

Effects of chemical reaction, heat and mass transfer along a wedge with heat source and concentration in the presence of suction or injection

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Abstract

An approximate numerical solution for the steady laminar boundary-layer flow over a wall of the wedge with suction or injection in the presence of species concentration and mass diffusion has been obtained by solving the governing equations using numerical technique. The fluid is assumed to be viscous and incompressible. Numerical calculations up to third level of truncation are carried out for different values of dimensionless parameters and an analysis of the results obtained shows that the flow field is influenced appreciably by the chemical reaction, heat source and suction or injection at the wall of the wedge.

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1. Introduction

Combined heat and mass transfer problems with chemical reaction are of importance in many processes and have, therefore, received a considerable amount of attention in recent years. In processes such as drying, evaporation at the surface of a water body, energy transfer in a wet cooling tower and the flow in a desert cooler, heat and mass transfer occur simultaneously. Possible applications of this type of flow can be found in many industries. For example, in the power industry, among

the methods of generating electric power is one in which electrical energy is extracted directly from a moving conducting fluid.

Many practical diffusive operations involve the molecular diffusion of a species in the presence of chemical reaction within or at the boundary. There are two types of reactions. A homogeneous reaction is one that occurs uniformly throughout a given phase. The species generation in a homogeneous reaction is analogous to internal source of heat generation. In contrast, a heterogeneous reaction takes place in a restricted region or within the boundary of a phase. It can therefore be treated as a boundary condition similar to the constant heat flux condition in heat transfer. The study of heat and mass transfer with chemical reaction is of great practical importance to engineers and scientists because of its

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almost universal occurrence in many branches of science and engineering. The flow of a fluid past a wedge is of fundamental importance since this type of flow constitutes a general and wide class of flows in which the free stream velocity is proportional to the power of the length coordinate measured from the stagnation point.

All industrial chemical processes are designed to transform cheaper raw materials to high value products (usually via chemical reaction). A 'reactor', in which such chemical transformations take place, has to carry out several functions like bringing reactants into intimate contact, providing an appropriate environment (temperature and concentration fields) for adequate time and allowing for the removal of products. Fluid dynamics plays a pivotal role in establishing relationship between reactor hardware and reactor performance. For a specific chemistry/catalyst, the reactor performance is a complex function of the underlying transport processes. The first step in any reaction engineering analysis is formulating a mathematical framework to describe the rate (and mechanisms) by which one chemical species is converted into another in the absence of any transport limitations (chemical kinetics). Once the intrinsic kinetics is available, the production rate and composition of the products can be related, in principle, to reactor volume, reactor configuration and mode of operation by solving mass, momentum and energy balances over the reactor. This is the central task of a reaction and reactor engineering activity. Analyses of the transport processes and their interaction with chemical reactions are quite difficult and is intimately connected to the underlying fluid dynamics. Such a combined analysis of chemical and physical processes constitutes for core of chemical reaction engineering. The recent advances in understanding physics of flows and computational flow modeling (CFM) can make tremendous contributions in chemical engineering.

We are particularly interested in cases in which diffusion and chemical reaction occur at roughly the same speed. When diffusion is much faster than chemical reaction, then only chemical factors influence the chemical reaction rate; when diffusion is not much faster than reaction, the diffusion and kinetics interact to produce very different effects. The study of heat generation or absorption effects in moving fluids is important in view of several physical problems, such as fluids undergoing exothermic or endothermic chemical reaction. Due to the fast growth of electronic technology, effective cooling of electronic equipment has become warranted and cooling of electronic equipment ranges from individual transistors to main frame computers and from energy suppliers to telephone switch boards and thermal diffusion effect has been utilized for isotopes separation in the mixture between gases with very light molecular weight (hydrogen and helium) and medium molecular weight.

