Dispersion:-

To monitor dispersion parameter or process variability different control charts are used. R-chart & S-chart

R-chart:-

To monitor dispersion parameter σ R-chart is a very simple chart used for this purpose. R-chart is based on sample range.

Consider $X_1, X_2, \cdots, X_n$ be a random sample from dist^n with mean μ & variance σ^2. let $X_{(1)}$ is the smallest observation & X_n is the largest observation in the sample. Then range can be defined as:

$$R = X_n - X_1$$

Range = Max value - Min value

This R will be used as plotting statistic.

Now consider

$$W = \frac{R}{\sigma}$$

be a relative range

$$E(w) = E\left(\frac{R}{\sigma}\right) = d_2$$

$$E\left(\frac{R}{\sigma}\right) = d_2$$

$$\frac{E(R)}{\sigma} = d_2$$

$$E(R) = d_2\sigma$$

$$\mu_R = d_2\sigma$$

where d_2 is a function of sample size n i.e. $d_2 = f(n)$ & this value can be found from the table from the appendix of the book.

$$SD(w) = SD\left(\frac{R}{\sigma}\right) = d_3$$

$$S.D\left(\frac{R}{\sigma}\right) = d_3$$

$$\frac{S.D(R)}{\sigma} = d_3$$

$$S.D(R) = d_3\sigma$$

$$\sigma_R = d_3\sigma$$

where d_3 is a function of "n".

Now the control limits for R-chart are as

$$UCL = E(R) + 3SD(R) = d_2\sigma + 3d_3\sigma$$

$$CL = E(R) = d_2\sigma$$

$$LCL = E(R) - 3SD(R) = d_2\sigma - 3d_3\sigma$$

D_2

$$UCL = (d_2 + 3d_3)\bar{\sigma} = D_2\bar{\sigma}$$

$$CL = d_2\bar{\sigma} = d_2\bar{\sigma}$$

D_1

$$LCL = (d_2 - 3d_3)\bar{\sigma} = D_1\bar{\sigma}$$

where D_1 & D_2 are function of sample size n & can be found from table.

Now if any sample range falls out of these limits process is considered to be out of control otherwise process is in-control