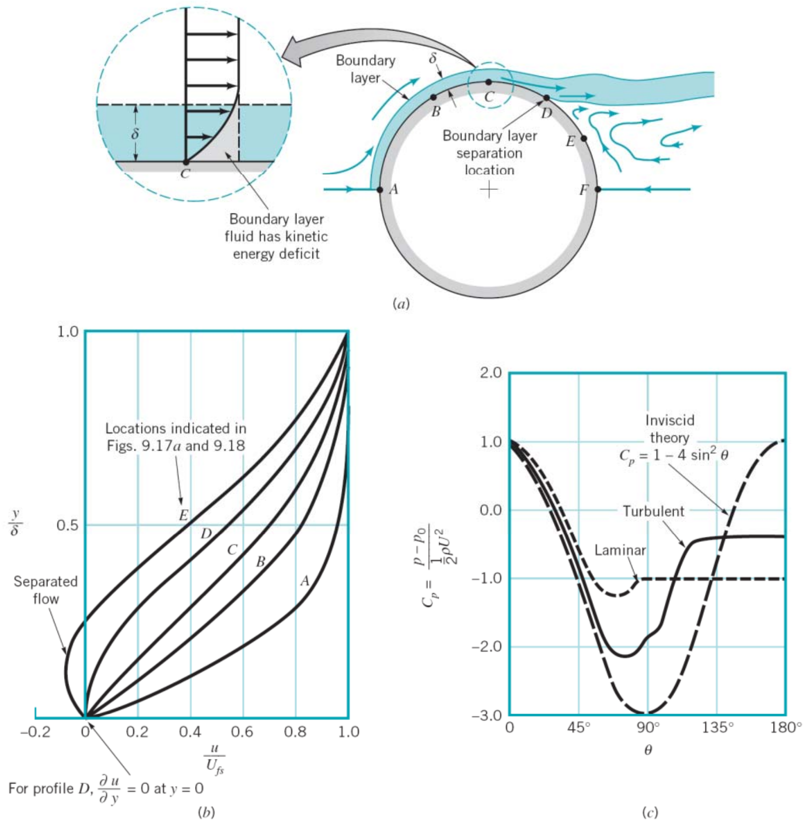
No matter how small the viscosity, there will be a boundary layer that separates from the surface, giving a drag that is independent of the value of μ.

This leads to what has been termed *d'Alembert's paradox*, the drag on an object in an inviscid fluid is zero, but the drag on an object in a fluid with vanishingly small (but nonzero) viscosity is not zero.



# Flows Past Circular Cylinder
## Viscous Flow 1/5

- Consider a fluid particle within the boundary layer. In its attempt to flow from A to F.
- Because of the viscous effects involved, the particle in the boundary layer experiences a loss of energy as it flow along.
- This loss means that the particle does not have enough energy to coast all of the way up the pressure hill ( from C to F) and to reach point F at the rear of the cylinder.



# Flows Past Circular Cylinder

## Viscous Flow 2/5

- The kinetic energy deficit is seen in the velocity profile detail at Point C.
  - ⇨ The situation is similar to a bicyclist coasting down a hill and up the other side of the valley. If there were no friction, the rider starting with zero speed could reach the same height from which he started. Clearly friction, making it impossible for a rider to reach the height from which he started without supplying additional energy.
- The fluid within the boundary layer does not have such an energy supply. Thus, the fluid flows against the increasing pressure as far as it can, at which point the boundary layer separates from (lifts off) the surface.

# Flows Past Circular Cylinder
## Viscous Flow 3/5

- At the separation location D, the velocity gradient at the wall and the wall shear stress are zero.
- Beyond that separation location (from D to E) there is reverse flow in the boundary layer.
- Because of the boundary layer separation, the average pressure on the rear half of the cylinder is considerably less than on the front half.
- Thus, a large pressure drag is developed, even though the viscous shear drag may be quite small.

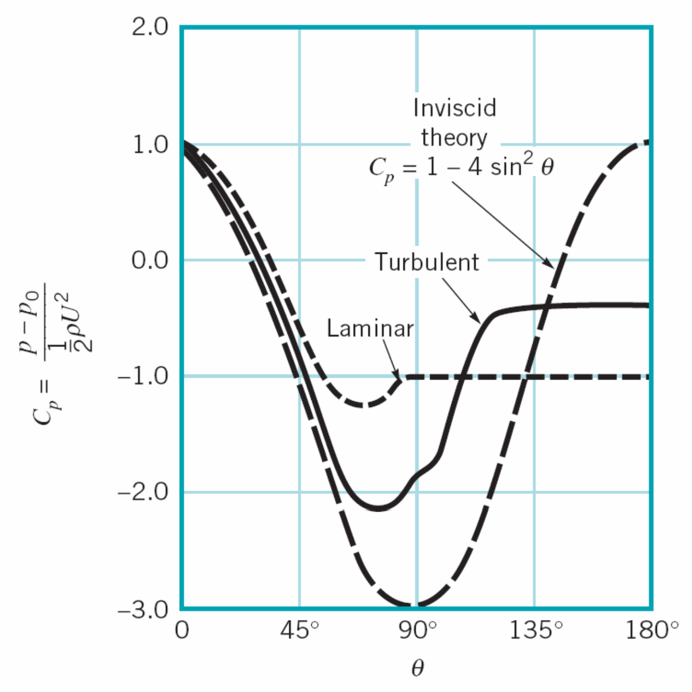
**Drag**= friction drag + pressure drag

# Flows Past Circular Cylinder Viscous Flow 4/5

- The location of separation, the width of the wake region behind the object, and the pressure distribution on the surface depend on the nature of the boundary layer flow.
- Compared with a laminar boundary layer, the turbulent layer flow has more kinetic energy and momentum. Thus, the **turbulent** boundary layer can **flow farther** around the cylinder before it separates than can the laminar boundary layer.

# Flows Past Circular Cylinder Viscous Flow 5/5

Separation occurs when the momentum of fluid layers near the surface is reduced to zero by the combined action of pressure and viscous forces.



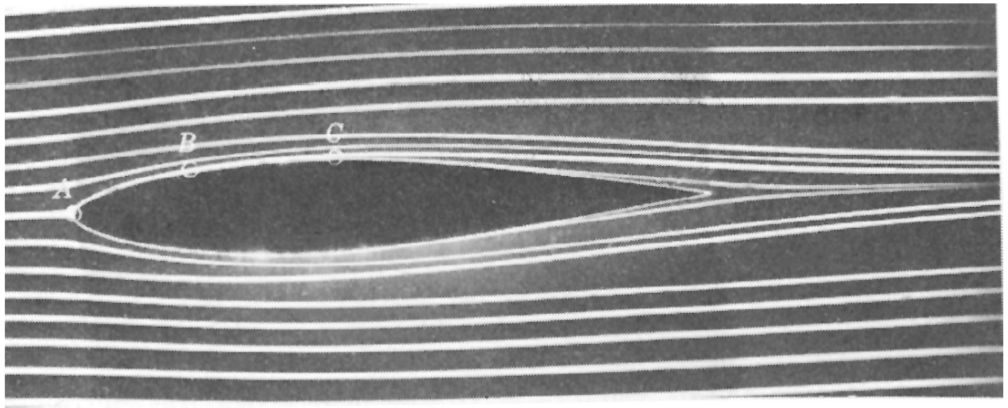

The momentum flux within the turbulent boundary layer is greater than within the laminar layer.

The turbulent layer is better able to resist separation in an adverse pressure gradient.

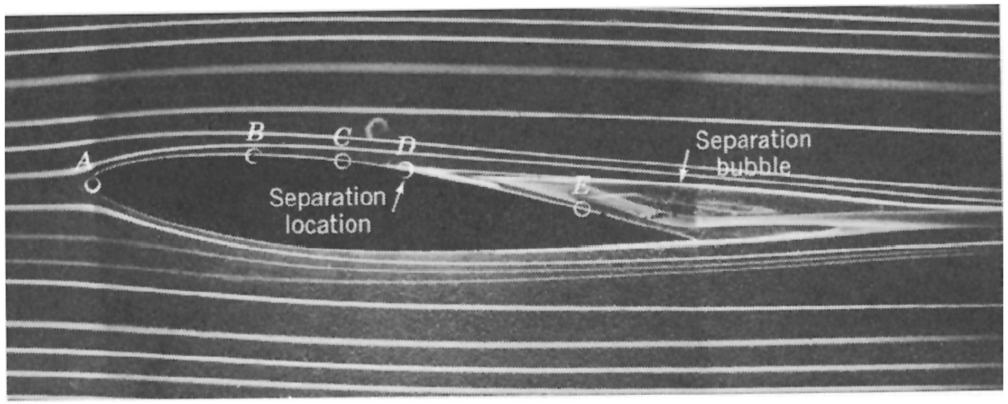
**Drag**= friction drag + pressure drag

# Flow Past an airfoil

Flow visualization photographs of flow past an airfoil: (*a*) zero angle of attack, no separation, (*b*) 5° angle of attack, flow separation.