In these types of problems, the well known Falkner–Skan transformation is used to reduce boundary-layer equations into ordinary differential equations for similar flows [1]. It can also be used for non-similar flows for convenience in numerical work because it reduces, even if it does not eliminate, depends on the x -coordinate. The solutions of the Falkner–Skan equations are sometimes referred to as wedge flow solutions with only two of the wedge flows being common in practice [2]. The dimensionless parameter, m plays an important role in such type of problems because it denotes the shape factor of the velocity profiles. It has been shown [3] that when $m < 0$ (increasing pressure), the velocity profiles have point of inflexion whereas when $m > 0$ (decreasing pressure), there is no point of inflexion. This fact is of great importance in the analysis of the stability of laminar flows with a pressure gradient. Yih [4] presented an analysis of the forced convection boundary layer flow over a wedge with uniform suction/blowing, whereas Watanabe [5] investigated the behavior of the boundary layer over a wedge with suction/injection in forced flow. Recently, laminar boundary layer flow over a wedge with suction/injection has been discussed by Kafoussias and Nanousis [6] and Anjali Devi and Kandasamy [7] analyzed the effects of thermal stratification on laminar boundary layer flow over a wedge with suction/injection.

Since no attempt has been made to analyze nonlinear boundary layer flow with chemical reaction, heat and mass transfer over a wedge with suction or injection at the wall in the presence of heat source, we have investigated it in this article. The similarity transformation has been utilized to convert the governing partial differential equations into ordinary differential equations and then the numerical solution of the problem is drawn using R.K. Gill method. Numerical calculations upto third level of truncation were carried out for different values of dimensionless parameters of the problem under consideration for the purpose of illustrating the results graphically. Examination of such flow models reveals the influence of chemical reaction on velocity, temperature and concentration profiles. The analysis of the results obtained shows that the flow field is influenced appreciably by the presence of chemical reaction, heat source and suction or injection at the wall of the wedge.

2. Mathematical analysis

Two-dimensional laminar boundary-layer flow of a viscous and incompressible fluid over a wall of the wedge with suction or injection is analysed. As shown in Fig. 1 [9], x -axis is parallel to the wedge and y -axis is taken normal to it. The fluid properties are assumed to be constant in a limited temperature range. The concentration of diffusing species is very small in comparison to other chemical species, the concentration of

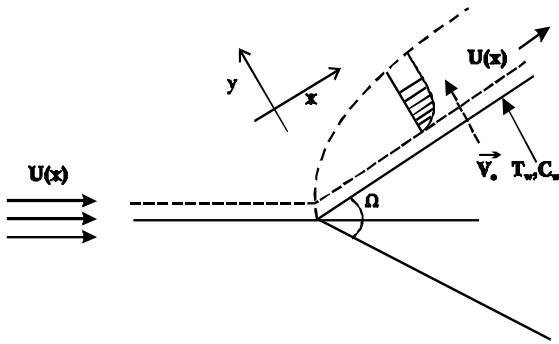


Fig. 1. Flow analysis along the wall of the wedge.

species far from the wall, C_∞ , is infinitesimally small [8] and hence the Soret and Dufour effects are neglected. The chemical reactions are taking place in the flow and all thermophysical properties are assumed to be constant except the density in the buoyancy terms of the linear momentum equation which is approximated according to the Boussinesq's approximation. Under these assumptions, the equations that describe the physical situation are given by

$$\partial u / \partial x + \partial v / \partial y = 0 \tag{1}$$

$$u \partial u / \partial x + v \partial u / \partial y = v \partial^2 u / \partial y^2 + U dU / dx + g\beta(T - T_\infty) \sin(\Omega/2) + g\beta^*(C - C_\infty) \sin(\Omega/2) \tag{2}$$

$$u \partial T / \partial x + v \partial T / \partial y = \alpha \partial^2 T / \partial y^2 + Q(T_\infty - T) \tag{3}$$

$$u \partial C / \partial x + v \partial C / \partial y = D \partial^2 C / \partial y^2 - k_1 C \tag{4}$$

where the term $Q(T_\infty - T)$ is assumed to be the amount of heat generated or absorbed per unit volume. Q is a constant, which may take on either positive or negative values. When the wall temperature T_w exceeds the free stream temperature T_∞ , the source term represents the heat source when $Q < 0$ and heat sink when $Q > 0$. For the condition that $T_w < T_\infty$, the opposite relationship is true [10]. The constant k_1 is the first order chemical reaction rate (when $k_1 < 0$ generating reactant and $k_1 > 0$ destructive reactant) and D is the effective diffusion coefficient.

