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Conjugate Effect of Heat and Mass Transfer in Natural Convection Flow from an Isothermal Sphere with Chemical Reaction†

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Natural convection flow from an isothermal sphere immersed in a viscous incompressible fluid in the presence of species concentration and chemical reaction has been investigated. The governing boundary layer equations are transformed into a non-dimensional form and the resulting nonlinear system of partial differential equations are reduced to local non-similarity boundary layer equations, which are solved numerically by very efficient implicit finite difference method together with Keller box scheme. Numerical results are presented by velocity, temperature and species concentration profiles of the fluid as well as the local skin-friction coefficients, local heat transfer rate and the local species concentration transfer rate for a wide range of chemical reaction parameter γ ($= 0.0; 0.5; 1.0; 2.0; 4.0$), buoyancy parameter w ($= 0.0; 0.2; 0.6; 1.0$), Schmidt number Sc ($=0.7; 10.0; 50.0; 100.0$) and Prandtl number Pr ($=0.7; 7.0$).

* * *

Nomenclature

a	radius of the sphere;
C_p	specific heat at constant pressure;
C_{fx}	local skin-friction;
D	molecular diffusivity;
f	dimensionless stream function;
g	acceleration due to gravity;
Gr	Grashof number;
J_w	concentration flux;
k	thermal conductivity;
Nu_x	local Nusselt number;
Pr	Prandtl number;

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q_w	heat flux at the surface;
Sc	Schimdt number;
Sh_x	local Sherwood number;
T	temperature of the fluid in the boundary layer;
T_∞	temperature of the ambient fluid;
T_w	temperature at the surface;
u, v	the dimensionless $x-$ and $y-$component of the velocity;
$\hat{u}, \hat{v}$	the dimensional $\hat{x}-$ and $\hat{y}-$component of the velocity;
w	buoyancy parameter;
x, y	axis in the direction along and normal to the surface.

Greek Symbols

β	volumetric coefficient of thermal expansion;
ψ	stream function;
τ_w	shear stress;
γ	chemical reaction parameter;
ρ	density of the fluid;
μ	viscosity of the fluid;
ϕ	dimensionless concentration function;
θ	dimensionless temperature function.

Introduction

The application of boundary layer techniques to mass transfer has been of considerable assistance in developing the theory of separation processes and chemical kinetics. Some of interesting problems that have been studied are mass transfer from droplets and free convection on electrolysis in non-isothermal boundary layer. The heat, mass and momentum transfer on a continuously moving or stretching plate has several applications in electro-chemistry and polymer processing [1–4].

Gebhart and Pera [5] investigated the nature of vertical natural convection flow resulting form the combined buoyancy effects of thermal and mass diffusion. Diffusion and chemical reaction in an isothermal laminar flow along a soluble flat plate was studied and an appropriate mass-transfer analogue to the flow along a flat plate that contains a species *A* slightly soluble in the fluid *B* were discussed by Fairbanks and Wick [6]. Hossain and Rees [7] investigated the combined effect of thermal and mass diffusion in natural convection flow along a vertical wavy surface. Anjalidevi and Kandasamy [8] investigated the effects of chemical reaction, heat and mass transfer on laminar flow along a semi-infinite horizontal plate.

By taking advantage of the mathematical equivalence of the thermal boundary layer problem with the concentration analogue, results obtained for heat transfer characteristics can be carried directly over to the case of mass transfer by replacing the Prandtl number Pr by the Schimdt number Sc. However, the presence of a chemical reaction term in the mass diffusion equation generally destroys the formal equivalence with the thermal energy problem and, moreover, generally prohibits the construction of the otherwise attractive similarity solutions. Takhar et al. [9] for example, considered diffusion of a chemical reactive species from a stretching sheet.

The boundary layer theory with chemical reaction has been applied to some problems of free and mixed convection flow from the surface of simple geometry by the above authors. Chiang et al. [10], Huang and Chen [11] investigated the laminar free convection from a sphere. Pop et

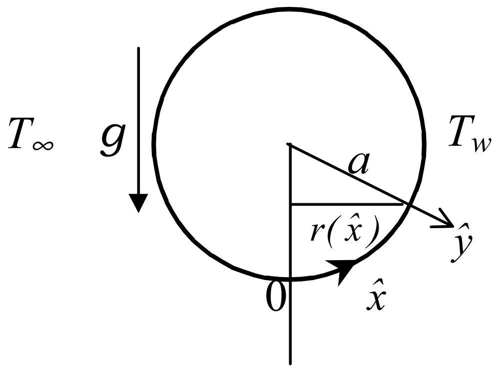

Fig. 1. Physical model and coordinate system.

al. [12] studied the free convection boundary layer on an isothermal sphere in a micropolar fluid. The analysis of mixed convection about a sphere was done by Chen and Mocoglu [13].