(*a*)



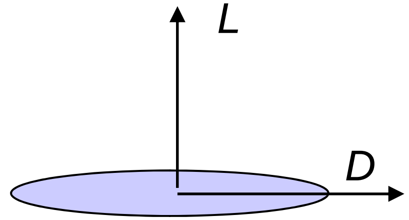

(*b*)

# NEXT…

- DRAG
  - Friction Drag
  - Pressure Drag
  - Drag Coefficient Data and Examples
- LIFT

# Lift and Drag Concepts 2/3

- The resultant force in the downstream direction (by the fluid) is termed the **DRAG**, and the resultant force normal to the upstream velocity is termed the **LIFT**, both of which are surface forces.




$$Drag = D = \underline{e}_x \cdot \int \underline{\sigma}_{(\underline{n})} dA = \underline{e}_x \cdot \int (-p+\tau_{rr})\underline{e}_r + \tau_{r\theta}\underline{e}_\theta dA$$

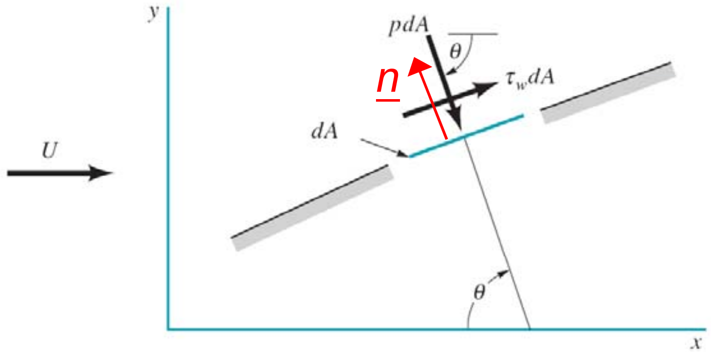
$$= \int (-p+\tau_{rr})(-\cos\theta) + \tau_{r\theta}\sin\theta dA$$

$$Lift = L = \underline{e}_y \cdot \int \underline{\sigma}_{(\underline{n})} dA = \underline{e}_y \cdot \int (-p+\tau_{rr})\underline{e}_r + \tau_{r\theta}\underline{e}_\theta dA$$

$$= \int (-p+\tau_{rr})(\sin\theta) + \tau_{r\theta}\cos\theta dA$$

$$\underline{e}_x \cdot \underline{e}_r = \cos(\pi-\theta) = -\cos\theta, \underline{e}_x \cdot \underline{e}_\theta = \cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta$$

$$\underline{e}_y \cdot \underline{e}_r = \cos\left(\frac{\pi}{2}-\theta\right) = \sin\theta, \underline{e}_y \cdot \underline{e}_r = \cos\theta$$



# DRAG

- Drag $F_D$ is the stream wise component of surface force exerted by a fluid on a body.

$$\text{Drag} = D = \int dF_x = \int p\cos\theta dA + \int \tau_w \sin\theta dA$$

$$Drag = D = \underline{e}_x \cdot \int \underline{t}_{(\underline{n})} dA = \underline{e}_x \cdot \int (-p + \tau_{rr})\underline{e}_r + \tau_{r\theta}\underline{e}_\theta dA$$

$$= \int [(-p + \tau_{rr})(-\cos\theta) + \tau_{r\theta}\sin\theta] dA = \int [(p\cos\theta) + \tau_{r\theta}\sin\theta] dA$$

$$= \int p\cos\theta dA + \int \tau_{r\theta}\sin\theta dA$$

- The drag coefficient $C_D$

$$C_D = \frac{D}{\frac{1}{2}\rho U^2 A}$$

where A is the cross sectional area.

- The **drag coefficient** is a function of object shape, Reynolds number,Re, Mach number, Ma, Froude number, Fr, and relative roughness of the surface, $\varepsilon / \ell$

$$C_D = f(shape, Re, Fr, Ma, \varepsilon / \ell)$$

# Friction Drag

❖ Friction drag is due to the shear stress on the object

$$D_f = \frac{1}{2}\rho U^2 b\ell C_{Df}$$

**$C_{Df}$=f (shear stress, orientation of the surface on which it acts)**

$$C_{Df} = \frac{D_f}{\frac{1}{2}\rho U^2 A}$$ is the friction drag coefficient.

$$\Rightarrow C_{Df} = f\left(Re_\ell, \varepsilon / \ell\right) \quad Re_\ell = \frac{\rho U \ell}{\mu}$$

# Pressure Drag (Form Drag)

- Pressure drag is due to the pressure difference on the object.

$$D_p = \int p \cos\theta \, dA$$

The pressure drag coefficient $C_{Dp}$

$$C_{Dp} = \frac{D_p}{\frac{1}{2}\rho U^2 A} = \frac{\int p \cos\theta \, dA}{\frac{1}{2}\rho U^2 A} = \frac{\int C_p \cos\theta \, dA}{A}$$

Dynamic pressure

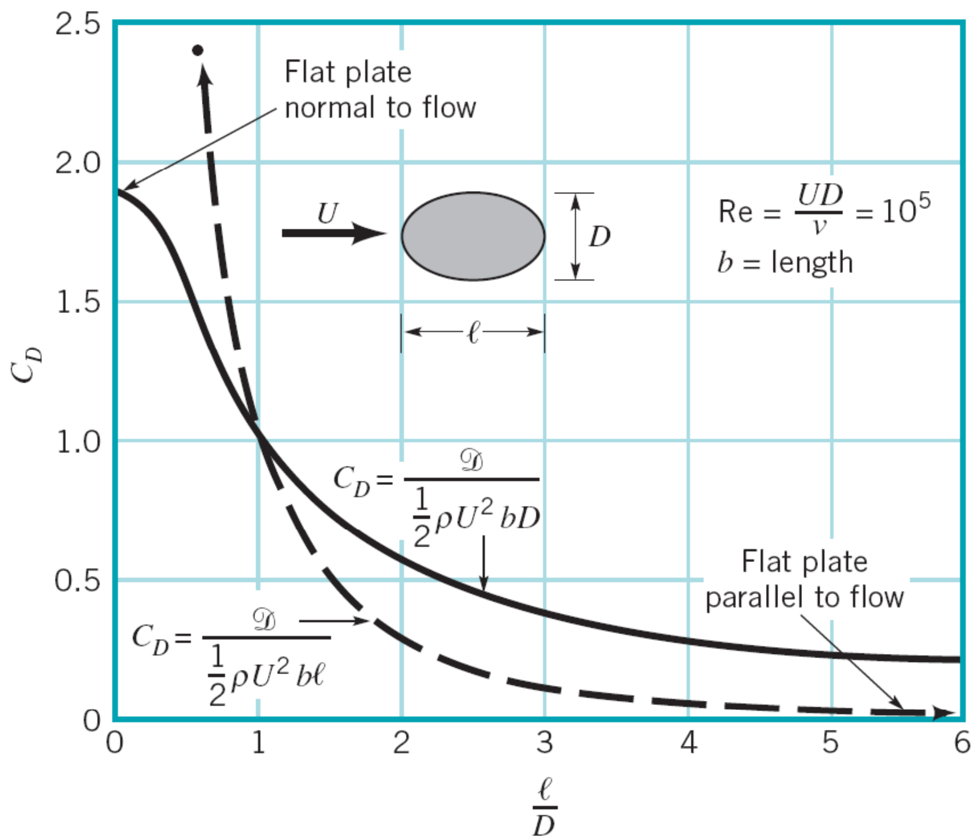
$$C_p = (p - p_0) / (\rho U^2 / 2)$$

$$\Rightarrow C_{Dp} = f(Re_\ell, \varepsilon / \ell)$$

# Drag Coefficient Data and Example

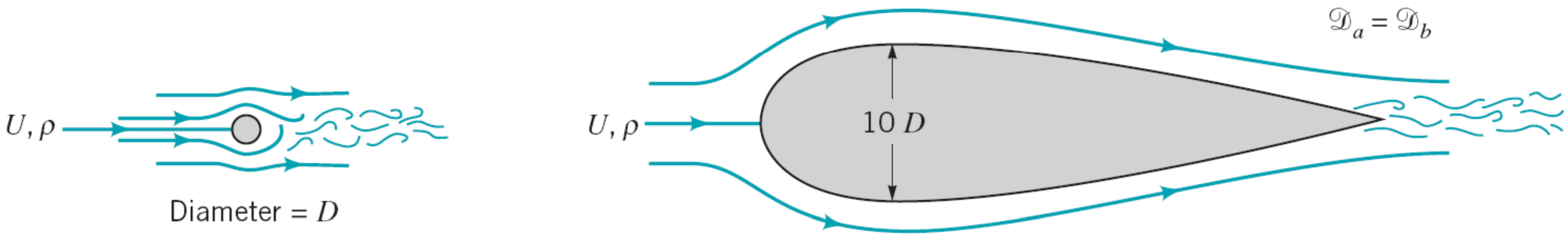
# $C_D$ – Shape Dependence

- The drag coefficient for an object depends on the shape of the object, with shapes ranging from those that are streamlined to those that are blunt.
- Drag coefficient for an ellipse with the characteristic area either the frontal area, A=bD, or the planform area, A=bℓ.



