

DEFINITION OF MOMENTS OF INERTIA FOR AREAS, RADIUS OF GYRATION OF AN AREA

- a) Define the moments of inertia (MoI) for an area.
- b) Determine the moment of inertia of basic shapes.



APPLICATIONS



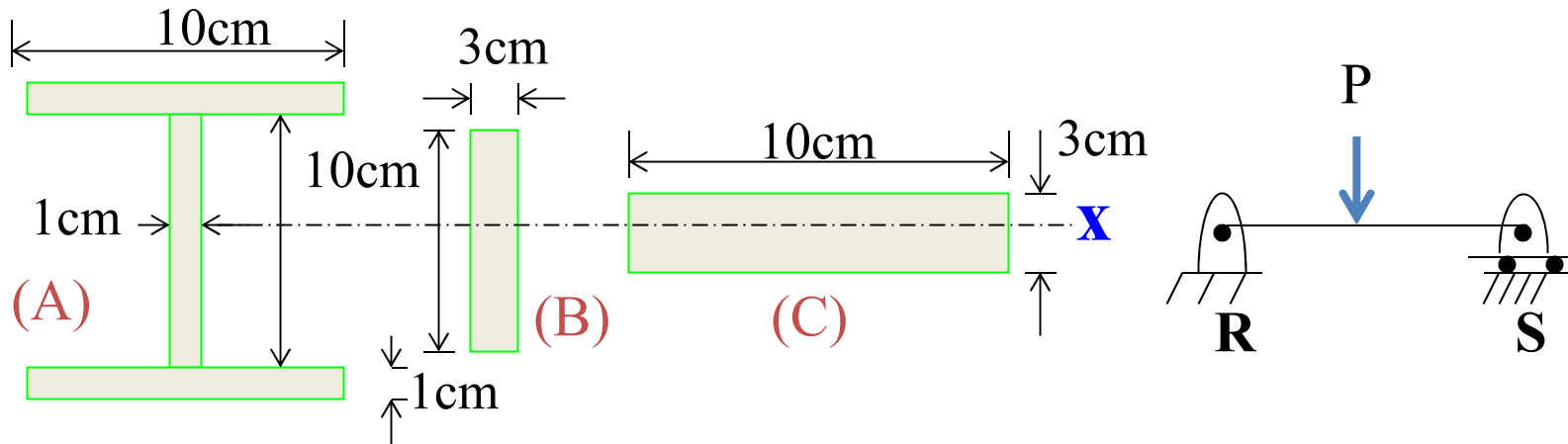
Many structural members like beams and columns have cross sectional shapes like an I, H, C, etc..

Why do they usually not have solid rectangular, square, or circular cross sectional areas?

What **primary property of these members** influences design decisions?



DEFINITION OF MOMENTS OF INERTIA FOR AREAS

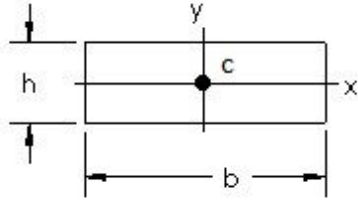


Consider three different possible cross sectional shapes and areas for the beam RS. All have the same total area and, assuming they are made of same material, they will have the same mass per unit length.

For the given vertical loading P on the beam, which shape will develop less internal stress and deflection?

The answer depends on the **MoI of the beam about the x-axis**. It turns out that **Section A has the highest MoI because most of the area is farthest from the x axis**. Hence, it has the least stress and deflection.