This investigation is focused on the boundary layer regime promoted by the combined events on a sphere with chemical reaction when the surface is at a uniform temperature and a uniform mass diffusion. Here it has been assumed that the level of species concentration is very low and that the heat generated during chemical reaction can be neglected. The basic equations are transformed to local non-similarity boundary layer equations, which are solved numerically using a very efficient finite-difference scheme together with Keller-box method [14]. To the best of our knowledge, this problem has not been considered before. The situation where the buoyancy forces assist the natural convection flow for various combination of the chemical reaction parameter γ, conjugate buoyancy parameter w, Prandtl number Pr and Schimdt number Sc is studied. The results allow us to predict different behavior that can be observed when the relevant parameters are varied.

1. Formulation of the Problem

A steady two-dimensional laminar free convective flows from an isothermal sphere of radius a, which is immersed in a viscous and incompressible fluid. It is assumed that the surface temperature of the sphere is T_w, where $T_w > T_\infty$. Here T_∞ is the ambient temperature of the fluid and T is the temperature of the fluid. The configuration considered is as shown in Fig. 1.

Under the usual Bousinesq approximation, the equations governing the flow are:

$$\frac{\partial}{\partial \hat{x}}\left(\hat{r}\hat{u}\right) + \frac{\partial}{\partial \hat{y}}\left(\hat{r}\hat{v}\right) = 0, \tag{1}$$

$$\rho\left(\hat{u}\frac{\partial \hat{u}}{\partial \hat{x}} + \hat{v}\frac{\partial \hat{u}}{\partial \hat{y}}\right) = \mu\frac{\partial^2 \hat{u}}{\partial \hat{y}^2} + \rho\,\mathrm{g}\beta_T\left(T - T_\infty\right)\sin\left(\frac{\hat{x}}{a}\right) + \rho\,\mathrm{g}\beta_c\left(C - C_\infty\right)\sin\left(\frac{\hat{x}}{a}\right), \tag{2}$$

$$\hat{u}\frac{\partial T}{\partial \hat{x}} + \hat{v}\frac{\partial T}{\partial \hat{y}} = \frac{k}{\rho c_p}\frac{\partial^2 T}{\partial y^2}, \tag{3}$$

$$\hat{u}\frac{\partial C}{\partial \hat{x}} + \hat{v}\frac{\partial C}{\partial \hat{y}} = D\frac{\partial^2 C}{\partial y^2} - K_0 C. \tag{4}$$

The boundary conditions (1) to (4) are:

$$\hat{u} = \hat{v} = 0, \qquad T = T_w, \qquad C = C_w \qquad \text{at} \qquad \hat{y} = 0; \tag{5}$$

$$\hat{u} \to 0, \qquad T \to T_\infty, \qquad C \to 0 \qquad \text{as} \qquad \hat{y} \to \infty, \tag{6}$$

where $\hat{u}$, $\hat{v}$ are velocity components along the $\hat{x}$, $\hat{y}$-axes; g is the acceleration due to gravity; ρ is the density; k is the thermal conductivity; β_T is the coefficient of thermal expansion; β_C is the coefficient of concentration expansion; μ is the viscosity of the fluid; C_p is the specific heat due to constant pressure and D is the molecular diffusivity of the species concentration. Here $r(\hat{x}) = a\sin(\hat{x}/a)$. An appropriate mass transfer analogue to the problem discussed from the surface of a sphere that contains a species *A* slightly soluble in fluid *B*. The concentration of the reactant is maintained at a constant value C_w at the surface of the sphere and is assumed to vanish far away from the surface (i. e., $C_\infty = 0.0$). Let the reaction of a species *A* with *B* be the first order homogeneous chemical reaction with constant rate K_0.

We now introduce the following non-dimensional variables:

$$\begin{gathered} x = \frac{\hat{x}}{a}, \qquad y = \mathrm{Gr}^{1/4}\left(\frac{\hat{y}}{a}\right), \qquad u = \frac{a}{\nu}\,\mathrm{Gr}^{-1/2}\hat{u}, \qquad v = \frac{a}{\nu}\,\mathrm{Gr}^{-1/4}\hat{v}, \\ \theta = \frac{T - T_\infty}{T_w - T_\infty}, \qquad \phi = \frac{C}{C_w}, \qquad \mathrm{Gr} = \mathrm{Gr}_{LT} + \mathrm{Gr}_{LC}, \\ \mathrm{Gr}_{LT} = \frac{\mathrm{g}\beta_T\left(T_w - T_\infty\right)a^3}{\nu^2}, \qquad \mathrm{Gr}_{LC} = \frac{\mathrm{g}\beta_c C_w a^3}{\nu^2}, \end{gathered} \tag{7}$$

where $\nu = \mu/\rho$ is the reference kinematic viscosity; Gr_{LT} is the Grashof number; Gr_{LC} is the modified Grashof number; θ is the non-dimensional temperature and ϕ is the non-dimensional species concentration function.