# $C_D$ – Shape Dependence

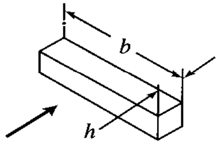
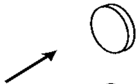
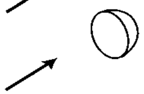
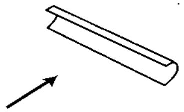
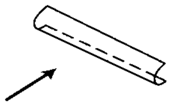
❖ Two objects of considerably different size that have the same drag force: (a) circular cylinder $C_D$=1.2, (b) streamlined strut $C_D$=0.12




(a) (b)

# $C_D$ – Shape Dependence Fox

- Drag coefficient for flow past a variety of objects.
- Re>1000

| Object | Diagram | | $C_D(Re \gtrsim 10^3)$ |
|---|---|---|---|
| Square prism |  | $b/h = \infty$ | 2.05 |
| | | $b/h = 1$ | 1.05 |
| Disk |  | | 1.17 |
| Ring |  | | $1.20^b$ |
| Hemisphere (open end facing flow) |  | | 1.42 |
| Hemisphere (open end facing downstream) |  | | 0.38 |
| C-section (open side facing flow) |  | | 2.30 |
| C-section (open side facing downstream) |  | | 1.20 |

Source: Fox et al.

# $C_D$ – Shape Dependence Fox

- For conventional section NACA0015, the pressure gradient becomes adverse at x/c=0.13, near the point of maximum thickness. The drag coefficient $C_D$=0.0061.
- For laminar-flow section NACA $66_2$-015, the pressure gradient becomes adverse at x/c=0.63. Thus the bulk of the flow is laminar; $C_D$=0.0035.

NACA: National Advisory Committee for Aeronautics

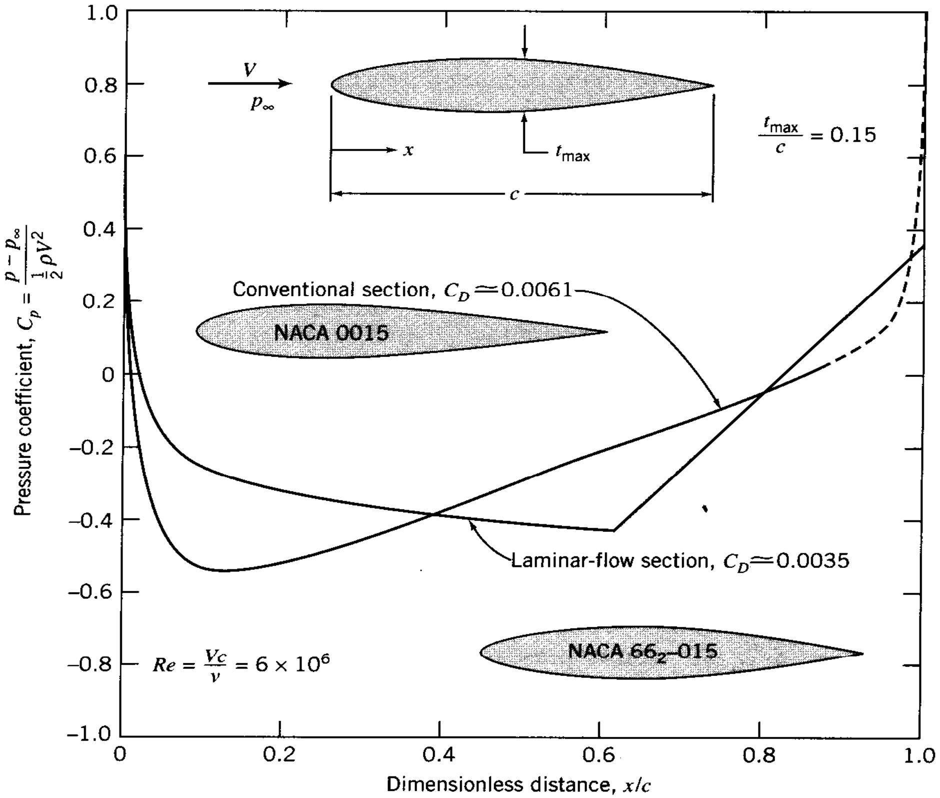
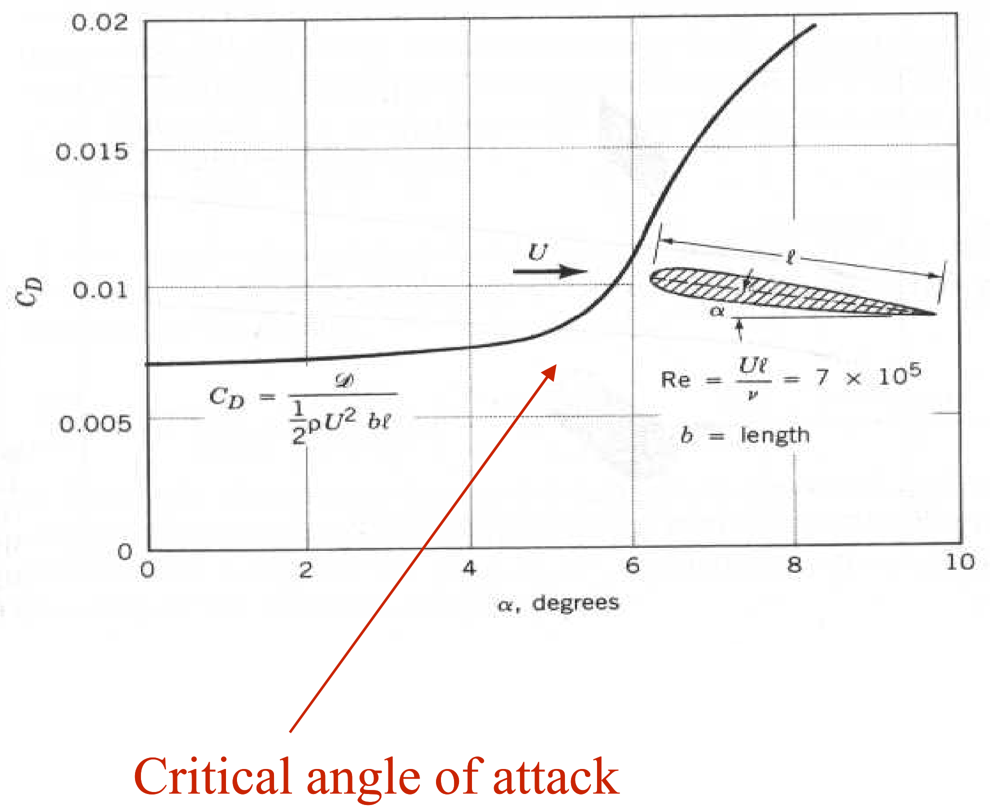



**Fig. 9.15** Theoretical pressure distributions at zero angle of attack for two symmetric airfoil sections of 15 percent thickness ratio. (Data from [21].)

Source: Fox et al.

