1.0 Outline 7

1.1 Basic Concepts

Space The *region* occupied by bodies. Their *positions* and *orientations* can be described by linear and angular measurements relative to a specified *coordinate system*.

Time The measure of the *succession of events*. Often, the change in the physical quantities are described with respect to time, e.g., $\bar{v} = \frac{d\bar{r}}{dt}$

Mass The measure of the *inertia of a body*, which indicates the resistance to a change in its velocity.

Force The measure of the *attempt to move a body*. It is a *fixed vector*.

1.1 Basic Concepts

Particle A body of *negligible dimensions*. This is a *relative matter* to the surrounding effect. Hence, rotation effect is insignificant.

Rigid body A body whose *relative movement between its parts* are *negligible* relative to the *gross motion of the body*.

Nonrigid body A body whose *relative movement between its parts* are *significant* relative to the *gross motion of the body*. Mechanics of the deformable material.

Particles	Objects
differential element analysis of the body	Molecular effects in the body
Flight speed of the airplane	Yawing, pitching motion of the plane

Rigid bodies	Nonrigid bodies
Tension in the truss structure	Material for constructing the truss
A stiff linkage of the robot	An n-joint robot arm

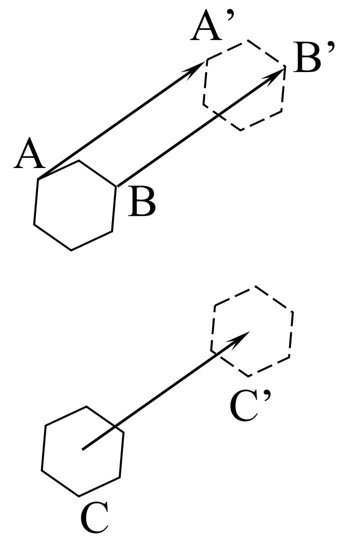
The body-fixed inertia of the nonrigid body is not constant.

1.2 Scalars and Vectors

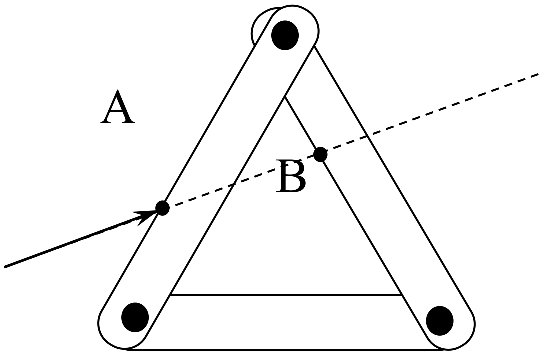
Scalars Quantities for which only the magnitude can describe completely. Time, volume, density, speed, energy,
and mass are some examples.

Vectors Quantities for which both the magnitude and the direction must be used to describe. For example, displacement, velocity, acceleration, force, moment, momentum.

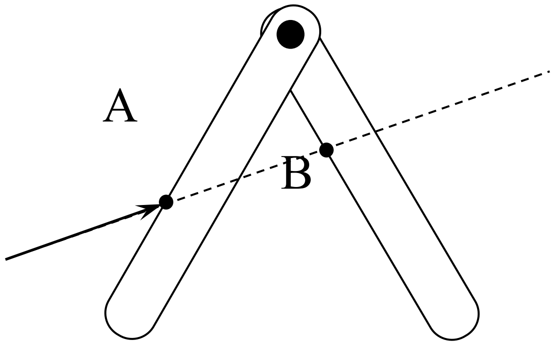
Free vector A vector whose action is not confined with a unique line in space, i.e., only its *magnitude* and *direction* do matter. For example, displacement vector of a pure translational rigid object, or couple vector of a rigid body.



Sliding vector A vector whose *line of action* must be specified in addition to its *magnitude* and *direction*.
For example, external force/moment acting on the rigid body.



Fixed vector A vector whose *magnitude*, *direction*, *line of action*, and *point of application* are all important. For example, external force/moment applied onto the nonrigid body. Deformable effect of the object is important.