Substituting variable (7) into Eqs (1) – (4) leads to the following non-dimensional equations:

$$\frac{\partial}{\partial x}(ru) + \frac{\partial}{\partial y}(rv) = 0, \tag{8}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial^2 u}{\partial y^2} + \frac{\theta + N\phi}{1 + N}\sin x, \tag{9}$$

$$u\frac{\partial \theta}{\partial x} + v\frac{\partial \theta}{\partial y} = \frac{1}{\mathrm{Pr}}\frac{\partial^2 \theta}{\partial y^2}, \tag{10}$$

$$u\frac{\partial \phi}{\partial x} + v\frac{\partial \phi}{\partial y} = \frac{1}{\mathrm{Sc}}\frac{\partial^2 \phi}{\partial y^2} - \gamma\phi. \tag{11}$$

With the boundary conditions (4) become:

$$u = v = 0, \qquad \theta = 1, \qquad \phi = 1 \qquad \text{at} \qquad y = 0, \tag{12}$$

$$u \to 0, \qquad \theta \to 0, \qquad \phi \to 0 \qquad \text{as} \qquad y \to \infty, \tag{13}$$

where N is the ratio of the buoyancy forces due to the temperature and concentration; γ is the chemical reaction parameter, which are defined respectively as:

$$N = \frac{\mathrm{Gr}_{LT}}{\mathrm{Gr}_{LC}} \qquad \text{and} \qquad \gamma = \frac{K_0 a^2}{\nu \mathrm{Gr}}. \tag{14}$$

In order to solve Eqs (8) – (11), subject to the boundary conditions (12), (13), we assume the following variables:

$$\psi = xr(x)f(x,y), \qquad \theta = \theta(x,y), \qquad \phi = \phi(x,y), \tag{15}$$

where ψ is the non-dimensional stream function defined in the usual way as:

$$u = \frac{1}{r}\frac{\partial \psi}{\partial y}, \qquad v = -\frac{1}{r}\frac{\partial \psi}{\partial x}. \tag{16}$$

Substituting Eq. (15) into Eqs (8) – (11), after some algebra the following transformed equations take the form:

$$\begin{gathered}\frac{\partial^3 f}{\partial y^3} + \left(1 + \frac{x}{\sin x}\cos x\right) f \frac{\partial^2 f}{\partial y^2} - \left(\frac{\partial f}{\partial y}\right)^2 + \frac{\sin x}{x}\left[(1-w)\,\theta + w\phi\right] \\ = x\left(\frac{\partial f}{\partial y}\frac{\partial^2 f}{\partial x \partial y} - \frac{\partial f}{\partial x}\frac{\partial^2 f}{\partial y^2}\right),\end{gathered} \tag{17}$$

$$\frac{1}{\mathrm{Pr}}\frac{\partial^2 \theta}{\partial y^2} + \left(1 + \frac{x}{\sin x}\cos x\right) f \frac{\partial \theta}{\partial y} = x\left(\frac{\partial f}{\partial y}\frac{\partial \theta}{\partial x} - \frac{\partial \theta}{\partial y}\frac{\partial f}{\partial x}\right), \tag{18}$$

$$\frac{1}{\mathrm{Sc}}\frac{\partial^2 \phi}{\partial y^2} + \left(1 + \frac{x}{\sin x}\cos x\right) f \frac{\partial \phi}{\partial y} - \gamma\phi = x\left(\frac{\partial f}{\partial y}\frac{\partial \phi}{\partial x} - \frac{\partial \phi}{\partial y}\frac{\partial f}{\partial x}\right). \tag{19}$$

Along with boundary conditions:

$$f = \frac{\partial f}{\partial y} = 0, \qquad \theta = 1, \qquad \phi = 1 \qquad \text{at} \qquad y = 0, \tag{20}$$

$$\frac{\partial f}{\partial y} \to 0, \qquad \theta \to 0, \qquad \phi \to 0 \qquad \text{as} \qquad y \to \infty. \tag{21}$$

In Eq. (17) w is termed as the conjugate buoyancy parameter and is defined by $w = N/(1+N)$. We also see that for $N = 0$, $w = 0$ and as $N \to \infty$, $w = 1$.

