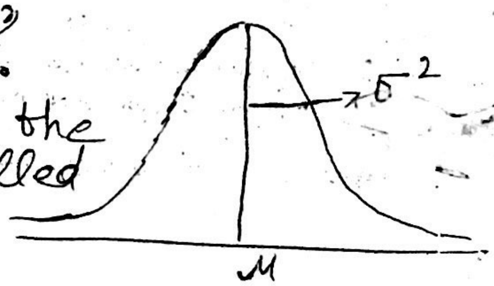
Normal Distribution

■ Introduction:-

The most important continuous probability distribution in the entire field of statistics is the normal distribution. Its graph called the normal curve is the bell shaped curve which describes approximately many phenomena that occur in nature, industry and research. Physical measurements in areas such as rainfall studies and measurements of manufactured parts are often more than adequately explained with a normal distribution. It is also known as "Guassian Distribution".

A continuous r.v. "X" having the bell shaped distribution is called a normal r.v.



■ Normal Distribution:-

The density of the normal r.v. X, with mean μ and variance σ^2, is

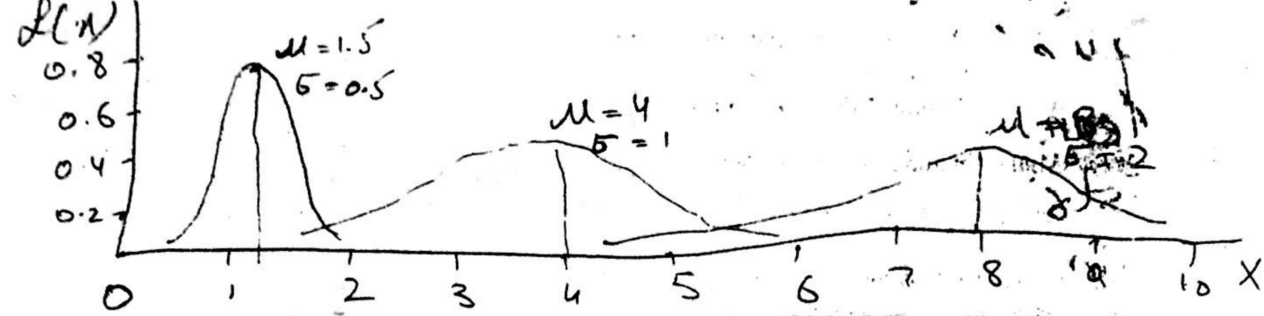
$$f(x;\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2} \qquad -\infty < X < \infty$$

where $\pi = 3.14159\ldots$ and

$e = 2.71828$

where μ is location parameter and σ^2 is shape parameter.

■ Standard Normal Distribution:- A norm probability distribution depends on the values of the parameters μ and σ^2 and the various possible values for these two parameters will result in an unlimited number of different normal distributions.



Then r.v. $Z = \frac{X-\mu}{\sigma}$ has zero mean and unit variance.

$$f(x;\mu,\sigma^2) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2\sigma^2}(x-\mu)^2} \quad -\infty < x < \infty$$

So
$$f(z;0,1) = \frac{1}{1\sqrt{2\pi}} e^{-\frac{1}{2(1)}(z-0)^2}$$

$$= \frac{1}{\sqrt{2\pi}} e^{-z^2/2} \quad -\infty < z < \infty$$

The distribution of a normal r.v. with mean "0" and variance "1" is called a standard normal distribution. Graph of normal distribution depends on 2 factors mean & S.D. Mean determine the location of the center of graph & S.D determine height & width of graph

■ Properties :-

(i) The total area under the normal curve is unity. $\int_{-\infty}^{\infty} f(x)\,dx = 1$

(ii) The mean and variance of the normal distribution are μ and σ^2 respectively.

(iii) The median and the mode of the normal distribution are each equal to μ.

(iv) Mean = Median = Mode, So the normal distribution is symmetrical and unimodal.

(V) The sum of independent normal variables is a normal variable.
If $X_1 \sim N(\mu_1, \sigma_1^2)$ and $X_2 \sim N(\mu_2, \sigma_2^2)$
then $X_1 + X_2 \sim N(\mu_1 + \mu_2, \sigma_1^2 + \sigma_2^2)$

(Vi)
(a) $\mu \pm \sigma$ will always contain 68.26%
(b) $\mu \pm 2\sigma$ will always contain 95.44%
(c) $\mu \pm 3\sigma$ will always contain 99.73%

Vii) The normal curve approaches, but never really touches the horizontal axis on either side of the mean towards $+\infty$ & $-\infty$, that is the curve is asymptotic to the horizontal axis as $x \rightarrow \pm\infty$