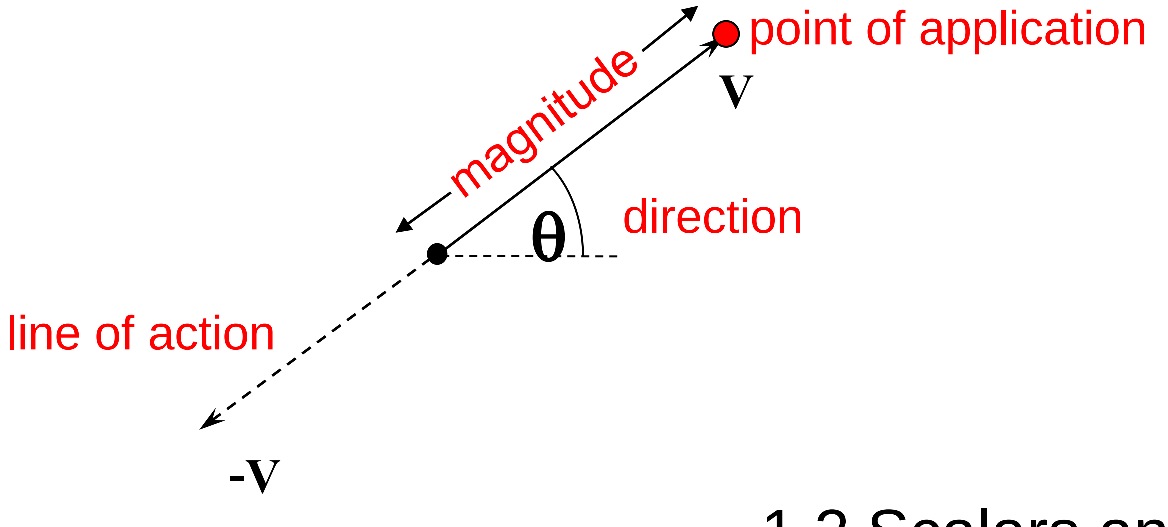


Representation of the Vector

Notation vector $\mathbf{v}, \bar{v}, \vec{v}, \underline{v}$

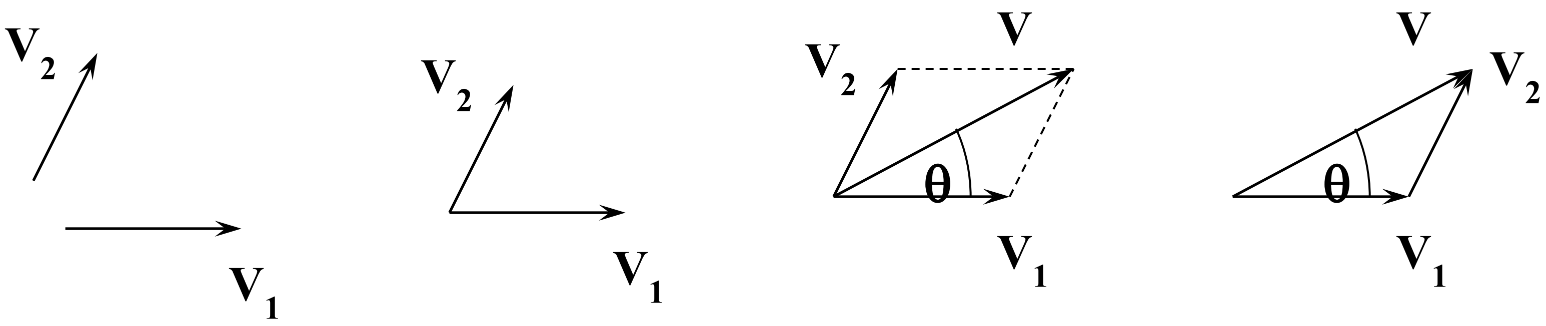
magnitude $|\mathbf{v}|, v$

Graphical Representation by a scaled line segment, with an arrowhead to indicate *magnitude*, *direction*, *line of action*, and *point of application*.



Vector Operations

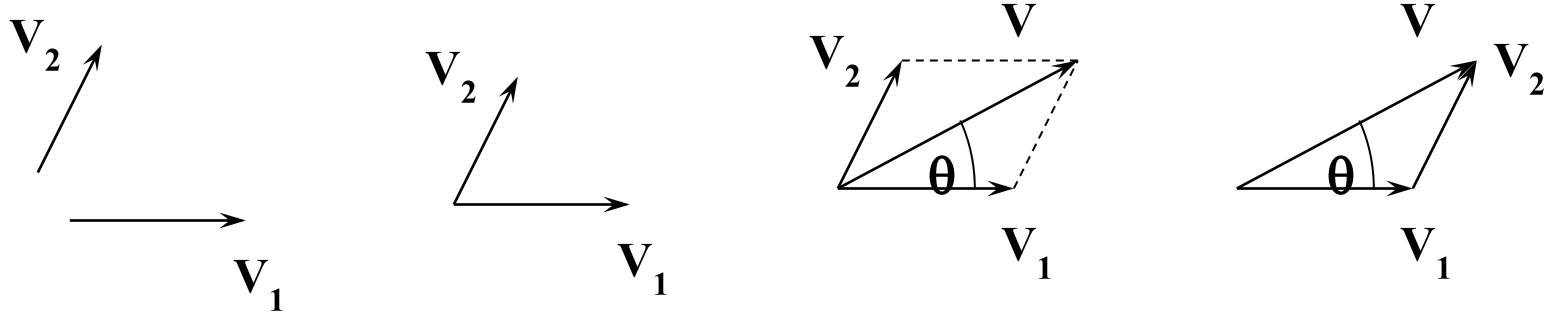
- Addition $\mathbf{V} = \mathbf{V}_1 + \mathbf{V}_2$