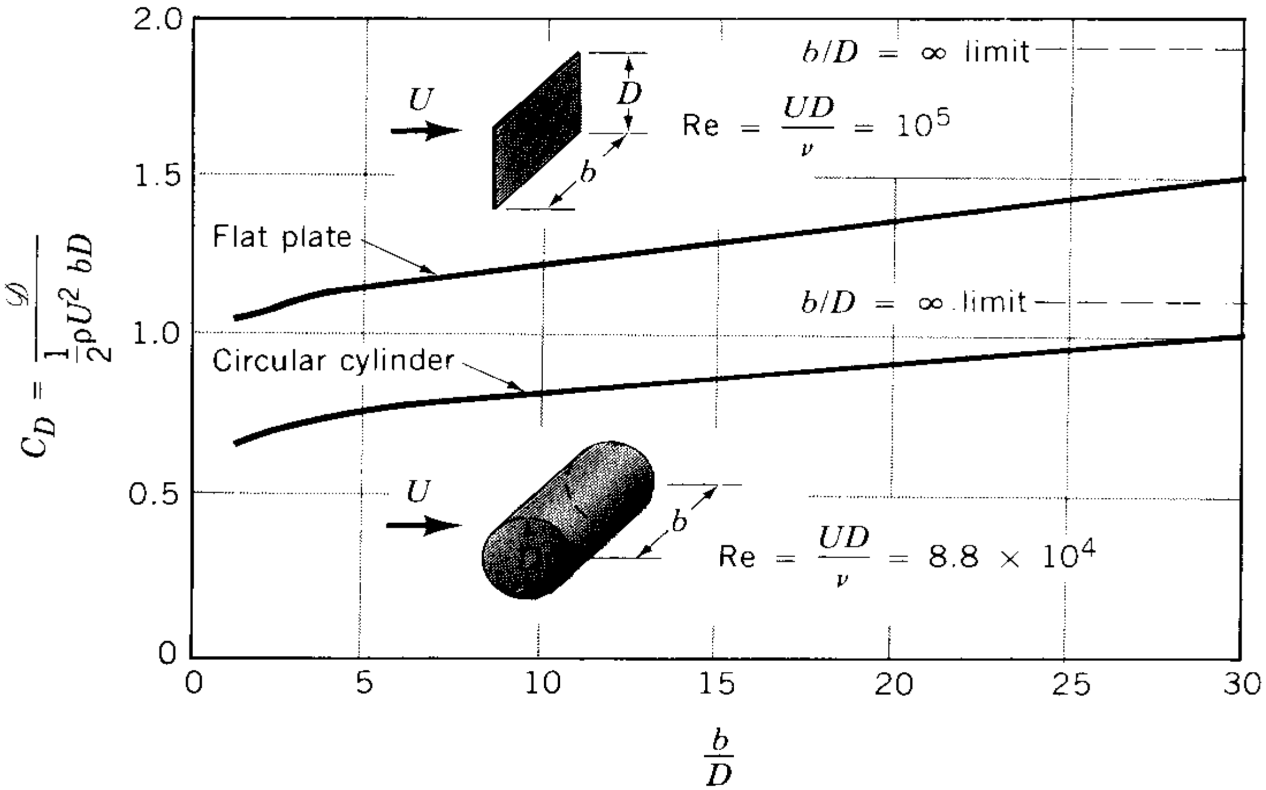
# $C_D$ – Shape Dependence Fox

- The variation of drag coefficient as a function of angle of attack for an airfoil.
- The angle of attack is small, the boundary layer remain attached to the airfoil, and the drag is relatively small.
- For angles larger than critical angle the body appears to the flow as if it were a blunt body, and the drag increases greatly.



# $C_D$ – Shape Dependence Fox

❖ The variation of drag coefficient as a function of aspect ratio for a flat plate normal to the upstream flow and a circular cylinder.



# $C_D$ – Reynolds Number Dependence

- The main categories of Reynolds number dependence are (1) very low Reynolds number flow, (2) Moderate Reynolds number flow, and (3) very large Reynolds number flow.
- For Low Reynolds number flows (Re<1) $\quad D = f(U, \ell, \mu)$

Dimensional analysis ⇒ $D = C\mu\ell U$

$$C_D = \frac{D}{\frac{1}{2}\rho U^2 \ell^2} = \frac{2C\mu\ell U}{\rho U^2 \ell^2} = \frac{2C}{Re}$$

For a sphere $\underline{D} = 6\pi\mu a \underline{U}$ or $\underline{D} = -6\pi\mu a \underline{u}$
where $\underline{u}$ is a velocity of a sphere,
$\underline{U}$ is a free stream velocity,
and $a$ is a radius.

- For moderate Reynolds number

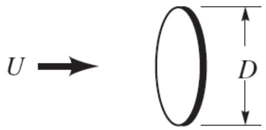
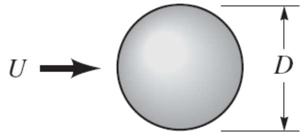
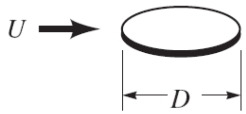
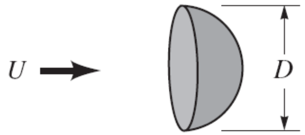
$$C_D \sim Re^{1/2}$$

# $C_D$ – Reynolds Number Dependence

❖ Drag coefficient **for low Reynolds number** flow past a variety of objects.

**TABLE 9.4**

**Low Reynolds Number Drag Coefficients (Ref. 7) ($\mathrm{Re} = \rho UD/\mu, A = \pi D^2/4$)**

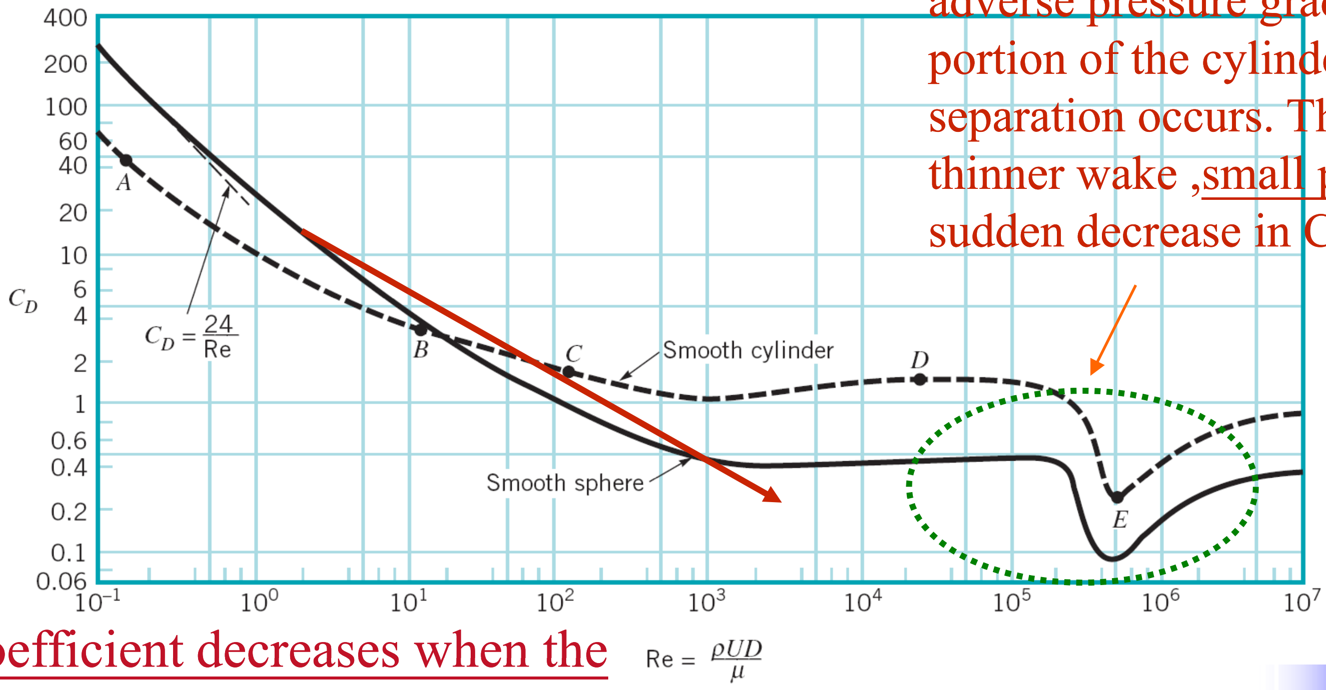
| Object | $C_D = \mathscr{D}/(\rho U^2 A/2)$ (for Re $\lesssim$ 1) | Object | $C_D$ |
|---|---|---|---|
| a. Circular disk normal to flow<br> | 20.4/Re | c. Sphere<br> | 24.0/Re |
| b. Circular disk parallel to flow<br> | 13.6/Re | d. Hemisphere<br> | 22.2/Re |

$$C_D = \frac{D}{\frac{1}{2}\rho U^2 \frac{1}{4}\pi D^2} = \frac{3\pi\mu DU}{\frac{1}{2}\rho U^2 \frac{1}{4}\pi D^2} = \frac{24}{\mathrm{Re}}$$

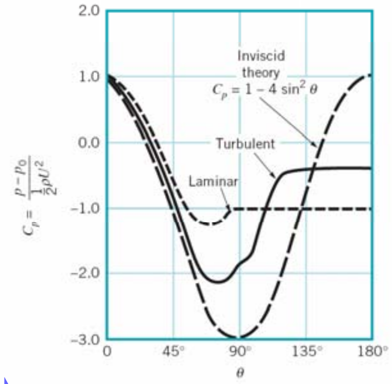
# $C_D$ – Reynolds Number Dependence

❖ Character of the drag coefficient as a function of Reynolds number for a smooth circular cylinder and a smooth sphere.

The turbulent boundary layer travels further along the surface into the adverse pressure gradient on the rear portion of the cylinder before separation occurs. This results a thinner wake ,small pressure drag ,and sudden decrease in $C_D$.



