S-Chart:-

S-Chart is used to moniter process despersion or variability it is generally applied when we have sample size greater than 10. R-chart & S-chart both can be apply when we have a sample of size less then 10. But due to simplicity R-chart is prefered. keeping in mind efficiency S-chart is better.

As we know that $S^2 = \frac{\Sigma(X-\bar{X})^2}{n-1}$ is an unbised estimator of popn standard daviation or process standard daviation (σ^2).

$$E(S^2) = \sigma^2$$

But sample standard daviation

$$S = \sqrt{\frac{\Sigma(X-\bar{X})^2}{n-1}}$$

is not an unbised estimator of σ

$$E(S) \neq \sigma$$

$P = \frac{S}{\sigma}$

$E(P) = E\left(\frac{S}{\sigma}\right) = C_4$

$= E(S) = \sigma C_4$

If the quality characteristic follows ~~norm~~ $N(\mu, \sigma^2)$ then samples standard deviation (s) actually estimates $C_4 \sigma$

$\mu_s = E(s) = C_4 \sigma$

where C_4 is a constt & depends upon ~~small~~ sample size n. And we know that by definition

$$Var(S) = E(S^2) - [E(S)]^2$$

$$= \sigma^2 - (C_4\sigma)^2$$

$$= \sigma^2 - C_4^2\sigma^2$$

$$Var(S) = \sigma^2(1 - C_4^2)$$

So that

$$Sd(S) = \sigma\sqrt{(1 - C_4^2)}$$

$$\sigma_s = \sigma\sqrt{1 - C_4^2}$$

Now the control limits of S-Chart become

$$UCL = E(S) + 3Sd(S)$$

$$CL = E(S)$$

$$LCL = E(S) - 3Sd(S)$$

Replace the values

$$UCL = C_4\sigma + 3\sigma\sqrt{1 - C_4^2} = B_6\sigma$$

$$CL = C_4\sigma = C_4\sigma$$

$$LCL = C_4\sigma - 3\sigma\sqrt{1 - C_4^2} = B_5\sigma$$

where

$$B_6 = C_4 + 3\sqrt{1 - C_4^2}$$

$$B_5 = C_4 - 3\sqrt{1 - C_4^2}$$

where C_4, B_5 & B_6 are functions of n & can be found from the Appendix of the book.

Now for any standard deviation falls out of the limits process is said to be out of control, otherwise process is in-control.

Note:-

If the behaviour of the plotting statistic is symmetric or approximately symmetric $k\sigma$ limits approach is generally used. And if the behaviour or distⁿ of plotting statistic is skewed or asymmetric then prob. limits are prefered.