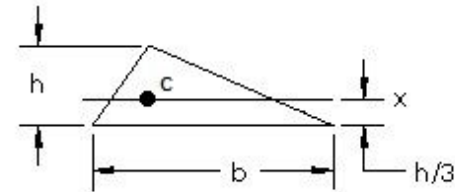




$$A = bh$$

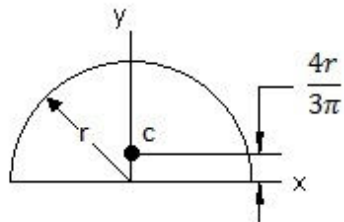
$$I_x = \frac{1}{12}bh^3$$

$$I_y = \frac{1}{12}hb^3$$



$$A = \frac{1}{2}bh$$

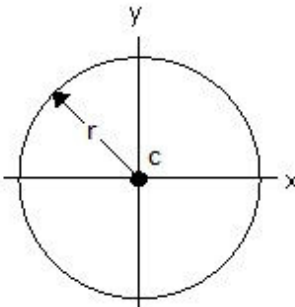
$$I_x = \frac{1}{36}bh^3$$



$$A = \frac{\pi r^2}{2}$$

$$I_x = \frac{1}{8}\pi r^4$$

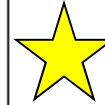
$$I_y = \frac{1}{8}\pi r^4$$



$$A = \pi r^2$$

$$I_x = \frac{1}{4}\pi r^4$$

$$I_y = \frac{1}{4}\pi r^4$$



Moment of Inertia of Some Basic Geometric Shapes

PARALLEL-AXIS THEOREM FOR AN AREA & MOMENT OF INERTIA FOR COMPOSITE AREAS

1. Apply the parallel-axis theorem.
2. Determine the moment of inertia (MoI) for a composite area.



APPLICATIONS

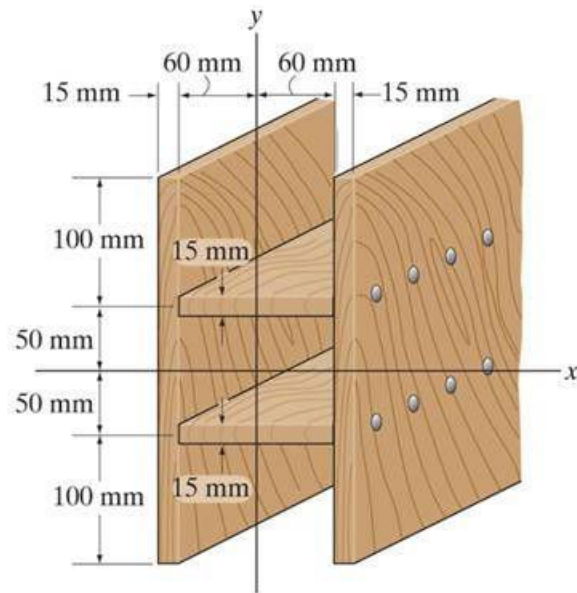


Cross-sectional areas of structural members are usually made of **simple shapes or combination** of simple shapes. **To design** these types of members, we need to find the **moment of inertia** (MoI).

It is helpful and efficient if you can do a simpler method for determining the MoI of such cross-sectional areas as compared to the integration method.



APPLICATIONS (continued)

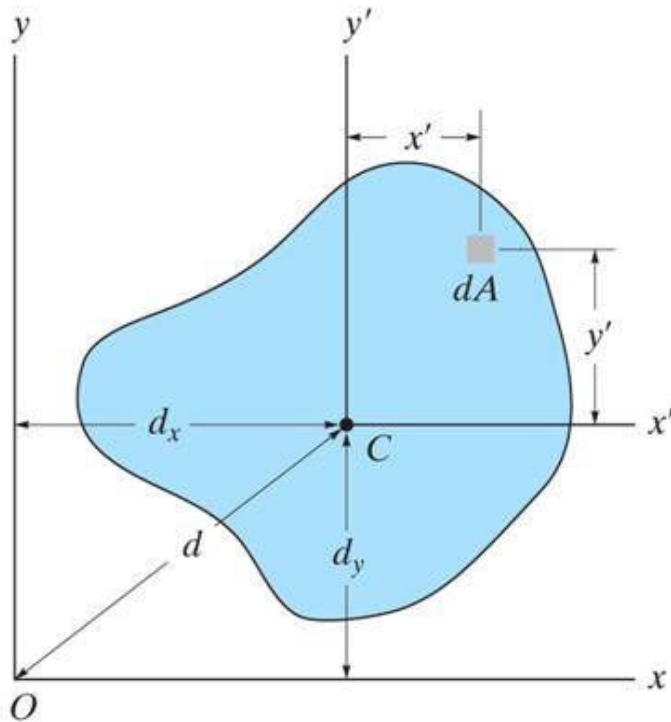


This is another example of a structural member with a composite cross-area. Such assemblies are often referred to as a “**built-up**” beam or member.

Design calculations typically require use of the MoI for these cross-sectional areas.