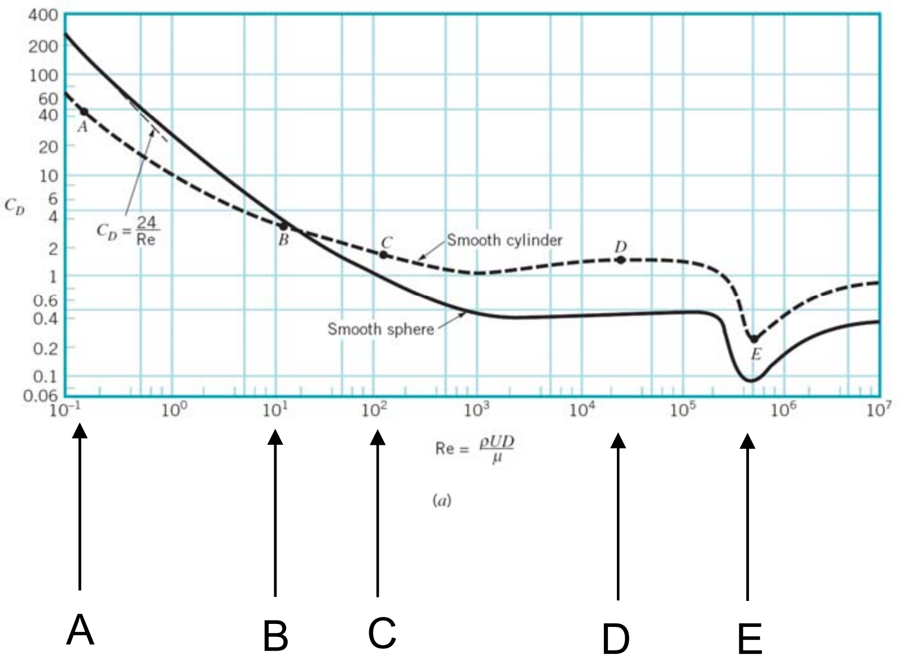
(a)

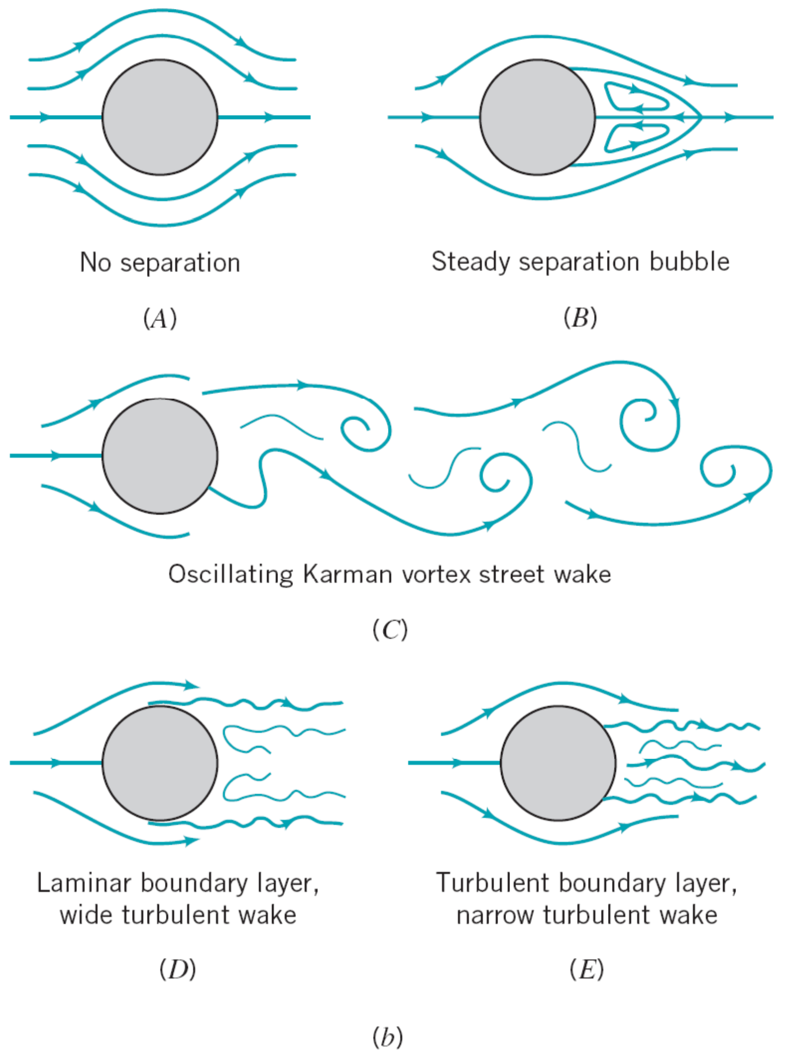



The drag coefficient decreases when the boundary layer becomes turbulent.

# Flow Patterns for Various Reynolds Numbers

☞ The structure of the flow field at selected Reynolds number.

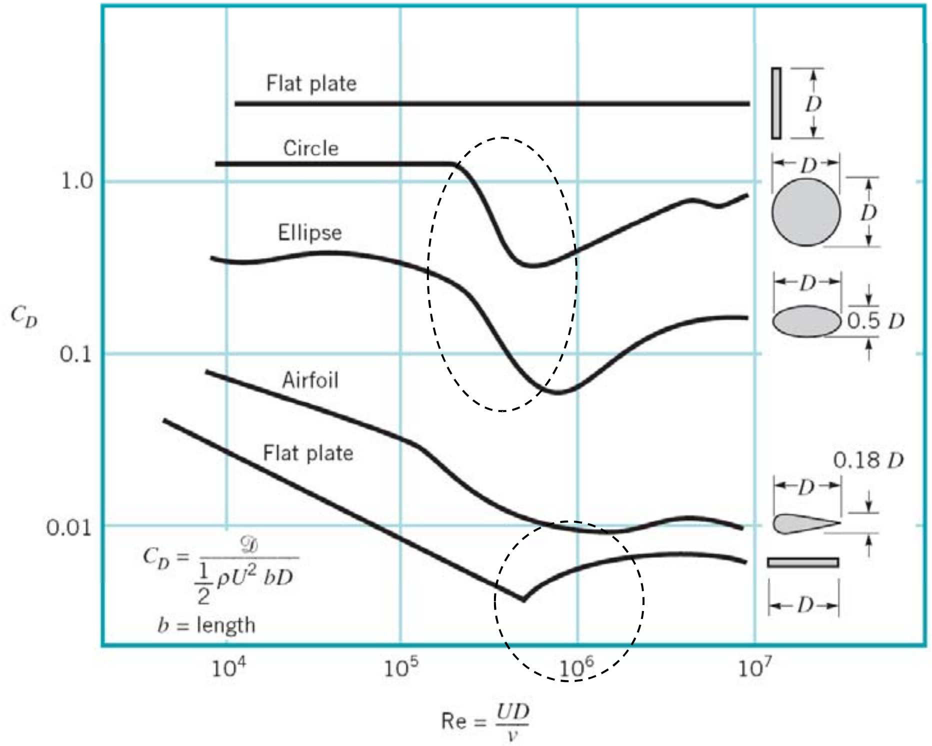


# $C_D$ – Reynolds Number Dependence

❖ Character of the drag coefficient as a function of Reynolds number for objects with various degrees of streamlining, from a flat plate normal to the upstream flow to a flat plate parallel to the flow.

❖On a flat plate with a sharp leading edge in a typical air-stream, **$Re_{xcr}$=5×10⁵**.




For blunt bodies, the drag coefficient decreases when the boundary layer becomes turbulent. (pressure drag decreases but friction drag increases -> total drag decreases)
For streamlined bodies, the drag coefficient increases when the boundary layer becomes turbulent. (friction drag increases)



