Control Chart for $\bar{X}$-Chart:-

Consider the quality characteristics say X follows ~~a~~ normal distⁿ with mean μ & σ^2 that is $X \sim N(\mu, \sigma^2)$. Then the control limits are as

$UCL = \mu + k\sigma$

$CL = \mu$

$LCL = \mu - k\sigma$

Consider the prob of type I error or FAR is α then the limits become

$UCL = \mu + Z_{\alpha/2}\sigma$

$CL = \mu$

$LCL = \mu - Z_{\alpha/2}\sigma$

Now the limits are called prob. limits otherwise the limits are $k\sigma$ limits. We can say that

$P(LCL < X < UCL) = 1-\alpha$ level of confidence

$P(\mu - Z_{\alpha/2}\sigma < X < \mu + Z_{\alpha/2}\sigma) = 1-\alpha$

& if

$P(X < LCL \text{ or } X > UCL) = \alpha$

$P(X < LCL) + P(X > UCL) = \alpha$

~~$P(X < Z_\alpha$~~

$P(X < \mu - Z_{\alpha/2}\sigma) + P(X > \mu + Z_{\alpha/2}\sigma) = \alpha$ FAR

Now consider we have a random sample of size n that is $X_1, X_2, \ldots, X_n$ from normal dist with μ & σ^2 then

$$\bar{X} = \frac{X_1 + X_2 + \cdots + X_n}{n}$$

$$\bar{X} \sim N(\mu, \sigma^2/n) \rightarrow \bar{X} \sim N(\mu, \sigma^2/n)$$

Then the control limits become

$$UCL = \mu + K\,\sigma/\sqrt{n}$$
$$CL = \mu$$
$$LCL = \mu - K\,\sigma/\sqrt{n}$$

If we replace K by $Z_{\alpha/2}$ the limits are called prob. limits

$$UCL = \mu + Z_{\alpha/2}\cdot\sigma/\sqrt{n}$$
$$CL = \mu$$
$$LCL = \mu - Z_{\alpha/2}\cdot\sigma/\sqrt{n}$$

And in prob. terms

$$P(\mu - Z_{\alpha/2}\,\sigma/\sqrt{n} < \bar{X} < \mu + Z_{\alpha/2}\,\sigma/\sqrt{n}) = 1-\alpha$$

And

$\cancel{P(\mu -}$

$$P\left(\bar{X} < \mu - Z_{\alpha/2}\frac{\sigma}{\sqrt{n}} \text{ or } \bar{X} > \mu + Z_{\alpha/2}\frac{\sigma}{\sqrt{n}}\right) = \alpha$$

$$P(\bar{X} < \mu - Z_{\alpha/2}\,\sigma/\sqrt{n}) + P(\bar{X} > \mu + Z_{\alpha/2}\,\sigma/\sqrt{n}) = \alpha$$

or we can say that prob. that

$$P(\bar{X} < \mu - Z_{\alpha/2}\,\sigma/\sqrt{n}) = \alpha/2$$

$$P(\bar{X} > \mu + Z_{\alpha/2}\,\sigma/\sqrt{n}) = \alpha/2$$

Now consider $K\sigma$ limits

$$UCL = \mu + K\sigma$$
$$CL = \mu$$
$$LCL = \mu - K\sigma$$

If we replace $K = 3$ the limits are called 3σ limits and the limits become

$UCL = \mu + 3\sigma$

$CL = \mu$

$LCL = \mu - 3\sigma$

$P(\mu - 3\sigma < X < \mu + 3\sigma) = 0.9973$

OR we can say we have $FAR = 0.0027$

$P(X < \mu - 3\sigma$ or $X > \mu + 3\sigma) = 0.0027$

$P(X < \mu - 3\sigma) + P(X > \mu + 3\sigma) = 0.0027$ $\quad \because 0.0027/2 = 0.00135$

$P(X < \mu - 3\sigma) = 0.00135$

$P(X > \mu + 3\sigma) = 0.00135$

Now consider we have taken a random sample of size n, $X_1, X_2, \dots, X_n$ then the distⁿ of

$\bar{X} \sim N(\mu, \sigma/\sqrt{n})$

Then 3σ limits becomes

$UCL = \mu + 3.\sigma/\sqrt{n}$

$CL = \mu$

$LCL = \mu - 3.\sigma/\sqrt{n}$

And the prob

$P(\mu - 3\sigma/\sqrt{n} < \bar{X} < \mu + 3\sigma/\sqrt{n}) = 0.9973$

$P(\bar{X} < \mu - 3\sigma/\sqrt{n}) = 0.00135$

$P(\bar{X} > \mu + 3.\sigma/\sqrt{n}) = 0.00135$

All the above discussion is true only if we know the values of μ & σ that means if the values of parameters μ & σ are known.

pop^n val

$UCL = \mu + A\sigma$ where $A = \frac{3}{\sqrt{n}}$

$CL = \mu$

$LCL = \mu - A\sigma$ where A is a function of n.

This value can be find from the index of book

$\bar{X}$ chart is used to moniter the location parame
or the mean value of the process. Also X-control
chart is used to moniter process mean or
location parameter (μ).