PARALLEL-AXIS THEOREM FOR AN AREA (Section 10.2)



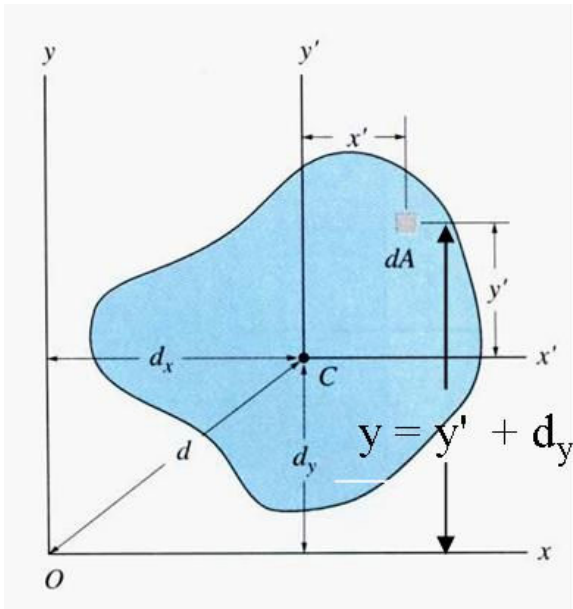
This theorem relates the moment of inertia (MoI) of an area about an axis passing through the area's centroid to the MoI of the area about a corresponding parallel axis. This theorem has many practical applications, especially when working with composite areas.

Consider an area with centroid C. The x' and y' axes pass through C. The MoI about the x -axis, which is parallel to, and distance d_y from the x' axis, is found by using **the parallel-axis theorem**.



PARALLEL-AXIS THEOREM

(continued)



$$\begin{aligned} I_X &= \int_A y^2 dA = \int_A (y' + d_y)^2 dA \\ &= \int_A y'^2 dA + 2 d_y \int_A y' dA + d_y^2 \int_A dA \end{aligned}$$

Using the definition of the centroid:

$$\bar{y}' = (\int_A y' dA) / (\int_A dA) . \text{ Now}$$

since C is at the origin of the $x' - y'$ axes,

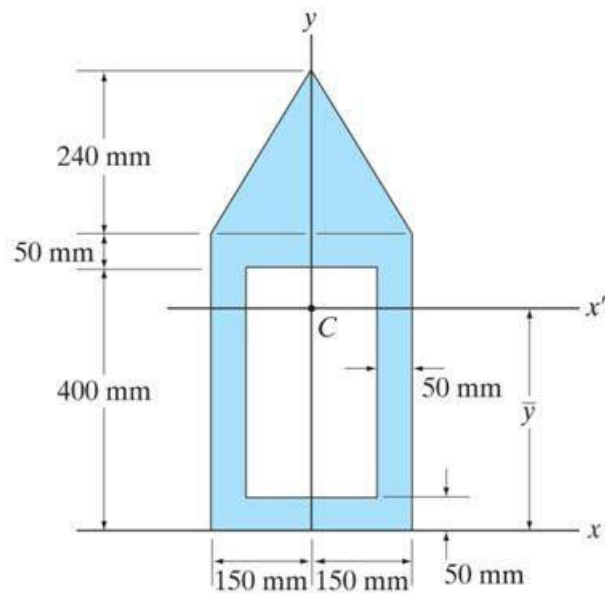
$$\bar{y}' = 0 , \text{ and hence } \int_A y' dA = 0 .$$

$$\text{Thus } I_X = \bar{I}_X' + A d_y^2$$

$$\text{Similarly, } I_Y = \bar{I}_Y' + A d_x^2$$



MOMENT OF INERTIA FOR A COMPOSITE AREA (Section 10.4)



A **composite area** is made by adding or subtracting a series of “**simple**” shaped areas like rectangles, triangles, and circles.

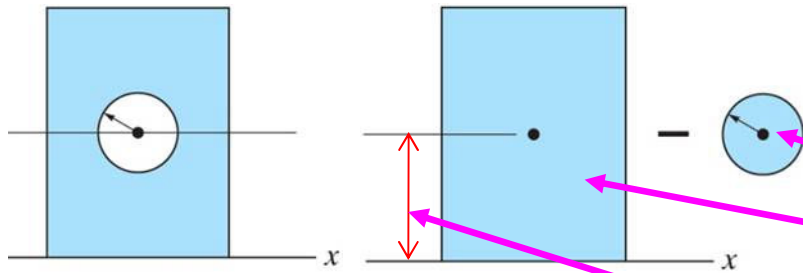
For example, the area on the left can be made from a rectangle minus a triangle and circle.