It can be seen that near the lower stagnation point of the sphere i. e., $x \approx 0$, Eqs (17) – (19) reduce to the following ordinary differential equations:

$$f''' + 2ff'' - f'^2 + \left[(1-w)\,\theta + w\phi\right] = 0, \tag{22}$$

$$\frac{1}{\mathrm{Pr}}\theta'' + 2f\theta' = 0, \tag{23}$$

$$\frac{1}{\mathrm{Sc}}\phi'' + 2f\phi' - \gamma\phi = 0. \tag{24}$$

Subject to the boundary conditions:

$$f(0) = f'(0) = 0, \qquad \theta(0) = 1, \qquad \phi(0) = 1, \tag{25}$$

$$f' \to 0, \qquad \theta \to 0, \qquad \phi \to 0 \qquad \text{as} \qquad y \to \infty. \tag{26}$$

In the above equations primes denote differentiation with respect to y.

In practical applications, the physical quantities of principle interest are the shearing stress, the rate of heat transfer and the rate of species concentration transfer in terms of the skin-friction coefficients C_f, Nusselt number Nu and Sherwood number Sh, respectively, which can be written as:

$$C_f = \frac{\mathrm{Gr}^{-3/4} a^2}{\mu_\infty \nu_\infty}\, \tau_w, \qquad \mathrm{Nu} = \frac{a\mathrm{Gr}^{-1/4}}{k(T_w - T_\infty)}\, q_w, \qquad \mathrm{Sh} = \frac{a\mathrm{Gr}^{-1/4}}{DC_w}\, J_w, \tag{27}$$

where

$$\tau_w = \mu \left(\frac{\partial \hat{u}}{\partial \hat{y}}\right)\bigg|_{\hat{y}=0}, \qquad q_w = -\, k\left(\frac{\partial T}{\partial \hat{y}}\right)\bigg|_{\hat{y}=0}, \qquad J_w = -\, D\left(\frac{\partial C}{\partial y}\right)\bigg|_{\hat{y}=0}. \tag{28}$$

Using the variables (7), (15) and the boundary condition (20) in Eqs (27), (28), we get:

$$C_{fx} = x f''(x,0), \tag{29}$$

$$\mathrm{Nu}_x = -\theta'(x,0), \tag{30}$$

$$\mathrm{Sh}_x = -\phi'(x,0). \tag{31}$$

We also discuss the effect of the chemical reaction parameter γ on the velocity, temperature and concentration distribution. The values of the velocity, temperature and concentration distribution are calculated respectively from the following relations:

$$u = \frac{\partial f}{\partial y}, \qquad \theta = \theta(x,y), \qquad \phi = \phi(x,y). \tag{32}$$

2. Results and Discussion

Eqs (17) – (19) subject to the boundary conditions were solved numerically using a very efficient implicit finite different method together with Keller box scheme, which is described by Cebeci and Bradshow [15]. The numerical solutions start at the lower stagnation point of the sphere, i. e., at $x \approx 0.0$, with initial profiles given by the Eqs (22) – (24) along with boundary conditions Eqs (25), (26) and proceed round the sphere up to $x \approx \pi/2$. The solutions are obtained for fluid having Prandtl number Pr $(= 0.7; 7.0)$, Schimdt number Sc $(= 0.7; 10.0; 50.0; 100.0)$, buoyancy ratio parameter w $(= 0.0; 0.2; 0.6; 1.0)$ and for a wide range of the values of chemical reaction parameter γ $(= 0.0; 0.5; 1.0; 2.0; 4.0)$.

Since the values of $f''(x,0)$, $[-\theta'(x,0)]$ and $[-\phi'(x,0)]$ in terms of the local skin-friction C_{fx}, the local Nusselt number Nu_x and the local Sherwood number Sh_x respectively are calculated from the Eqs (17) – (21) for the surface of the sphere. The numerical values of C_{fx}, Nu_x and Sh_x are depicted in Tables 1, 2 and Figs 2 – 5. A comparison of the local Nusselt number Nu_x, obtained in the present work and obtained earlier by Huang and Chen [11] and Pop et al. [12] has been shown in Table 1. It is clearly seen that there is an excellent agreement among the respective results.

The influence of the parameter γ, on the reduced local skin-friction coefficient C_{fx}, local Nusselt number Nu_x and the local Sherwood number Sh_x are illustrated in Fig. 2a–c respectively while $\mathrm{Pr} = \mathrm{Sc} = 0.7$, $w = 0.5$. It can be seen that an increase of the chemical reaction parameter γ $(= 0.0; 0.5; 1.0; 2.0; 4.0)$, leads to an increase in the local skin-friction and the local Sherwood number and a decrease in the local Nusselt number. This may be attributed to the fact that the increase of the values of γ implies more interaction of species concentration with the momentum boundary layer and less interaction with the thermal boundary layer.