**Drag**: 1. friction drag 2. pressure drag

# $C_D$ – Surface Roughness 1/3

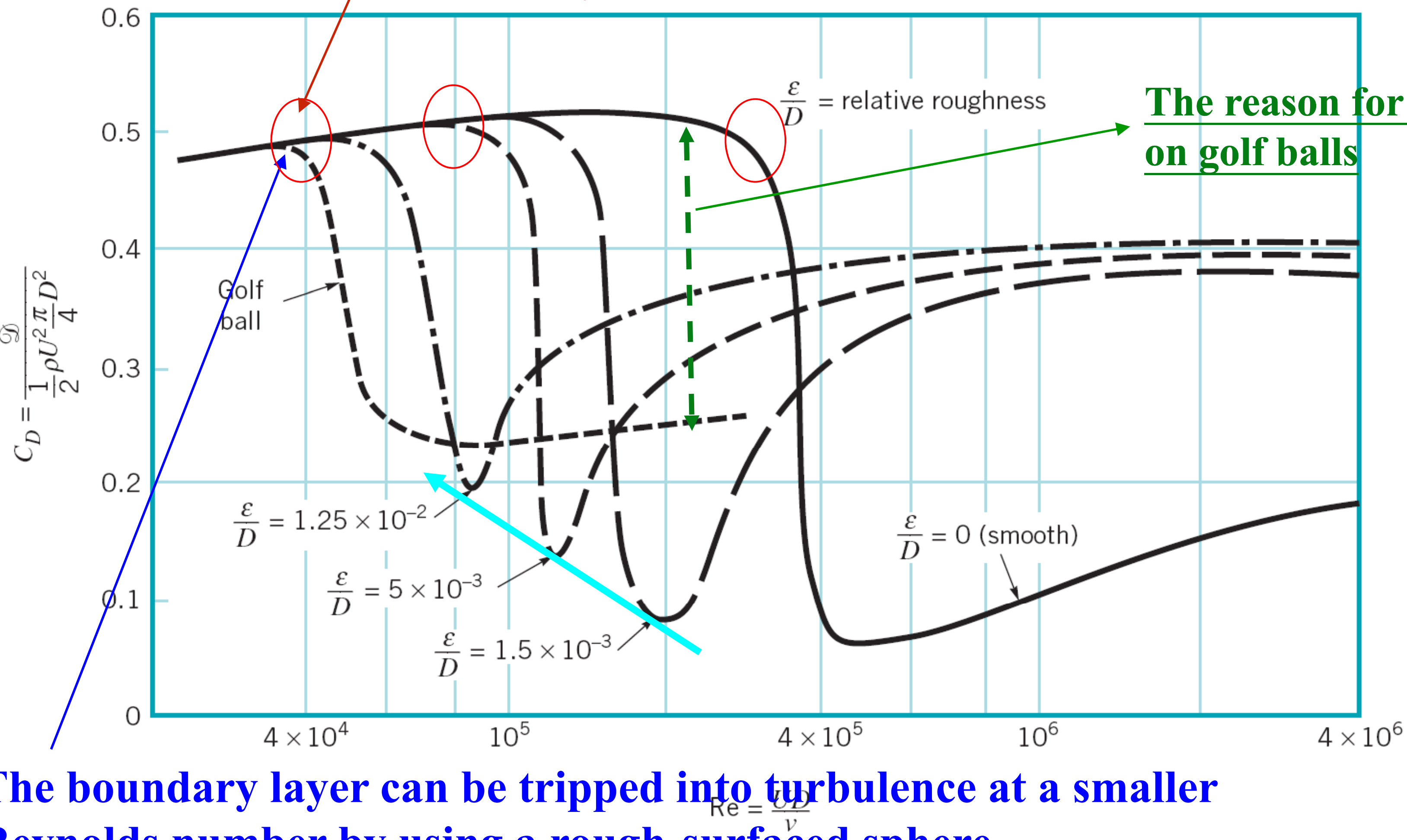
- Surface roughness protrudes through the laminar sub-layers adjacent to the surface and alters the wall shear stress.
- In addition to the increased turbulent shear stress, **surface roughness can alter the Reynolds number at which the boundary layer becomes turbulent**.
- A rough flat plate may have a larger portion of its length covered by a turbulent boundary layer than the corresponding smooth plate.
- For **streamlined** bodies, the **drag increases with increasing surface roughness** because turbulent shear stress is much greater than laminar shear stress.

# $C_D$ – Surface Roughness 2/3

- For extremely blunt body, such as a flat plate normal to the flow, the drag is independent of the surface roughness.
- For **blunt** bodies like a circular cylinder or sphere, **an increase in surface roughness can actually cause a decrease in the drag** - **a considerable drop in pressure drag with a slight increase in friction drag, combining to give a smaller overall drag.**
- The boundary layer can be tripped into turbulence at a smaller Reynolds number by using a rough-surfaced sphere. For example, the critical Reynolds number for a golf bass is approximately $Re=4\times10^4$. In the range of $4\times10^4 < Re < 4\times10^5$, the drag on the standard rough (i.e., dimpled) golf ball is considerably less than for the smooth ball.

# $C_D$ – Surface Roughness 3/3

Critical Reynolds number




The reason for dimples on golf balls

The boundary layer can be tripped into turbulence at a smaller Reynolds number by using a rough-surfaced sphere.

# LIFT 1/3

- Lift is defined as the component of surface force exerted by a fluid on a body perpendicular to the fluid motion.

$$\text{Lift} = L = \int dF_y = -\int p \sin\theta \, dA + \int \tau_w \cos\theta \, dA$$

- The lift coefficient, $C_L$, is defined as

$$C_L = \frac{L}{\frac{1}{2}\rho U^2 A}$$

$$Lift = L = \underline{e}_y \cdot \int \underline{t}_{(\underline{n})} dA = \underline{e}_y \cdot \int (-p + \tau_{rr})\underline{e}_r + \tau_{r\theta}\underline{e}_\theta dA$$

$$= \int [(-p + \tau_{rr})(\sin\theta) + \tau_{r\theta}\cos\theta] dA$$

$$= \int -p \sin\theta dA + \int \tau_{r\theta}\cos\theta dA$$

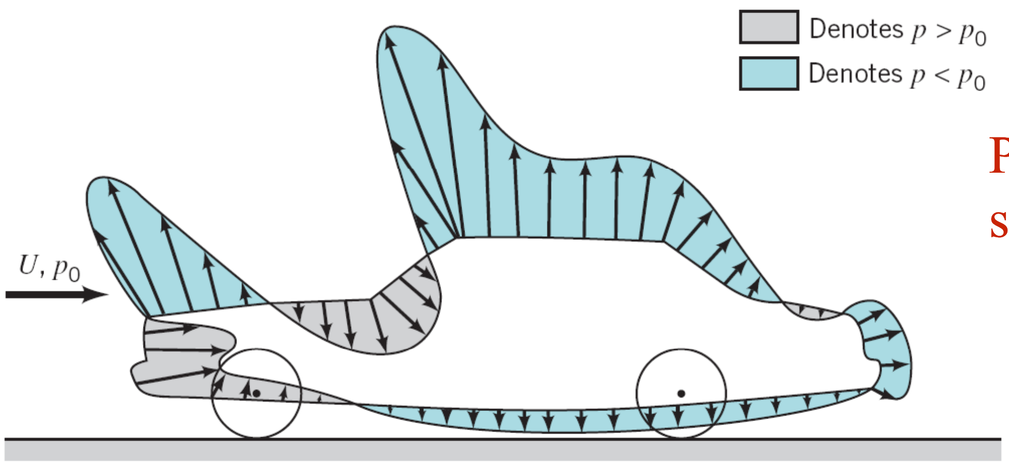
$$C_L = f\left(shape, Re, Fr, Ma, \varepsilon / \ell\right)$$

# LIFT 2/3

- Most common lift-generating devices (i.e., airfoils, fans, spoiler on cars, etc.) operate in the large Reynolds number in which the flow has a boundary layer character, with viscous effects confined to the boundary layers and wake regions.
- Most of the **lift comes from** the surface **pressure** distribution. The **wall shear stress contributes little** to the lift.
- The relative importance of shear stress and pressure effects depends strongly on the Reynolds number. For very low Reynolds number regimes, viscous effects are important, and the contribution of the shear stress to the lift may be as important as that of the pressure.

# LIFT 3/3

- For the most part, the pressure distribution on the surface of an automobile is consistent with simple Bernoulli equation analysis.
- Locations with **high-speed flow** (i.e., over the roof and hood) have **low pressure**, while locations with **low-speed flow** (i.e., on the grill and windshield) have **high pressure**.
- It is easy to believe that the integrated effect of this pressure distribution would provide a net upward force.



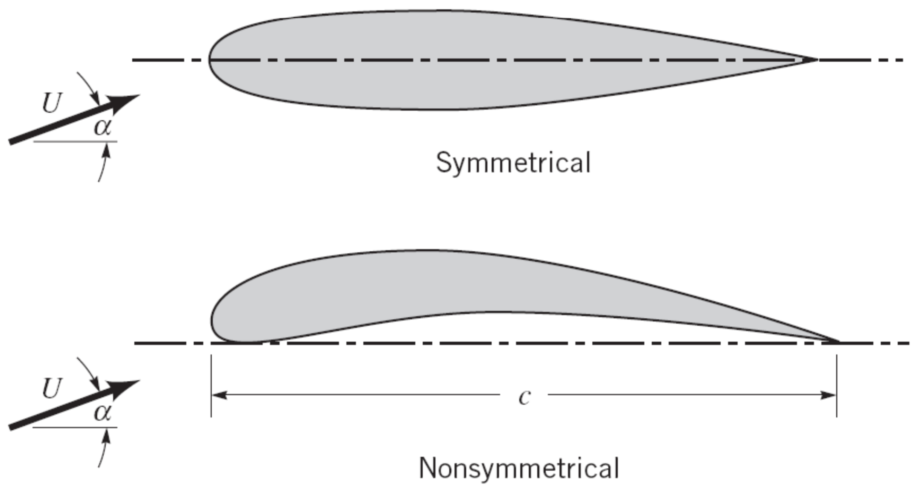

Pressure distribution on the surface of an automobile.