The MoI about their centroidal axes of these “**simpler**” shaped areas are found in most engineering handbooks as well as the inside back cover of the textbook.

Using these data and the **parallel-axis theorem**, the MoI for a composite area can easily be calculated.



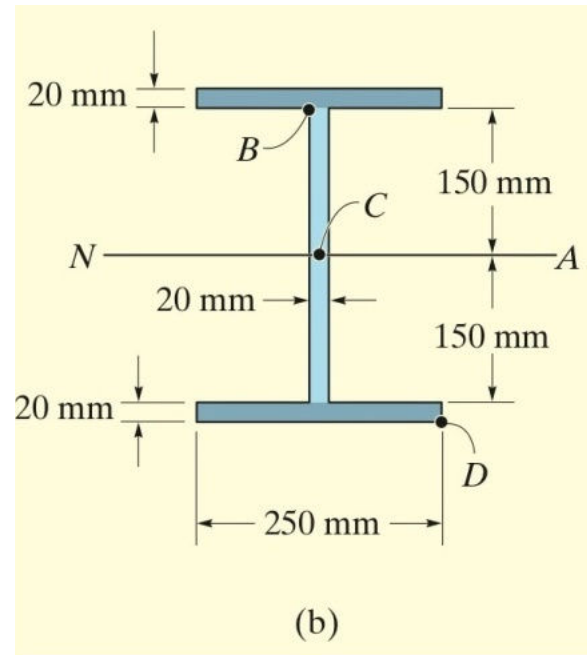
STEPS FOR ANALYSIS



1. Divide the given area into its simpler shaped parts.
2. Locate the centroid of each part and indicate the perpendicular distance from each centroid to the desired reference axis.
3. Determine the MoI of each “simpler” shaped part about the desired reference axis using the parallel-axis theorem ($I_X = I_{X'} + A (d_y)^2$).
4. The MoI of the entire area about the reference axis is determined by performing an algebraic summation of the individual MoIs obtained in Step 3. (Please note that MoI of a hole is **subtracted**).



Question



Find the moment of inertia of the section shown with respect to the neutral axis (NA).

Cross Section

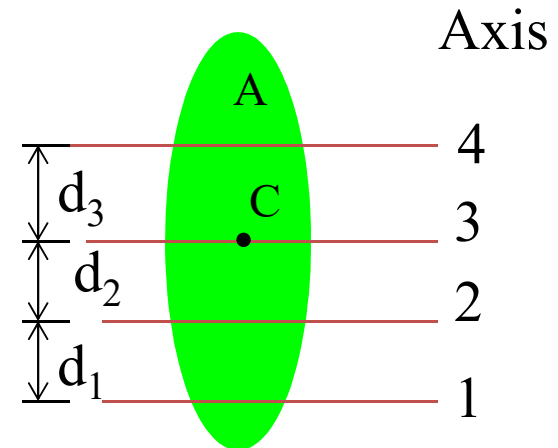
Solution

$$\begin{aligned} I &= \Sigma(\bar{I} + Ad^2) \\ &= 2 \left[\frac{1}{12} (0.25 \text{ m})(0.020 \text{ m})^3 + (0.25 \text{ m})(0.020 \text{ m})(0.160 \text{ m})^2 \right] \\ &\quad + \left[\frac{1}{12} (0.020 \text{ m})(0.300 \text{ m})^3 \right] \\ &= 301.3(10^{-6}) \text{ m}^4 \end{aligned}$$

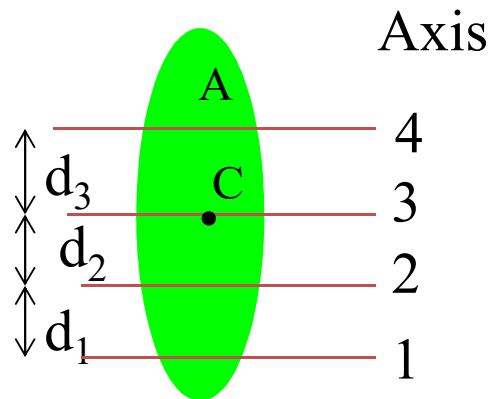
CONCEPT QUIZ

1. For the area A , we know the centroid's (C) location, area, distances between the four parallel axes, and the MoI about axis 1. We can determine the [MoI about axis 2](#) by applying the parallel axis theorem ____ .