Fig. 3a–c deal with the effect of chemical reaction parameter γ $(= 0.0; 0.5; 1.0; 2.0; 4.0)$ on the velocity, temperature and species concentration distribution at $x = \pi/4$, while $\mathrm{Pr} = \mathrm{Sc} = 0.7$,

$w = 0.5$. Here it is found that the velocity and concentration profiles decrease significantly and the non-dimensional temperature distribution increases slightly with the increase of chemical reaction parameter.

The values of $f''(x,0)$, $[-\theta'(x,0)]$ and $[-\phi'(x,0)]$ are given in Table 2 for different values of Prandtl number Pr ($= 0.7; 7.0$) while $\mathrm{Sc} = 0.7$ and $\gamma = w = 0.5$. For increasing axial distance parameter x, the values of skin-friction increase and the values of rate of heat transfer and the rate of species concentration decrease for both Prandtl number Pr. On the other hand, the values of skin-friction decrease and the values of rate of heat transfer and the rate of species concentration increase with the increase of values of Pr.

Fig. 4a–c show how variations in w ($= 0.0; 0.2; 0.6; 1.0$) affect the flow. When $w = 0.0$, the flow is induced entirely by thermal effects and the detailed concentration field is computed as a forced convection problem, whereas when $w = 1.0$, it is the concentration field which induced the boundary flow. It can be stated that an increase in the values of w leads to a decrease in the values of the local skin-friction coefficientsC_{fx}, local Nusselt number Nu_x and the local Sherwood number Sh_x. Variation in the Schimdt number Sc are considered in Fig. 5a–c, while $\mathrm{Pr} = 0.7$ and $\gamma = w = 0.5$. The local skin-friction coefficients C_{fx} and local Nusselt number Nu_x decrease and

Table 1
The values of $[-\theta'(x,0)]$ while $w = \gamma = 0.0$ for different values of Prandtl number Pr .

x (degrees)	$\mathrm{Pr} = 0.7$ Huang & Chen [11]	Pop et al. [12]	Present	$\mathrm{Pr} = 7.0$ Huang & Chen [11]	Pop et al. [12]	Present
0	0.4574	0.4576	0.4576	0.9581	0.9595	0.9582
10	0.4563	0.4565	0.4564	0.9559	0.9572	0.9558
20	0.4532	0.4533	0.4532	0.9496	0.9506	0.9492
30	0.4480	0.4480	0.4479	0.9389	0.9397	0.9383
40	0.4407	0.4405	0.4404	0.9239	0.9239	0.9231
50	0.4312	0.4308	0.4307	0.9045	0.9045	0.9034
60	0.4194	0.4189	0.4188	0.8805	0.8801	0.8791
70	0.4053	0.4046	0.4045	0.8518	0.8510	0.8501
80	0.3886	0.3879	0.3877	0.8182	0.8168	0.8161
90	0.3694	0.3684	0.3683	0.7792	0.7774	0.7768

Table 2
The values of $f''(x,0)$, $[-\theta'(x,0)]$ and $[-\phi'(x,0)]$ while $w = \gamma = 0.5$ and $\mathrm{Sc} = 0.7$
for different values of Prandtl number Pr .

x	$\mathrm{Pr} = 0.7$			$\mathrm{Pr} = 7.0$		
	$f''(x,0)$	$[-\theta'(x,0)]$	$[-\phi'(x,0)]$	$f''(x,0)$	$[-\theta'(x,0)]$	$[-\phi'(x,0)]$
0	0.00000	0.44204	0.71690	0.00000	1.06692	0.68480
$\pi/18$	0.12703	0.44088	0.71610	0.10299	1.06414	0.68416
$\pi/9$	0.25159	0.43764	0.71384	0.20396	1.05632	0.68233
$\pi/6$	0.37134	0.43231	0.71012	0.30098	1.04344	0.67932
$\pi/4$	0.53696	0.42028	0.70174	0.43506	1.01444	0.67256
$\pi/3$	0.67936	0.40327	0.68994	0.55017	0.97351	0.66306
$\pi/2$	0.86859	0.35308	0.65523	0.70293	0.85328	0.63530

