$Ab^x \neq 0$, $\quad b^2 - 4b + 4 = 0$

$$\Rightarrow (b-2)^2 = 0$$

$$b = 2, \quad b = 2$$

C.F is

$$y_x = (c_1 + c_2 x) 2^x$$

To find P.I, substitute $y_x = c_3 x^2 2^x$ in ①

①$\Rightarrow$

$$\cancel{2^x} c_3 \left[2^2(x^2+4x+4) - 2^3(x^2+2x+1) + 4x^2 \right] = 2 \cdot \cancel{2^x}$$

$$c_3 \left[4x^2 + \cancel{16x} + 16 - 8x^2 - \cancel{16x} - 8 + 4x^2 \right] = 2$$

$$c_3 (8) = 2$$

$$c_3 = 1/4$$

So

$$P.I = y_x^* = \frac{x^2}{4} \cdot 2^x$$

and the general solution to ① is,

$$Y_x = (c_1 + c_2 x) 2^x + \frac{1}{4} x^2 2^x$$ Answer.

Q3. Solve the difference equation

$$y_{k+2} - 6y_{k+1} + 7y_k = 3^k \quad \text{— ①}$$

Sol: Consider the corresponding homogeneous equation of ①

$$y_{k+2} - 6y_{k+1} + 7y_k = 0 \quad \text{— ②}$$

let $y_k = Ab^k$ be the non-trivial solution of ② so it must satisfies it. i.e.