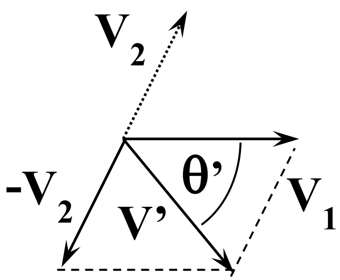
- Principle of transmissibility
- Parallelogram law
- Head to tail
- Line of action obtained separately (parallelogram law / principle of moment)
- point of application
- i.e., only vector magnitude and direction are ensured

Vector Operations

- Addition $\mathbf{v} = \mathbf{v}_1 + \mathbf{v}_2$



- $\mathbf{v}_1 + \mathbf{v}_2 = \mathbf{v}_2 + \mathbf{v}_1$ commutative law
- $\left(\mathbf{v}_1 + \mathbf{v}_2\right) + \mathbf{v}_3 = \mathbf{v}_1 + \left(\mathbf{v}_2 + \mathbf{v}_3\right)$ associative law
- $\mathbf{v}_1 - \mathbf{v}_2 = \mathbf{v}_1 + \left(-\mathbf{v}_2\right)$ vector subtraction

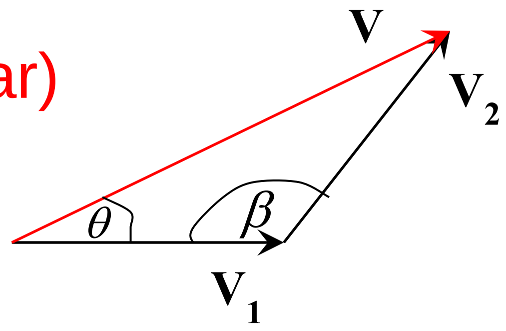


Magnitude and Direction Calculation (algebraic approach)

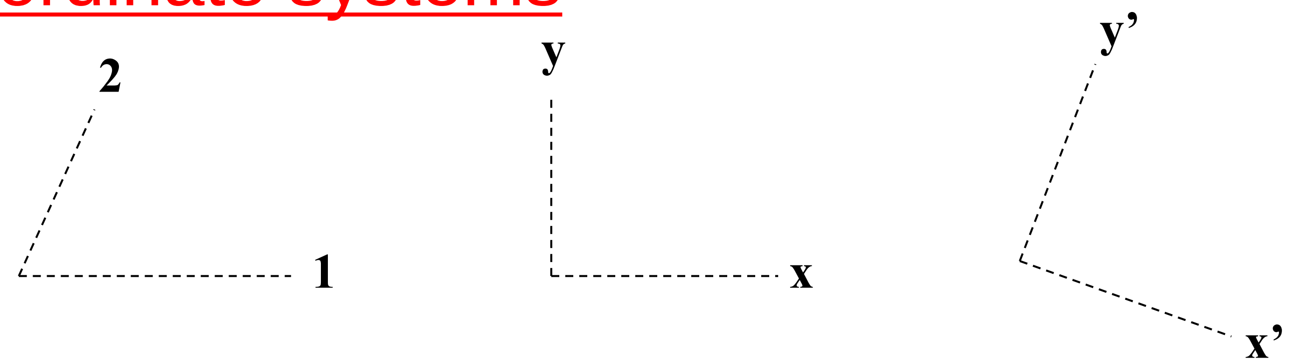
Cosine law: $v^2 = v_1^2 + v_2^2 - 2v_1v_2\cos\beta$

Sine law: $\frac{v}{\sin\beta} = \frac{v_2}{\sin\theta}$ + kill impossible angle

2 unknowns (scalar)