# Airfoil 1/6

- Airfoil is a typical device designed to produce lift.
- Lift is generated by a pressure distribution that is different on the top and bottom surface.
- For large Reynolds number flows, these pressure distribution are usually directly proportional to the dynamic pressure, $\rho U^2/2$, with viscous effects being of secondary importance.

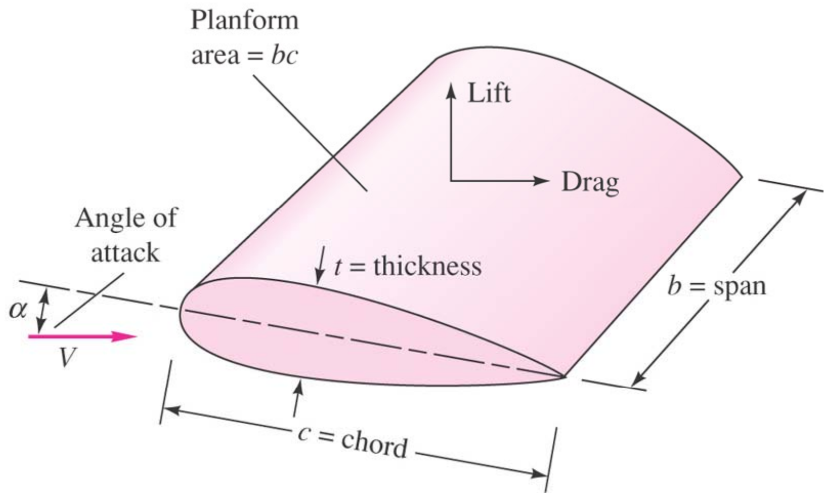
# Airfoil 2/6

- Symmetrical airfoil cannot produce lift if the angle of attack, α , is zero.
- Asymmetry of the nonsymmetrical airfoil could produce lift even with α =0.
- For certain value of α, the pressure distributions on the upper and lower surfaces are different, but their resultant pressure forces will be equal and opposite.



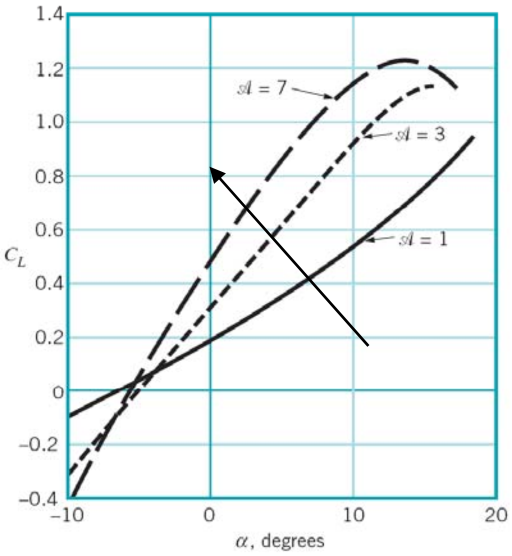
# Definition – Angle of Attack…

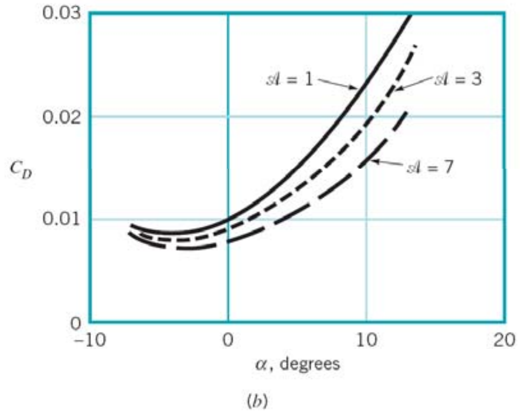
- The angle of attack (α) is the angle between the airfoil chord and the free stream velocity vector.
- The chord length (c) of an airfoil is the straight line joining the leading edge and the trailing edge.
- The aspect ration (A) is defined as the ratio of the square of the length of the airfoil (b = *l*) to the planform area ($A_p$=bc). A =$b^2/A_p$=b/c.



# Airfoil 3/6

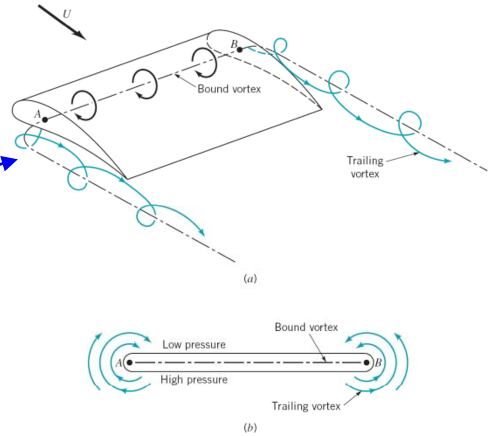
- The lift and drag coefficient is a function of angle of attack, α, and aspect ratio, **A**. The aspect ratio is defined as the ratio of the square of the wing length (b) to the planform area ($A_p$=bc) , **A**= $b^2/A_p$.
- The **lift coefficient increases and the drag coefficient decreases with an increase in aspect ratio (-> longer wings).**
- Long wings are more efficient because their wing tip losses are relatively minor than for short wings.



# Airfoil 4/6

- The increase in drag due to the finite length ($\mathbf{A} < \infty$) of the wing is often termed induced drag. It is due to the interaction of the complex swirling flow structure near the wing tips and the free stream.
- High performance soaring airplanes and highly efficient soaring birds (i.e., the albatross and sea gull) have long, narrow wings (-> large aspect ratio wings). Such wings, however, have considerable inertia that inhibits rapid maneuvers. Thus, highly maneuverable fighter or acrobatic airplanes and birds (i.e., the falcon) have small-aspect-ratio wings.



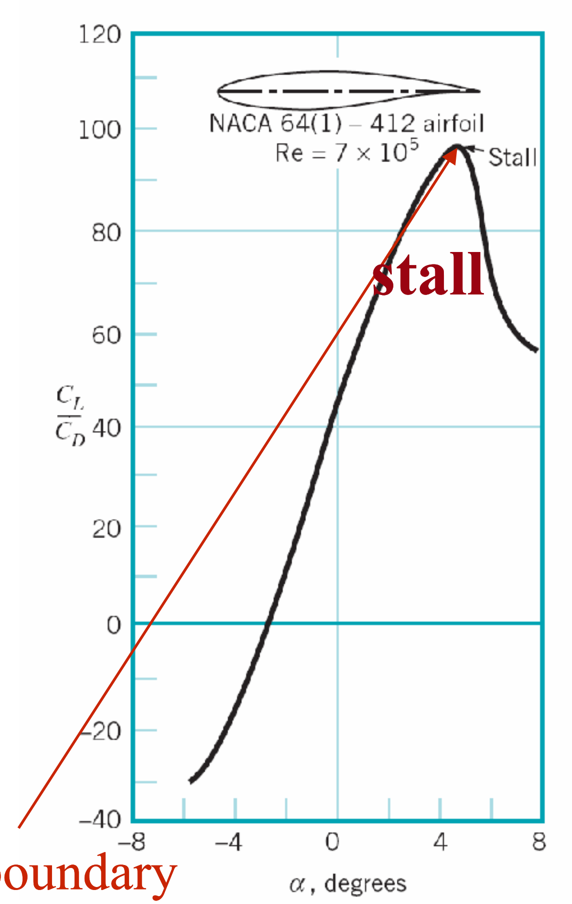
# Airfoil 5/6

- Although viscous effects and the wall shear stress contribute little to the direct generation of lift, they play an important role in the design and use of lifting devices.
- The viscosity-induced boundary layer separation can occur on non-streamlined bodies such as airfoils that have too large and angle of attack.
- As the angle of attack is increased, the boundary layer on the upper surface separates, the flow over the wing develops a wide, turbulent wake region, the lift decreases, and the drag increases.

