Estimation:-

"The procedure used to find out the unknown parameter is called estimation."

OR

Estimation is a procedure by which we obtain an estimate of the true but unknown parameter by using the sample observation $X_1, X_2, \ldots X_n$ from the population.

Estimate:-

Numerical value which comes from the procedure of estimation is called estimate or estimated value.

Estimator:-

Formula which is used for estimation is called estimator.

- Estimator is always a statistic.

$\because$ $\frac{\sum X_i}{n}$ → estimator

$\bar{X} = 1.5$ → estimate

→ There are two types of Estimates:-

(1) Point Estimation:-

(2) Interval Estimation:

1) Point Estimates:-

(2)

When an estimate for the unknown Population parameter is expressed by a single value then it is called point estimates.

2) Interval Estimates:-

An estimate expressed by a range of values within which the true value of the Population parameter is believed to lie, is reffered to as an interval estimate.

...ssical ... might be termed as properties of estimators

- Unbisedness:-

(3)

An unbised estimator is one if its expected value is equal to corresponding pop^n parameter otherwise the estimator is bised. Commonly suppose a statistic estimator $\hat{\theta}$ is used to estimate the pop^n parameter θ. The statistic $\hat{\theta}$ is said to be an unbised estimator of θ if

$$E(\hat{\theta}) = \theta$$

Otherwise if $E(\hat{\theta}) \neq \theta$ then is called "bised"

The bised of an estimator is defined as:

$$Bias(\hat{\theta}) = E(\hat{\theta}) - \theta$$

e.g:- $\bar{X}$ is an unbised estimator of pop^n mean μ when the pop^n distribution is normal with parameters μ & σ^2

(i) $E(c) = c$ ∵ c is const

(ii) $E(x) = E(x)$

i.e $E[E(x)] = E(x)$ ∵ $E(X) = \mu$ ✓

(iii) $E\{X - E(X)\} = 0$

∵ $E(X) - E(X) =$

$\mu - \mu = 0$

$\bar{X} = \frac{X_1 + X_2 + X_3 \pm X_n}{n}$

$E(\bar{X}) = E(X)$

(iv) $E(X \pm c) = E(X) \pm c$

(v) $E(aX) = aE(X)$

(vi) $E(aX \pm c) = aE(X) \pm c$

(vii) $E(X_1 + X_2 + X_3 + \cdots + X_n) = E(X_1) + E(X_2) + \cdots + E(X_n)$

(viii) $E(X_1 \cdot X_2) = E(X_1) \cdot E(X_2)$

Problem: (5)

Show that $\bar{x}$ of r.v $x_1, x_2, \ldots, x_n$ is an unbised estimator of popⁿ mean μ

$$\bar{x} = \frac{\sum^n x}{n}$$

For the purpose of unbisedness we apply expectation

$$E(\bar{x}) = E\left(\frac{\sum x}{n}\right)$$

$$= E\left(\frac{x_1 + x_2 + \cdots + x_n}{n}\right)$$

$$= \frac{E(x_1) + E(x_2) + \cdots + E(x_n)}{n}$$

$$= \frac{\mu + \mu + \cdots + \mu}{n}$$

$$= \frac{\cancel{n}\mu}{\cancel{n}}$$

$$E(\bar{x}) = \mu$$

So

$\bar{x}$ is an unbised estimator of popⁿ mean

OR

$$E(\bar{x}) = E\left(\frac{\sum x}{n}\right)$$

$$= \frac{1}{n} E(\sum x)$$

$$= \frac{1}{n} \sum E(x)$$

$$= \frac{1}{n} \sum \mu \Rightarrow \frac{1}{\cancel{n}} \cancel{n}\mu = \mu$$

$S^2 = \frac{\Sigma(X-\bar{X})^2}{n}$ or $\frac{\Sigma(X-\bar{X})^2}{n-1}$

(6)

Show than S^2 of the random sample $X_1, X_2, \ldots, X_n$ is bised estimator of pop^n variance σ^2 & s^2 is an unbised estimator of pop^n variance σ^2

$E(S^2) = \sigma^2$; $E(s^2) = \sigma^2$

$S^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i-\bar{X})^2$ — I

$= \frac{1}{n}\Sigma\left[X_i - \mu + \mu - \bar{X}\right]^2$ — II

$= \frac{1}{n}\Sigma\left\{(X_i-\mu)+(\mu-\bar{X})\right\}^2$ — III

$= \frac{1}{n}\Sigma\left\{(X_i-\mu)-(\bar{X}-\mu)\right\}^2$ — IV

$= \frac{1}{n}\Sigma\left\{(X_i-\mu)^2+(\bar{X}-\mu)^2-2(X_i-\mu)(\bar{X}-\mu)\right\}$ — V