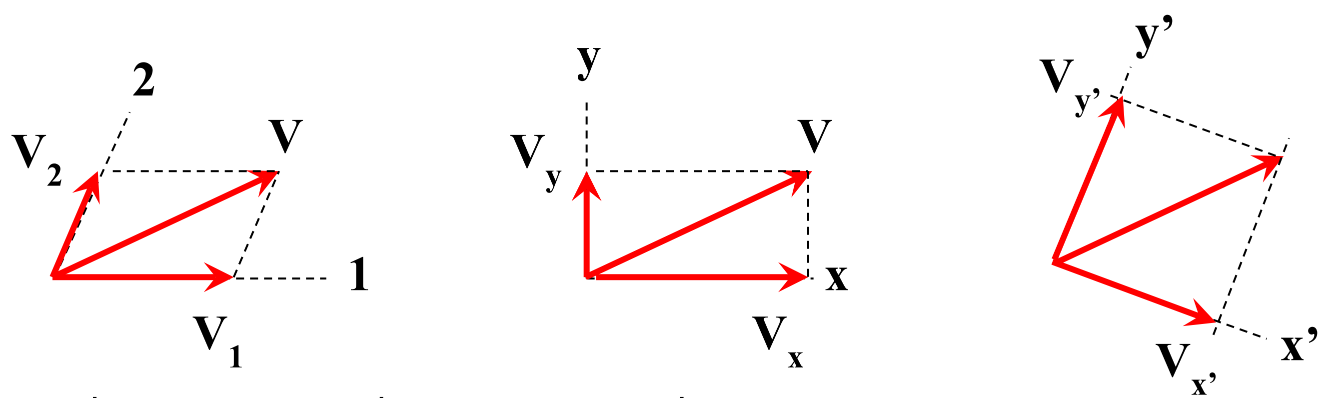


Coordinate systems



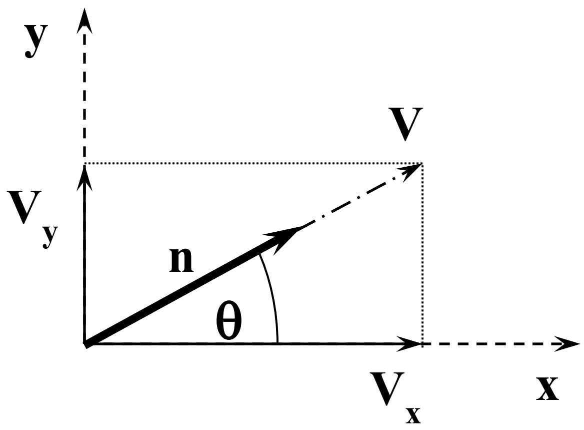
Rectangular coord. system is easier to manipulate

Vector Components along the coordinate axis directions



$$\mathbf{v} = \mathbf{v}_1 + \mathbf{v}_2 = \mathbf{v}_x + \mathbf{v}_y = \mathbf{v}_{x'} + \mathbf{v}_{y'}$$

2-D Rectangular Coordinate systems



$$\mathbf{v} = v\hat{\mathbf{n}} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}}$$

$$\hat{\mathbf{n}} = \cos\theta\,\hat{\mathbf{i}} + \sin\theta\,\hat{\mathbf{j}} = \frac{1}{v}\mathbf{v}$$

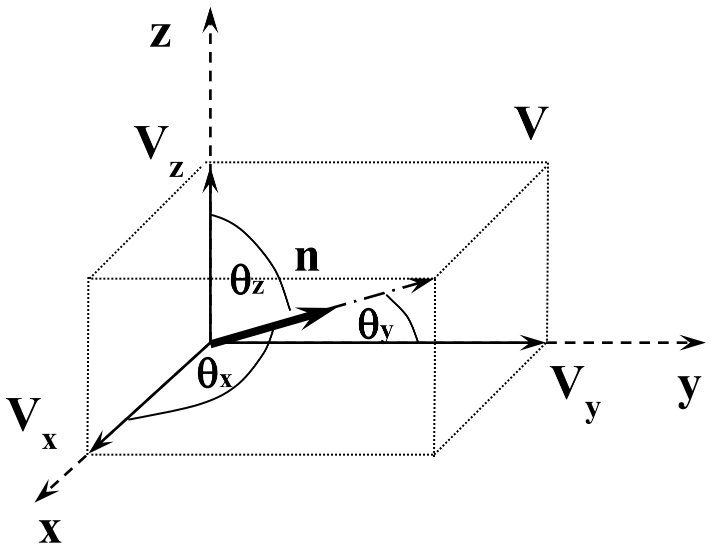
$$v^2 = v_x^2 + v_y^2$$

$$v_x = v\cos\theta$$

$$v_y = v\sin\theta$$

$$\theta = \arctan 2\left(v_y, v_x\right)$$

3-D Rectangular Coordinate systems



$$\mathbf{v} = v\hat{\mathbf{n}} = v_x\hat{\mathbf{i}} + v_y\hat{\mathbf{j}} + v_z\hat{\mathbf{k}}$$

