Hypergeometric Probability Distribution

HYPERGEOMETRIC EXPERIMENT

The experiment in which the condition of independence is violated and the probability of success does not remain constant for all trials, is called hypergeometric experiment.

PROPERTIES:

1- The outcomes of each trial may be classified into one of the two catagories, success and failure.
2- The probability of success changes on each trial
3- The successive trials are dependent.
4- The experiment is repeated a fixed number of times

HYPERGEOMETRIC DISTRIBUTION

The number of successes, X in a hypergeometric experiment is called a hypergeometric random variable and its probability distribution is called hypergeometric distribution.

PMF OF HYPERGEOMETRIC DISTRIBUTION

When hypergeometric r.v X assumes a value x, the hypergeometric p.d is given by formula:

$$P(X=x) = \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}} \qquad x = 0, 1, 2, \ldots, n \text{ and } x = 0, 1, 2, \ldots, K$$

N = Number of units in set or population
n = Number of units in subset or sample
K = Number of successes in set or population. (a positive (non-negative) integer less than or equal to N)

PARAMETERS OF HYPERGEOMETRIC P.d

It has three parameters

1- N

2- n

3- K

therefore it is denoted as $h(x;N,n,K)$

HYPERGEOMETRIC p.d IS APPROPRIATE WHEN

1- a random sample size n is drawn without replacement from a finite population of N units

2- K of the units are of one kind (success) and remaining $N-K$ of another kind (failure).

PROPERTES OF HYPERGEOMETRIC DISTRIBUTION:

1- PROPERTY OF COMPLETENESS (LEGTIMATE PROPERTY)

The sum of hypergeometric probabilities is one.

$$\sum_{x=0}^{n} P(X=x) = \sum_{x=0}^{n} \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$$

$$= \frac{1}{\binom{N}{n}} \sum_{x=0}^{n} \binom{K}{x}\binom{N-K}{n-x}$$

NUMERCAL PROBLEMS:

1. A Shipment of 8 similar micro-computers to retail outlet contains 3 Computers that are defected. If a school make a random purchase of two of these computers. Find the hypergeometric ~~probability~~ distribution for numbers of defective computers.

→ Total no. of Computers (Total set) $= N = 8$

→ defective computers (one kind) $= K = 3$.

→ B.No. of selected computers (subset) $= n = 2$.

→ Then the values of x are $0, 1, 2$

[The value of x starts from zero bcz it's also possible that the selected computers may be all the will not be defected].

X	$P(X=x)$	$= h(x;N,n,K)$	$\frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$
0	$P(X=0)$	$h(0;8,2,3)$	$\frac{\binom{3}{0}\binom{8-3}{2-0}}{\binom{8}{2}} = \frac{\binom{3}{0}\binom{5}{2}}{\binom{8}{2}}$ $= \frac{5}{14}$
1	$P(X=1)$	$h(1;8,2,3)$	$\frac{\binom{3}{1}\binom{8-3}{2-1}}{\binom{8}{2}} = \frac{\binom{3}{1}\binom{5}{1}}{\binom{8}{2}}$ $= \frac{15}{28}$
2	$P(X=2)$	$h(2;8,2,3)$	$\frac{\binom{3}{2}\binom{8-3}{2-2}}{\binom{8}{2}}$ $\frac{\binom{3}{2}\binom{5}{0}}{\binom{8}{2}}$ $= \frac{3}{28}$

here, we can write $\frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$

as $\frac{{}^{K}C_{x} \times {}^{N-K}C_{n-x}}{{}^{N}C_{n}}$

2. The names of 5 men and 5 women are written on slips of paper and placed in a box.
Four names are drawn. What is the probability that 2 are men and 2 are women?

Let X denote the number of men

Then

No. of $N = 5 + 5 = 10$ names

Selected names $= n = 4$.

No. of slips of paper with names of men $= K = 5$

hypergeometric formula;

$$h(x; N, n, K) = \frac{\binom{K}{x}\binom{N-K}{n-x}}{\binom{N}{n}}$$

$$= \frac{{}^{K}C_{x} \times {}^{N-K}C_{n-x}}{{}^{N}C_{n}}$$

here

if we want to find the probability for having 4 slips with names of 2 men and 2 women

And x denotes the number of men then we have

$\boxed{x = 2}$ → for two men

using this value our given condition is satisfied thus

$$h(2; 10, 4, 5) = \frac{\binom{5}{2}\binom{10-5}{4-2}}{\binom{10}{4}} = \frac{{}^{5}C_{2} \times {}^{10-5}C_{4-2}}{{}^{10}C_{4}}$$

$$= \frac{{}^{5}C_{2} \times {}^{5}C_{2}}{{}^{10}C_{4}} = \frac{10}{21}$$