- A) Directly between the axes 1 and 2.
- B) Between axes 1 and 3 and then between the axes 3 and 2.
- C) Between axes 1 and 4 and then axes 4 and 2.
- D) None of the above.



CONCEPT QUIZ (continued)



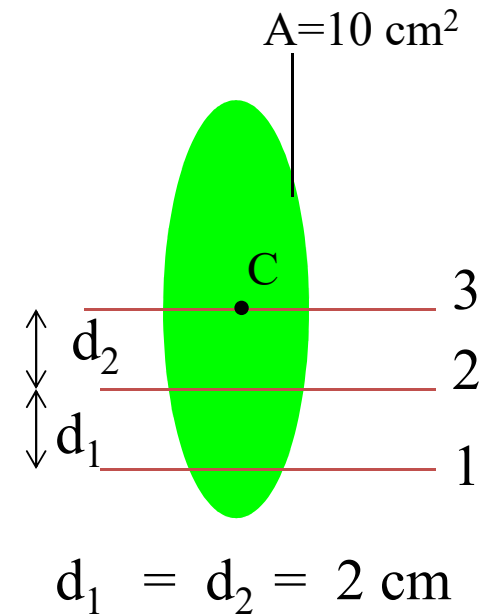
2. For the same case, consider the MoI about each of the four axes. About which axis will the MoI be the smallest number?
- A) Axis 1
 - B) Axis 2
 - C) Axis 3
 - D) Axis 4
 - E) Can not tell.



ATTENTION QUIZ

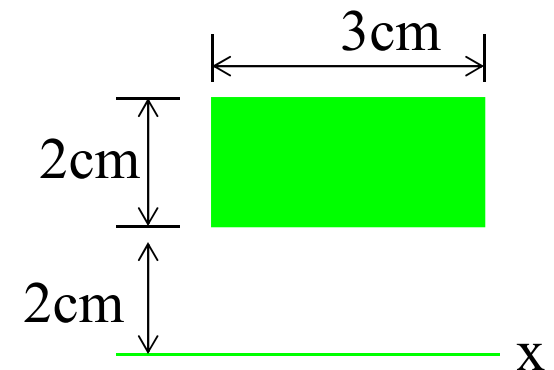
1. For the given area, the moment of inertia about axis 1 is 200 cm^4 . What is the MoI about axis 3 (the centroidal axis)?

- A) 90 cm^4 B) 110 cm^4
C) 60 cm^4 D) 40 cm^4

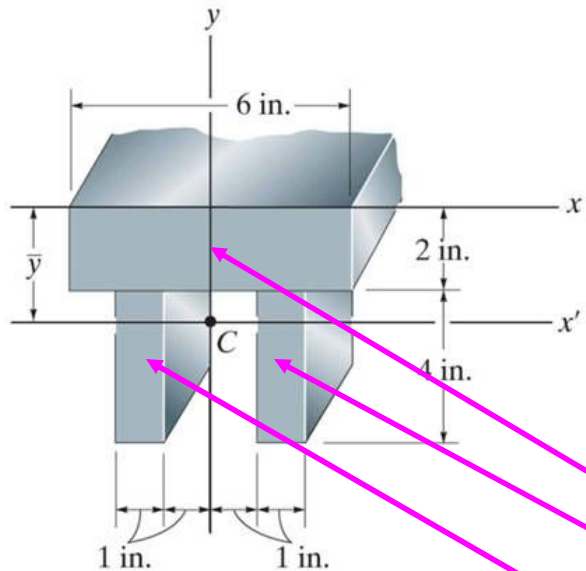


2. The moment of inertia of the rectangle about the x-axis equals

- A) 8 cm^4 . B) 56 cm^4 .
C) 24 cm^4 . D) 26 cm^4 .



EXAMPLE



Given: The beam's cross-sectional area.

Find: The moment of inertia of the area about the **y-axis**.

Plan: Follow the steps for analysis.