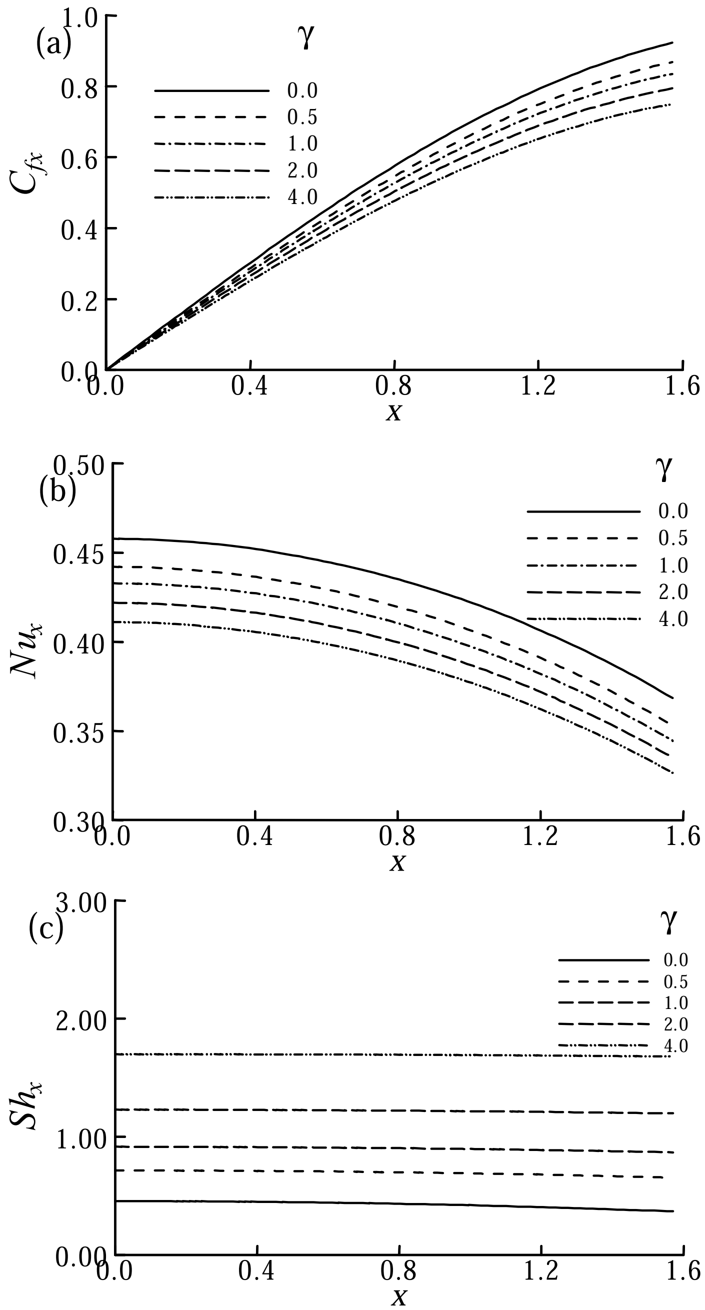

Fig. 2. a) skin-friction coefficient; b) rate of heat transfer; c) rate of species concentration for different values of γ while $\mathrm{Pr} = \mathrm{Sc} = 0.7$ and $w = 0.5$.

