Type : IV

When the R.H.S of the given non-homogeneous difference equation is of the form

$$f(x) = \alpha^x g(x)$$

where $g(x)$ is the polynomial in x and α, a constant. In order to find P.I, we shall make substitution

$$y_x = \alpha^x [a_0 + a_1 x + a_2 x^2 + \dots]$$ upto the highest power of x involved in the difference equation.

Example, Solve

$$y_{k+2} - 13 y_{k+1} + 36 y_k = 2^k (k^2+1). \quad \text{—} ①$$

Sol: Corresponding homogeneous equ to ① is

$$y_{k+2} - 13 y_{k+1} + 36 y_k = 0 \quad \text{—} ②$$

let $y_k = Ab^k$ be the non-~~homogeneous~~ trivial solution of equ ② so it must satisfy it

$$\Rightarrow \quad Ab^{k+2} - 13Ab^{k+1} + 36Ab^k = 0$$

$$Ab^k [b^2 - 13b + 36] = 0$$

$\because Ab^k \neq 0$,

$$b^2 - 13b + 36 = 0$$

$$(b-4)(b-9) = 0$$

$$b = 4, \quad b = 9$$

So C.F is,

$$y_k = c_1 (4)^k + c_2 (9)^k.$$

To find P.I, substitute

$$y_k = 2^k(a_0 + a_1 k + a_2 k^2) \quad \text{in } ①$$

① ⇒

$$2^{k+2}[a_0 + a_1(k+2) + a_2(k+2)^2] - 13(2^{k+1})[a_0 + a_1(k+1) + a_2(k+1)^2] + 36(2^k)[a_0 + a_1 k + a_2 k^2] = 2^k(k^2+1)$$

By comparison method.

$$\cancel{2^k}[4(a_0 + a_1 k + 2a_1 + a_2 k^2 + 4a_2 + 4ka_2) - 26(a_0 + a_1 k + a_1 + a_2 k^2 + a_2 + 2a_2 k) + 36(a_0 + a_1 k + a_2 k^2)] = \cancel{2^k}(k^2+1)$$

$$\Rightarrow k^2(4a_2 - 26a_2 + 36a_2) + k(4a_1 + 16a_2 - 26a_1 - 52a_2 + 36a_1) + (4a_0 + 8a_1 + 16a_2 - 26a_0 - 26a_1 - 26a_2 + 36a_0) = k^2 + 1.$$

By comparison,

$$\Rightarrow 14a_2 = 1 \quad \Rightarrow \quad \boxed{a_2 = \frac{1}{14}}$$

$$\Rightarrow 14a_1 - 36a_2 = 0$$

$$14a_1 - 36\left(\frac{1}{14}\right) = 0$$

$$14a_1 = 18/7$$

$$a_1 = \frac{18}{14 \times 7} = \frac{9}{49} \quad \Rightarrow \quad \boxed{a_1 = \frac{9}{49}}$$

$$\Rightarrow 14a_0 - 18a_1 - 10a_2 = 0$$

$$14a_0 - 18\left(\frac{9}{49}\right) - 10\left(\frac{1}{14}\right) = 0$$

$$\Rightarrow \boxed{a_0 = \frac{123}{343}}$$

So. $y_k^* = 2^k\left[\frac{123}{343} + \frac{9k}{49} + \frac{k^2}{14}\right]$ and

the general solution to ① is,

$$Y_k = c_1(4)^k + c_2(9)^k + 2^k\left[\frac{123}{343} + \frac{9k}{49} + \frac{k^2}{14}\right]$$ Ans.

Questions:

1. $y_{k+2} - 4y_{k+1} + 4y_k = 2^k(k^2+k+1)$ ①

Sol:- The homogeneous part is,

$$y_{k+2} - 4y_{k+1} + 4y_k = 0.$$

let $y_k = Ab^k$ be the non-trivial solution of equ ② so it must satisfy it. i.e.

$$Ab^{k+2} - 4Ab^{k+1} + 4Ab^k = 0$$
$$Ab^k[b^2 - 4b + 4] = 0$$
$\because Ab^k \neq 0$
$$b^2 - 4b + 4 = 0.$$
$$(b-2)^2 = 0.$$
$$b = 2, \quad b = 2.$$

So C.F. is,

$$y_k = (c_1 + c_2 k)2^k.$$

To find P.I. substitute

$y_k = 2^k k^2(a_0 + a_1 k + a_2 k^2)$ in equ ①

① $\Rightarrow$ $2^{k+2}(k+2)^2[a_0 + a_1(k+2) + a_2(k+2)^2] - 4\cdot(2)^{k+1}(k+1)^2[a_0 + a_1(k+1) + a_2(k+1)^2] + 4\cdot 2^k k^2[a_0 + a_1 k + a_2 k^2] = 2^k(k^2+k+1)$

$$\Rightarrow \cancel{2^k}\left[4(k+2)^2\{a_0+a_1k+2a_1+a_2k^2+4a_2+4a_2k\} - 8(k+1)^2\{a_0+a_1k+a_1+a_2k^2+a_2+2a_2k\} + 4k^2\{a_0+a_1k+a_2k^2\}\right] = \cancel{2^k}[k^2+k+1]$$

$$\Rightarrow [4a_0k^2+4a_1k^3+8a_1k^2+4a_2k^4+16a_2k^2+16a_2k^3 +16a_0k+16a_1k^2+32a_1k+16a_2k^3+64a_2k+ 64a_2k^2+16a_0+16a_1k+32a_1+16a_2k^2+64a_2 +64a_2k] - [8a_0k^2+8a_1k^3+8a_1k^2+8a_2k^4+ 8a_2k^2+16a_2k^3+16a_0k+16a_1k^2+16a_1k+16a_2k^3 +16a_2k+32a_2k^2+8a_0+8a_1k+8a_1+8a_2k^2+ +8a_2+16a_2k] + [4a_0k^2+4a_1k^3+4a_2k^4]$$

$$= k^2+k+1.$$

By comparing coefficients of k^2, k and constant terms we get a_0, a_1, a_2.

$k^2 \Rightarrow$ $8a_1+4a_0+16a_2+16a_1+64a_2+16a_2 -8a_2-8a_1-8a_2-16a_1-32a_2+4a_0-8a_2 = 1.$

i.e. $\boxed{a_2 = 1/48}$

$k \Rightarrow$ $16a_0+32a_1+64a_2+16a_1+64a_2-16a_0 -16a_1-16a_2-8a_1-16a_2 = 1$

$\Rightarrow$ $\boxed{a_1 = \frac{-1}{24}}$

constants ⇒ $16a_0 + 32a_1 + 64a_2 - 8a_0 - 8a_1 - 8a_2 = 1.$

$$\Rightarrow \boxed{a_0 = \frac{5}{48}}$$

So the P. solution is

$$y_k^* = 2^k k^2 \left[\frac{5}{48} + \left(\frac{-1}{24}\right)k + \frac{1}{48} k^2 \right]$$

General sol is

$$Y_k = 2^k \left[(c_1 + c_2 k) + k^2 \left\{ \frac{5}{48} - \frac{1}{24} k + \frac{1}{48} k \right. \right.$$

Ans

• **Type V:** When the R.H.S of the n... homogeneous difference equation has ... form as

$$f(x) = \sin Ax \quad \text{or} \quad \cos Bx.$$

where A and B are constants ... To find P.I, we shal...