  ⇨ **Airfoil stall results.**

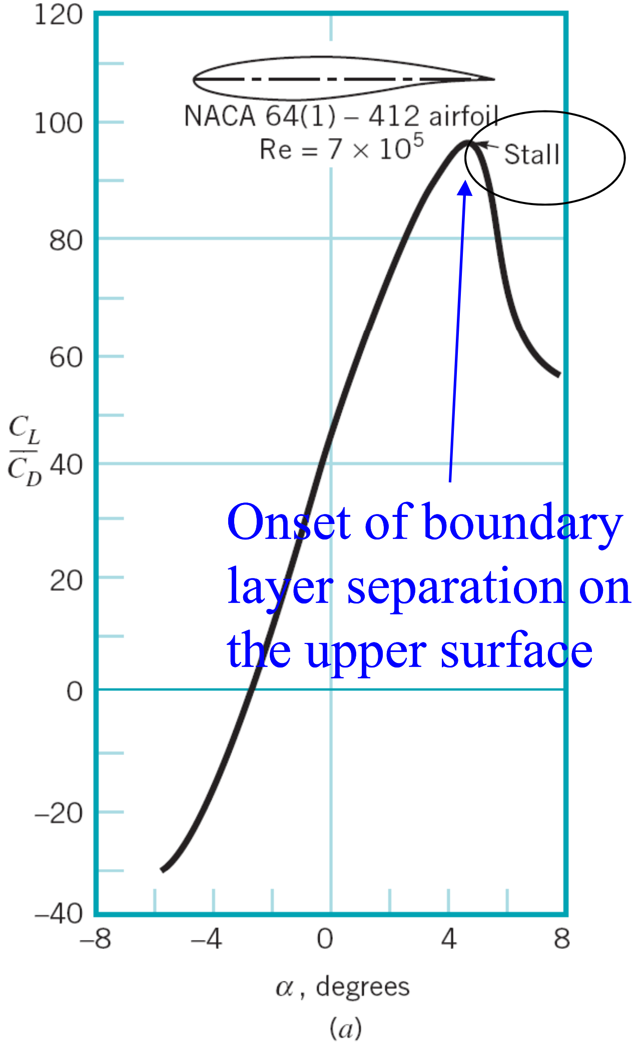
# Airfoil 6/6

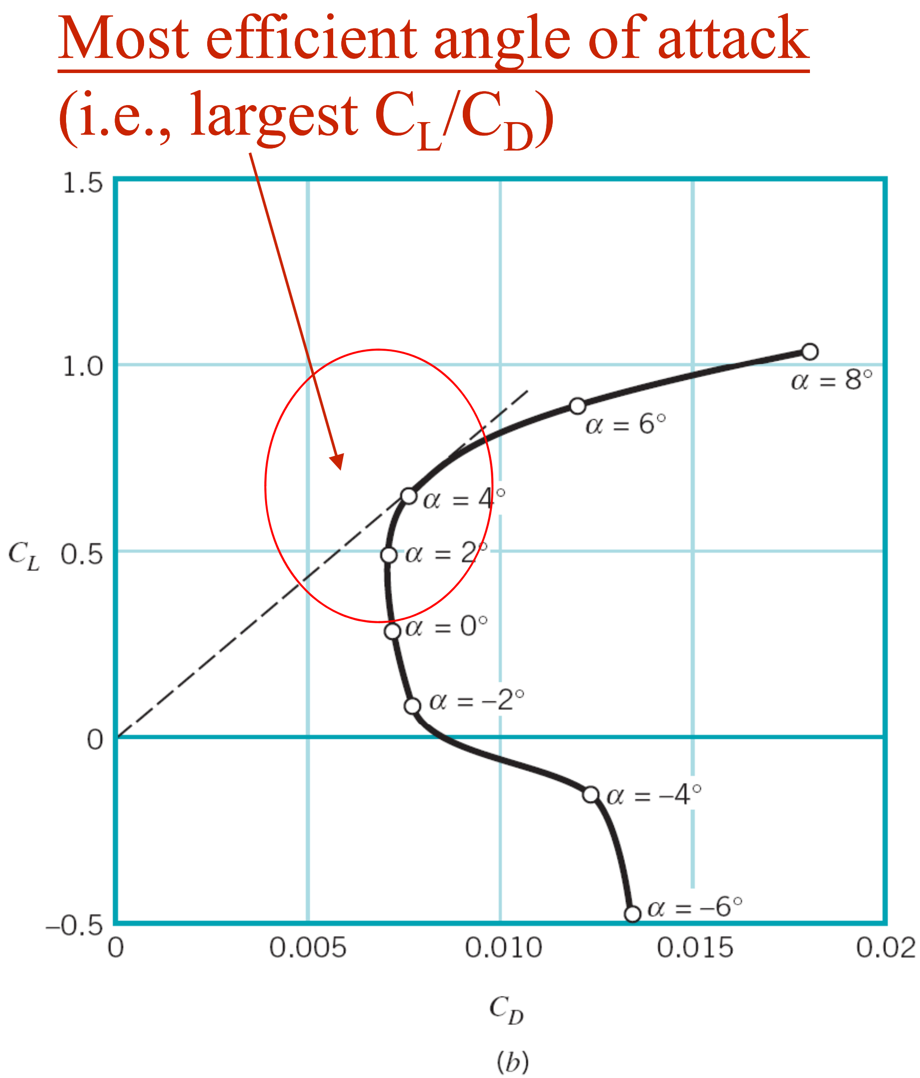
- Such conditions are extremely dangerous if they occur while the airplane is flying at a low altitude where there is not sufficient time and altitude to recover from the stall.
- As the angle of attack is increase, the △ p between the upper and lower surfaces increase, causing the lift coefficient to increase smoothly until a maximum is reached. Further increases in angle of attack produce a sudden decrease in $C_L/C_D$.




Onset of boundary layer separation on the upper surface

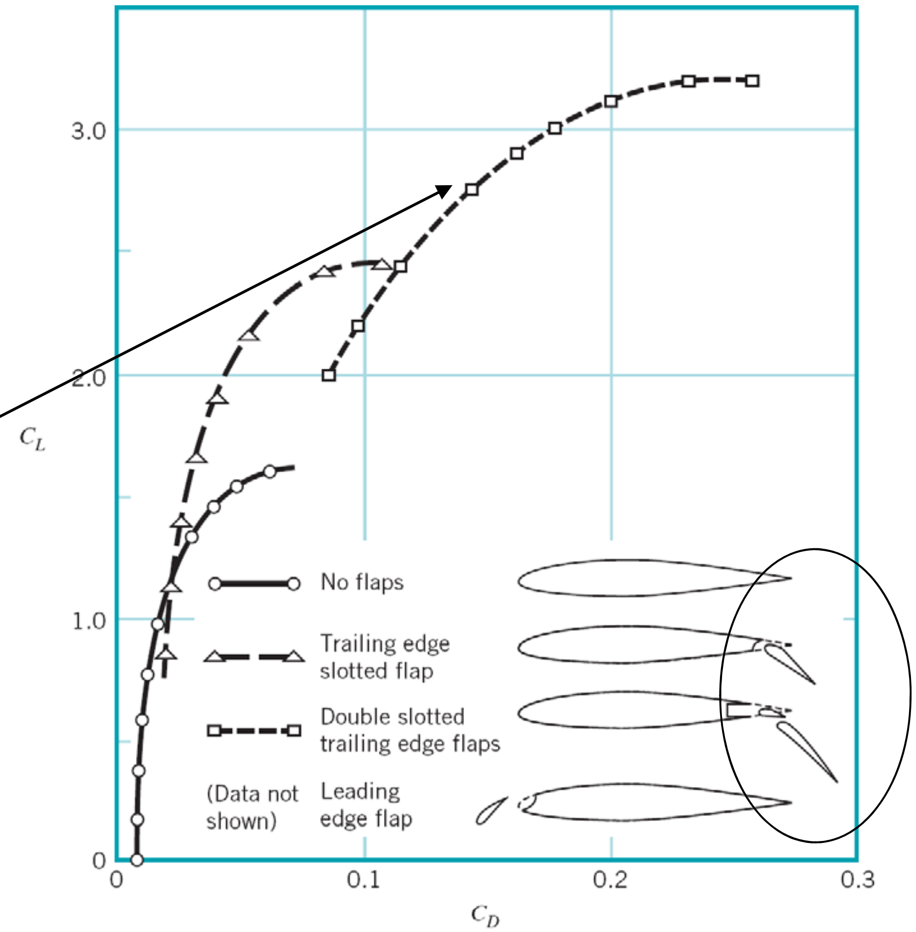
# $C_L/C_D$ vs. α, $C_L$ vs. $C_D$



# Lift Control Devices 1/2

- To generate necessary lift during the relatively low-speed landing and takeoff procedures, the airfoil shape is altered by extending special flaps on the front and/or rear portion of the wing.
- Use of the flaps considerably enhances the lift, although it is at the expense of an increase in the drag



# Lift Control Devices 2/2

- Application of high-lift boundary layer control devices to reduce takeoff speed of a jet transport aircraft.
- In the landing configuration, large slotted trailing-edge flaps roll out from under the wing and deflect downward to increase the lift coefficient. After touchdown, spoiler are raised in front of each flap to decrease lift and ensure that the plane remains on the ground.
- In the takeoff configuration, large slotted trailing-edge flaps deflect to increase the lift coefficient.