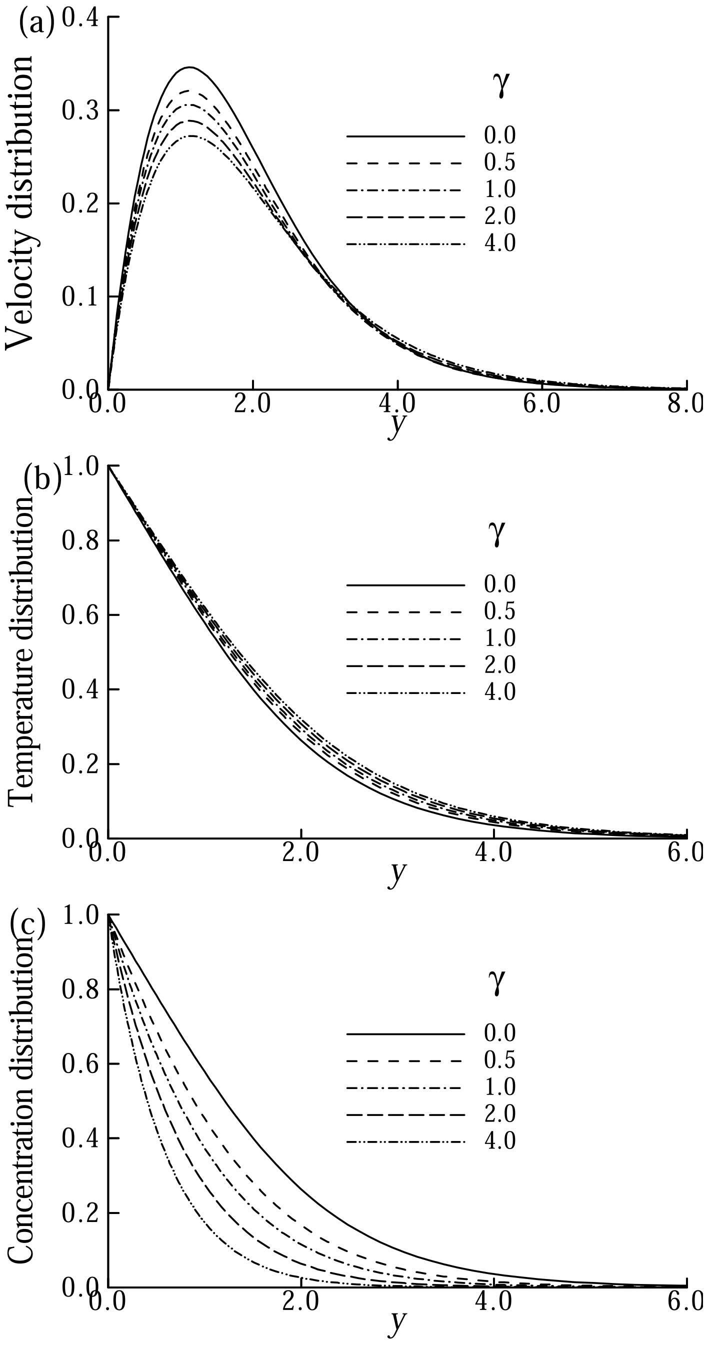

Fig. 3. a) velocity profiles; b) temperature profiles; c) concentration profiles for different values of γ while $\mathrm{Pr} = \mathrm{Sc} = 0.7$, $w = 0.5$ at $x = \pi/4$.

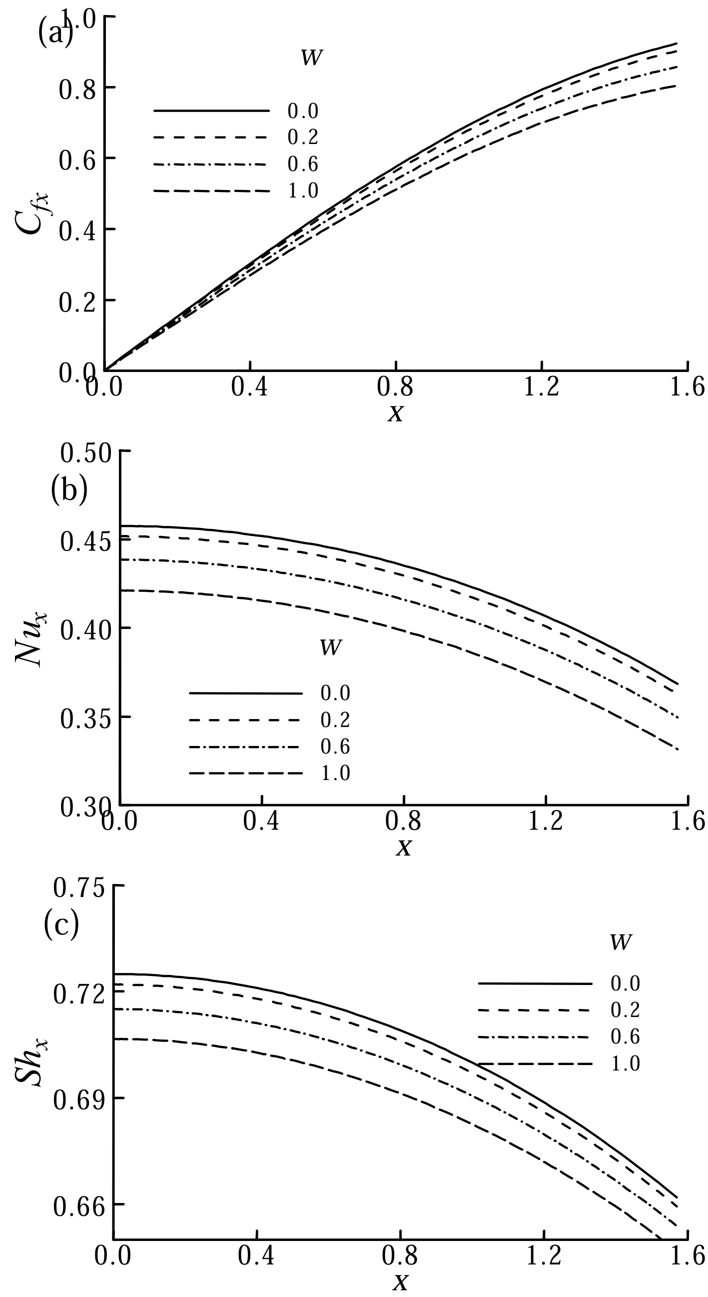

Fig. 4. a) skin-friction coefficient; b) rate of heat transfer; c) rate of species concentration for different values of w while $\mathrm{Pr} = \mathrm{Sc} = 0.7$ and $\gamma = 0.5$.