$$\hat{\mathbf{n}} = \cos\theta_x\hat{\mathbf{i}} + \cos\theta_y\hat{\mathbf{j}} + \cos\theta_z\hat{\mathbf{k}} = \frac{1}{v}\mathbf{v}$$

$$v^2 = v_x^2 + v_y^2 + v_z^2$$

$$v_x = v\cos\theta_x = \mathbf{v}\bullet\hat{\mathbf{i}}$$

$$v_y = v\cos\theta_y = \mathbf{v}\bullet\hat{\mathbf{j}}$$

$$v_z = v\cos\theta_z = \mathbf{v}\bullet\hat{\mathbf{k}}$$

$$\cos\theta_x = \frac{1}{v}\mathbf{v}\bullet\hat{\mathbf{i}}$$

$$\cos\theta_y = \frac{1}{v}\mathbf{v}\bullet\hat{\mathbf{j}}$$

$$\cos\theta_z = \frac{1}{v}\mathbf{v}\bullet\hat{\mathbf{k}}$$

$$\cos^2\theta_x + \cos^2\theta_y + \cos^2\theta_z = 1$$

Newton's Laws

1st Law: A particle remains at rest or continues to move in a straight line with a uniform velocity if there is no unbalanced force acting on it.

$$\sum \mathbf{F} = \mathbf{0} \Leftrightarrow \mathbf{a} = \mathbf{0}$$

Newton's Laws

2nd Law: The *absolute* acceleration of a particle is proportional to the resultant force acting on it and is in the direction of this resultant force.

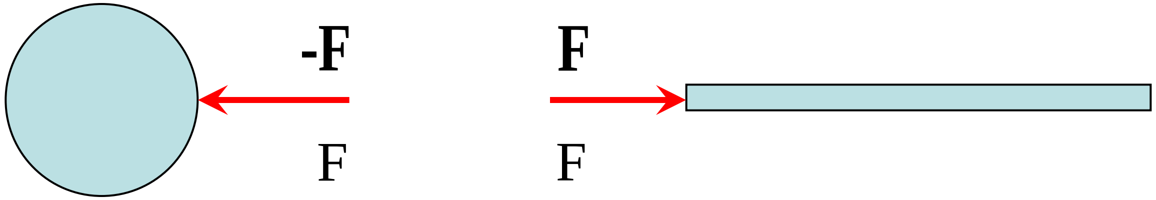
$$\sum \mathbf{F} = m\mathbf{a}$$

a = absolute acceleration

Newton's Laws

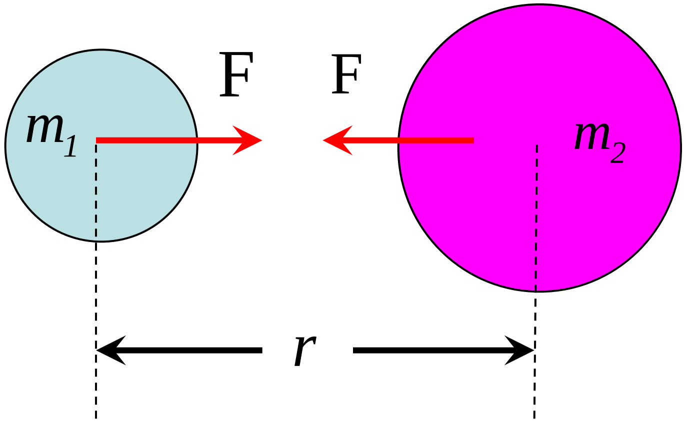
3rd Law: The forces of *action* and *reaction* between interacting bodies are equal in magnitude, opposite in direction, and collinear.

action force = -(reaction force)



FBD

Gravitational Law



$$F = G\frac{m_1 m_2}{r^2}$$

$$G = 6.673\times10^{-11}\ \mathrm{m}^3/\left(\mathrm{kg}\bullet\mathrm{s}^2\right)$$

Gravitational acceleration

Calculate from the gravitational force between the earth and the object. This attractive force is called the *weight* of the body.

$$\mathrm{W} = m\frac{\mathrm{G}m_e}{r^2} = mg$$

$g =$ free falling acceleration observed on the **moving** earth

$= 9.81 \text{ m/s}^2$ can be considered the absolute acceleration in the engineering problem on earth