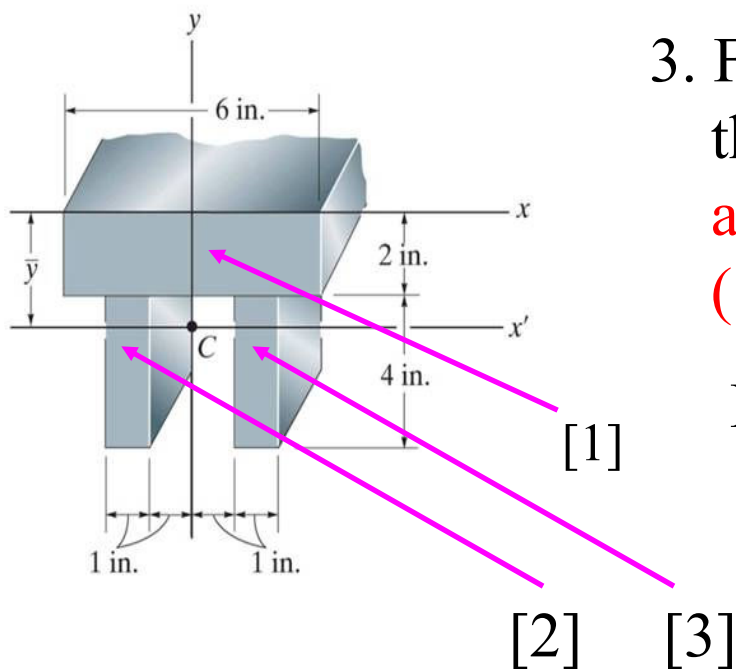
Solution

[2] [3] [1]

1. The cross-sectional area can be divided into three rectangles ([1], [2], [3]) as shown.
2. The centroids of these three rectangles are in their center. The distances from these centers to the y-axis are 0 in, 1.5 in, and 1.5 in, respectively.



EXAMPLE (continued)



3. From the inside back cover of the book, the MoI of a **rectangle** about its **centroidal axis** is $(1/12) b h^3$.

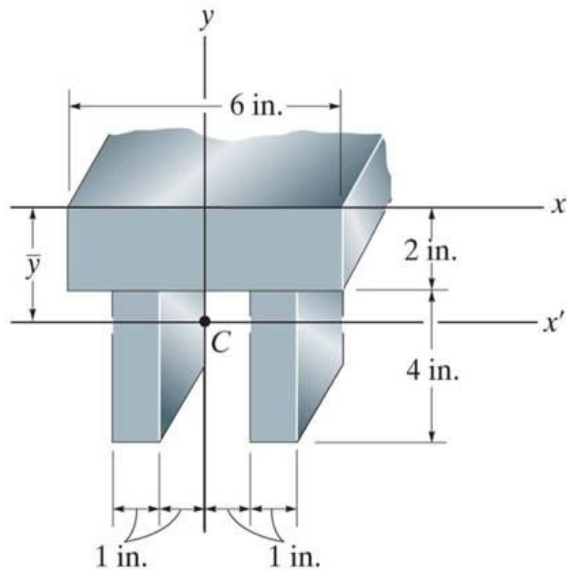
$$\begin{aligned} I_{y[1]} &= (1/12) (2 \text{ in}) (6 \text{ in})^3 \\ &= 36 \text{ in}^4 \end{aligned}$$

Using the parallel-axis theorem,

$$\begin{aligned} I_{Y[2]} &= I_{Y[3]} = \bar{I}_Y + A (d_X)^2 \\ &= (1/12) (4) (1)^3 + (4) (1) (1.5)^2 \\ &= 9.333 \text{ in}^4 \end{aligned}$$



