Newton Raphson Method

F(Xn+1)=f(Xn)+(X(n+1)-Xn)f’(xn)

F(Xn+1)=0

f(Xn)+(X(n+1)-Xn)f’(xn)=0

f(Xn)=-{X(n+1)-X(n)} f’(Xn)

f(Xn)/f’(Xn)= -X(n+1)+X(n)

X(n+1)=X(n)- f(Xn)/f’(Xn)

1st iteration

X(0)=1 guess

F(xn)=sinxn

F(x0)=sinx0=sin1= 0.017

F’(xn)=cosxn

F’(x0)=cos(x0)=cos(1)=0.99

 Xn+1=Xn-f(Xn)/f’(Xn) where n=0

 =1-0.017/0.99= 0.983

X(1)=0.983

2nd iteration

X(1)=0.983

F(xn)=sinxn

F(x1)=sin(x1)=sin(0.983)=0.017

F’(xn)=cosxn

F’(x1)=cos(x1)=cos(0.983)=0.99

 Xn+1=Xn-f(Xn)/f’(Xn) where n=1

 X2=0.983-0.017/0.99= 0.966

X2=0.966

3rd iteration

X(2)=0.966

F(xn)=sinxn

F(x1)=sin(x1)=sin(0.966)=0.0168=0.017

F’(xn)=cosxn

F’(x1)=cos(x1)=cos(0.966)=0.99

 Xn+1=Xn-f(Xn)/f’(Xn) where n=2

 X3=0.966-0.017/0.99= 0.949

X3=0.949

Next Guess is X0=0.5

 1st iteration

X0=0.5

F(xn)=sinxn

F(x1)=sin(0.5)=0.008

F’(xn)=cosxn

F’(x1)=cos(0.5)=0.99

 Xn+1=Xn-f(Xn)/f’(Xn) where n=0

 X1=0.5-0.008/0.99=0.492

X2=0.483

X3=0.474

X4=0.464

Question Number 2

X0=2

F(x)= x^2-2x-2

F’(x)=2x-2

Xn+1= Xn – f(Xn)/f’(Xn) ----------equation 1

Putting n=0 in equation 1

X1 = X0 – f(X0)/f’(X0)

F(x0)= x0^2-2x0-2

 =2^2-2x2-2= -2

F’(x0)=2X0-2= 2(2)-2= 2

X1 = X0 – f(X0)/f’(X0)

 =2- (-2)/2 =2 + 1

 =3

X2 = 2.75

X3 = 2.7321

X4 = 2.7320

X5 = 2. 7321

hence x=2.7321 is the root correct to 3 dp

Question number 3

X0=2

F(x)= 1-5/X^2

F(Xn) = 1- 5 Xn^-2

F’(Xn)= -5 (-2 Xn ^-3)

Xn+1 = Xn – f(Xn)/ f’ (Xn) --------equation

Xn+1 = Xn – (1- 5 Xn^-2)/ -5 (-2 Xn ^-3)

 =Xn- (1-5 Xn^-2)/ 10 Xn^ -3

 = Xn- 1/10(Xn^3 – 5 Xn)

 =(10 Xn - Xn^3 + 5Xn)/10

 = (15 Xn – Xn^3 )/10

Xn+1 = (15 Xn – Xn^3 )/10

n=0 Xo=2

X1 = (15 Xo – Xo^3)/10

 = (15 x 2 – 2^3)/10

 = (30 -8)/10

 = 2.2

n=1 X1=2.2

Xn+1 = (15 Xn – Xn^3 )/10

X2 = (15 X1 - X1^3)/10

 = (15 (2.2) - 2.2 ^ 3)/10

 =2.2352

X3 = 2.2360

X4 = 2.2360

 Day 2

Newton Raphson Method

F(x) = X^2 – 2X – 2

F’(Xn)= 2Xn -2

X0 = 2

Xn+1= Xn – F(Xn)/F’(Xn)

F(Xn) = Xn^2 – 2Xn – 2

**First iteration**

Putting n=0 and Xo=2

F(Xo)= Xo^ 2 – 2 Xo – 2 = 2^2 – 2x 2 – 2= -2

F’(Xo)= 2Xo -2 = 2x2-2= 2

Xn+1= Xn – F(Xn)/F’(Xn) n=0

X1= Xo – f(Xo)/f’(Xo)

 = 2 – (-2)/2 = 3

**2nd Iteration**

Putting n=1 and X1=3

X2=2.75

F(X1)= X1^ 2 – 2 X1 – 2 = 3^2 – 3x 2 – 2= 1

F’(X1)= 2Xo -2 = 2x3-2= 4

Xn+1= Xn – F(Xn)/F’(Xn) n=0

X2= X1– f(X1)/f’(X1)

 = 3 – (1)/4 = 2.75

**3rd Iteration**

X3=2.7321

**4th Iteration**

X3=2.7320

**5th Iteration**

X5=2.7321

Hence X= 2.7321 is the root correct to 3 dp

F(x) = X^2 – 2X – 2

F(2.7321)= (2.7321)^2 – 2(2.7321) -2= 0.0003188

Question 2

F(x)= 1- 5/x^2 3dp accuracy

X0=2

X1= 2.2

X2= 2.2352

X3= 2.2360

X4= 2.235

F(2.236)= 1- 5/(2.236)^2 = -0.00000005

Question

F(x)= 1- 5/x^2 = 1-5 x^-2

F’(x) = 10 x^-3 = 10/x^3

Xn+1 =Xn – F(x)/F’(x) = Xn – (1-5 xn^-2)/ 10 xn^-3

 =Xn –(Xn^3- 5 Xn)/10

 =(10 Xn – (Xn^3 -5 Xn))/10

 =(10 Xn- Xn^3 + 5Xn) /10

 Xn+1 =(15 Xn – Xn^3) /10

 n=0 Xo=2

Xn+1 =(15 Xn – Xn^3) /10

X1= (15 x 2- 2^3)/10

 X1=2.2