$= \frac{1}{n}\left\{\Sigma(X_i-\mu)^2+n(\bar{X}-\mu)^2-2\Sigma(X_i-\mu)(\bar{X}-\mu)\right\}$ — VI

$= \frac{1}{n}\left[\Sigma(X_i-\mu)^2+n(\bar{X}-\mu)^2-2n(\bar{X}-\mu)(\bar{X}-\mu)\right]$ — VII

$= \frac{1}{n}\left[\Sigma(X_i-\mu)^2+n(\bar{X}-\mu)^2-2n(\bar{X}-\mu)^2\right]$ — VIII

$= \frac{1}{n}\left[\Sigma(X_i-\mu)^2-n(\bar{X}-\mu)^2\right]$ IX

Taking Expectation

$E(S^2) = E\left[\frac{1}{n}\Sigma(X_i-\mu)^2 - \frac{1}{\cancel{n}}\cdot\cancel{n}(\bar{X}-\mu)^2\right]$ X

$= \frac{1}{n}\Sigma E(X_i-\mu)^2 - E(\bar{X}-\mu)^2$ XI

7

$$= \frac{1}{n}\sum Var(X_i) - Var(\bar{X})$$

$$\because \sum\sigma^2 = n\sigma^2$$

$$= \frac{1}{n}(n\sigma^2) - \frac{\sigma^2}{n}$$

$$= \sigma^2 - \frac{\sigma^2}{n}$$

$$= \sigma^2\left(1 - \frac{1}{n}\right)$$

$$= \sigma^2\left(\frac{n-1}{n}\right)$$

$$E(S^2) = \left(\frac{n-1}{n}\right)\sigma^2$$

S^2 is a bised estimator of σ^2.

To find S_2

$$E(S_2^2) = \frac{n}{n-1}E(S_1^2)$$

$$= \frac{\cancel{n}}{n-1}\cdot\frac{n-1}{\cancel{n}}\cdot\sigma^2$$

$$E(S_2^2) = \sigma^2$$

$$E(S_2^2) = \frac{n}{n-1}S_1^2$$

$$= \frac{\cancel{n}}{n-1}\cdot\frac{\sum(X_i-\bar{X})^2}{\cancel{n}}$$

$$\therefore s^2 = S_2^2 = \frac{\sum(X_i-\bar{X})^2}{n-1}$$

$$\sum(X_i - \bar{X})^2$$

$S^2 = \frac{\Sigma(X-\bar{X})^2}{n}$ or $\frac{\Sigma(X-\bar{X})^2}{n-1}$ (6)

Show than S^2 of random sample $X_1, X_2, \ldots, X_n$ is bised estimator of popⁿ variance σ^2 & s^2 is an unbised estimator of popⁿ variance σ^2

$E(S^2) = \sigma^2$; $E(s^2) = \sigma^2$

$S^2 = \frac{1}{n}\sum_{i=1}^{n}(X_i-\bar{X})^2$ — I

$= \frac{1}{n}\Sigma[X_i-\mu+\mu-\bar{X}]^2$ — II

$= \frac{1}{n}\Sigma\{(X_i-\mu)+(\mu-\bar{X})\}^2$ — III

$= \frac{1}{n}\Sigma\{(X_i-\mu)-(\bar{X}-\mu)\}^2$ — IV

$= \frac{1}{n}\Sigma[(X_i-\mu)^2+(\bar{X}-\mu)^2-2(X_i-\mu)(\bar{X}-\mu)]$ — V

$= \frac{1}{n}[\Sigma(X_i-\mu)^2+n(\bar{X}-\mu)^2-2\Sigma(X_i-\mu)(\bar{X}-\mu)]$ — VI

$\because \frac{\Sigma X_i}{n} = \bar{X}$

$= \frac{1}{n}[\Sigma(X_i-\mu)^2+n(\bar{X}-\mu)^2-2n(\bar{X}-\mu)(\bar{X}-\mu)]$ — VII

$= \frac{1}{n}[\Sigma(X_i-\mu)^2+n(\bar{X}-\mu)^2-2n(\bar{X}-\mu)^2]$ — VIII

$= \frac{1}{n}[\Sigma(X_i-\mu)^2-n(\bar{X}-\mu)^2]$ IX

Taking Expectation

$E(S^2) = E[\frac{1}{n}\Sigma(X_i-\mu)^2 - \frac{1}{\cancel{n}}\cdot\cancel{n}(\bar{X}-\mu)^2]$ X

$= \frac{1}{n}\Sigma E(X_i-\mu)^2 - E(\bar{X}-\mu)^2$ XI