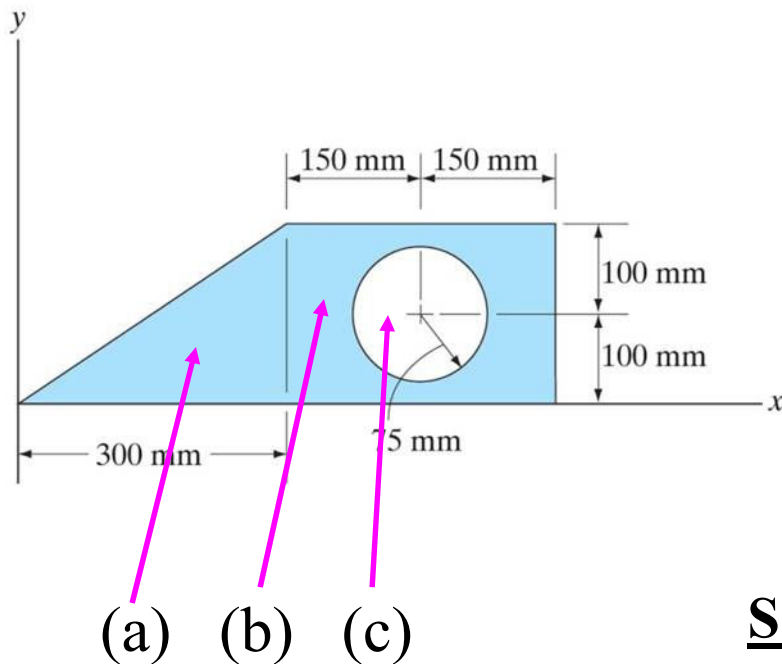
EXAMPLE (continued)



$$\begin{aligned} 4. \quad I_y &= I_{y1} + I_{y2} + I_{y3} \\ &= 54.7 \text{ in}^4 \end{aligned}$$



GROUP PROBLEM SOLVING



Given: The shaded area as shown in the figure.

Find: The moment of inertia for the area about the x-axis and the radius of gyration k_x .

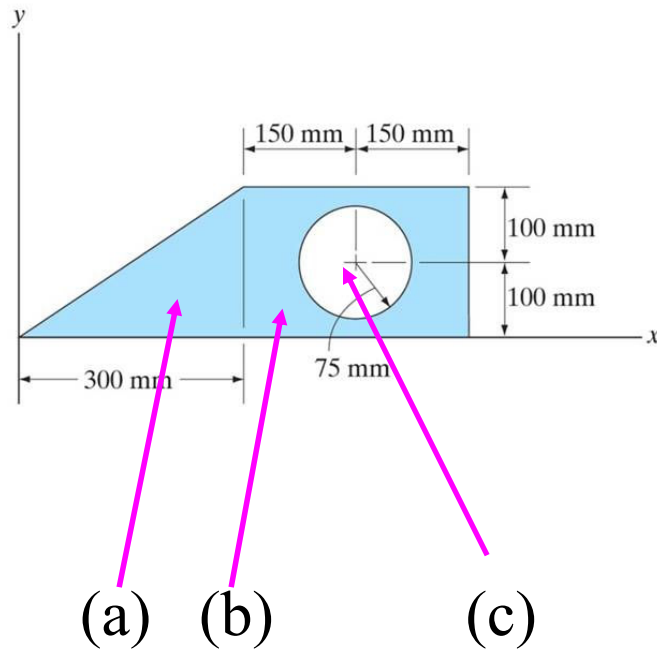
Plan: Follow the steps for analysis.

Solution

1. The given area can be obtained by subtracting the circle (c) from the sum of the triangle (a) and rectangle (b).
2. Information about the centroids of the simple shapes can be obtained from the inside back cover of the book. The perpendicular distances of the centroids from the x-axis are: $d_a = 67 \text{ mm}$, $d_b = 100 \text{ mm}$, and $d_c = 100 \text{ mm}$.



GROUP PROBLEM SOLVING (continued)



$$3. I_{Xa} = (1/36) (300) (200)^3 + (1/2) (300) (200) (67)^2 = 201.3 (10)^6 \text{ mm}^4$$

$$I_{Xb} = (1/12) 300(200)^3 + 300(200)(100)^2 = 800 (10)^6 \text{ mm}^4$$

$$I_{Xc} = (1/4) \pi (75)^4 + \pi (75)^2 (100)^2 = 201.5 (10)^6 \text{ mm}^4$$

$$I_X = I_{Xa} + I_{Xb} - I_{Xc} = 799.8 (10)^6 \text{ mm}^4$$

$$k_X = \sqrt{(I_X / A)}$$

$$A = (1/2) 300 (200) + 300 (200) - \pi (75)^2 = 7.234 (10)^4 \text{ mm}^2$$

$$k_X = \sqrt{\{799.8 (10)^6 / 7.233 (10)^4\}} = 105 \text{ mm}$$



End of the Lecture

Let Learning Continue