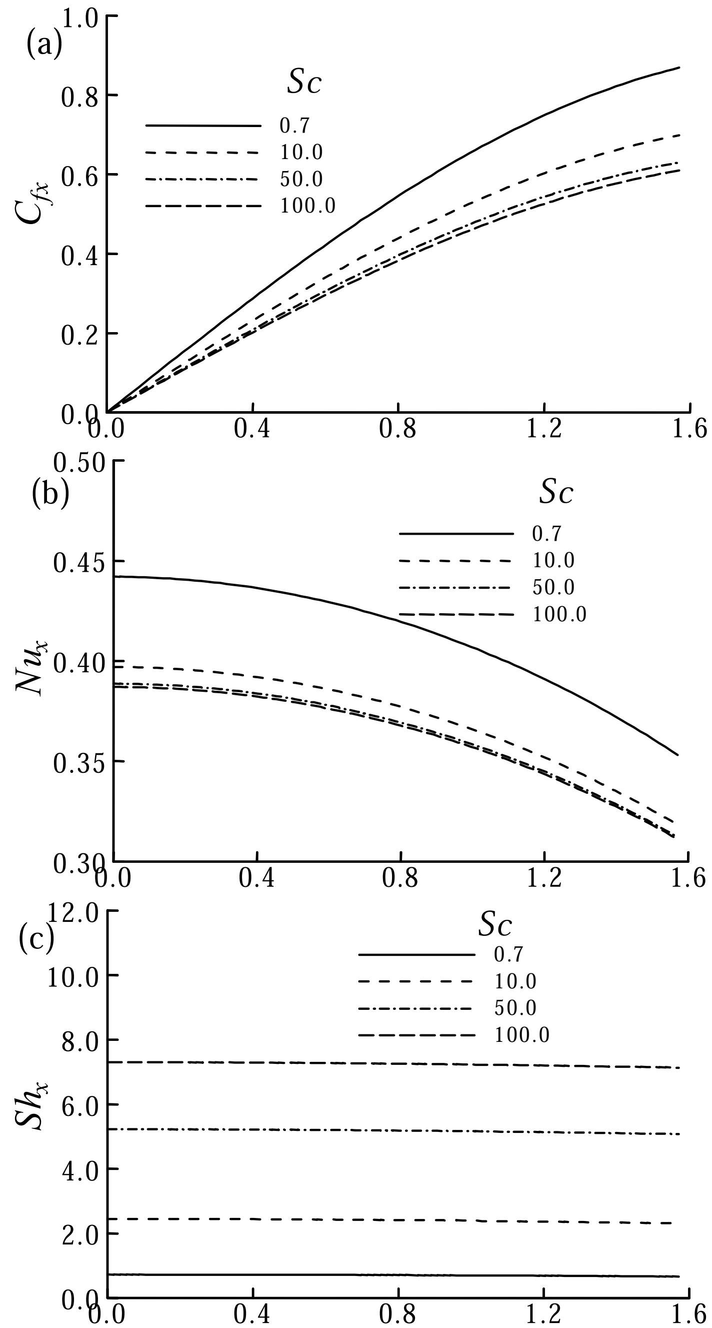

Fig. 5. a) skin-friction coefficient; b) rate of heat transfer; c) rate of species concentration for different values of Sc while $\mathrm{Pr} = 0.7$ and $w = \gamma = 0.5$.

the corresponding local Sherwood number Sh_x significantly increases with the increase of values of Sc, which was expected.

Conclusion

The effect of chemical reaction, heat and mass diffusion in natural convection flow from an isothermal sphere has been investigated numerically. New variables to transform the complex geometry into a simple shape and the resulting non-similarity boundary layer equations are solved by a very efficient implicit finite difference method together with Keller-box scheme [14].

From the present investigation the following conclusions may be drawn:

- The local skin-friction coefficient C_{fx} and the local Sherwood number Sh_x increase and in the local Nusselt number Nu_x decrease when the value of the chemical reaction parameter γ increases.

- As γ increases both the velocity and concentration distribution decrease significantly and the temperature distribution enhance slightly at $x = \pi/4$ of the surface.

- An increase in the values of w leads to a decrease in the values of the local skin-friction coefficients C_{fx}, local Nusselt number Nu_x and the local Sherwood number Sh_x.

- At increase of the values of Pr and Sc the skin-friction coefficient C_{fx} decreases and the rate of species concentration Sh_x increases within the boundary layer.

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