The boundary conditions are

$$u = 0, \quad v = v_0, \quad C = C_w, \quad T = T_w \text{ at } y = 0$$

$$u = U(x), \quad C = C_\infty, \quad T = T_\infty \text{ as } y \rightarrow \infty \tag{5}$$

Following the lines of Bansal [11], the following change of variables are introduced

$$\psi(x, y) = (2Uvx / (1 + m))^{1/2} f(x, \eta)$$

$$\eta(x, y) = y((1 + m)U / 2vx)^{1/2} \tag{6}$$

Under this consideration, the potential flow velocity can be written as

$$U(x) = cx^m, \quad \beta_1 = 2m / (1 + m) \tag{7}$$

where c is a constant and β_1 is the Hartree pressure gradient parameter that corresponds to $\beta_1 = \Omega / \pi$ for a total angle Ω of the wedge.

The velocity components are given by

$$u = \partial \psi / \partial y, \quad v = -\partial \psi / \partial x \tag{8}$$

It can be easily verified that the continuity Eq. (1) is identically satisfied and introduced the non-dimensional form of temperature and the concentration as

$$\theta = (T - T_\infty) / (T_w - T_\infty) \tag{9}$$

$$\phi = (C - C_\infty) / (C_w - C_\infty) \tag{10}$$

$$Re_x = Ux / \nu \quad (\text{Reynolds number}) \tag{11}$$

$$Gr = \nu g \beta (T_w - T_\infty) / U^3 \quad (\text{Grashof number}) \tag{12}$$

$$Gc = \nu g \beta^* (C_w - C_\infty) / U^3$$

(Modified Grashof number) $\tag{13}$

$$Pr = \mu C_p / K \quad (\text{Prandtl number}) \tag{14}$$

$$Sc = \nu / D \quad (\text{Schmidt number}) \tag{15}$$

$$S1 = (2xQ / U) \quad (\text{Heat source parameter}) \tag{16}$$

$$S = -v_0((1 + m)x / 2\nu U)^{1/2}$$

(Suction or injection parameter) $\tag{17}$

$$\gamma = \nu k_1 / U^2 \quad (\text{Chemical reaction parameter}) \tag{18}$$

Now Eqs. (2)–(4) becomes

$$\partial^3 f / \partial \eta^3 = -f \partial^2 f / \partial \eta^2 - (2m / (1 + m))(1 - (\partial f / \partial \eta)^2) - (2 / (1 + m))(Gc Re_x \phi + Gr Re_x \theta) \sin(\Omega/2) + (2x / (1 + m))(\partial f / \partial \eta)(\partial^2 f / \partial x \partial \eta) - \partial f / \partial x (\partial^2 f / \partial \eta^2) \tag{19}$$

$$\partial^2 \theta / \partial \eta^2 = -Pr f \partial \theta / \partial \eta + (2Pr / (1 + m))\theta \partial f / \partial \eta + Pr(2x / (1 + m))(\partial f / \partial \eta)(\partial \theta / \partial x) - \partial f / \partial x (\partial \theta / \partial \eta) + Pr S1 \theta \tag{20}$$

$$\partial^2 \phi / \partial \eta^2 = -Sc f \partial \phi / \partial \eta + (2Sc / (1 + m))Re_x \gamma \phi + (2Sc / (1 + m))\phi \partial f / \partial \eta + (2x Sc / (1 + m)) \times (\partial f / \partial \eta)(\partial \phi / \partial x) - \partial \phi / \partial \eta (\partial f / \partial x) \tag{21}$$

The boundary conditions (5) can be written as

$$\begin{aligned} \eta = 0: \partial f / \partial \eta = 0, (f/2)(1 + (x/U)dU/dx) + x \partial f / \partial x \\ = -v_0((1+m)x/2vU)^{1/2}, \quad \theta = 1, \quad \phi = 1 \\ \eta \rightarrow \infty: \partial f / \partial \eta = 1, \quad \theta = 0, \quad \phi = 0 \end{aligned} \quad (22)$$

where v_0 is the velocity of suction if $v_0 < 0$ and injection if $v_0 > 0$.

Eqs. (19)–(21) and boundary conditions (22) can be written as

$$\begin{aligned} \partial^3 f / \partial \eta^3 + (f + ((1-m)/(1+m))\xi \partial f / \partial \xi) \partial^2 f / \partial \eta^2 \\ - (((1-m)/(1+m))\xi \partial^2 f / \partial \xi \partial \eta) \partial f / \partial \eta \\ + (2m/(1+m))(1 - (\partial f / \partial \eta)^2) + (2/(1+m)) \\ \times (Gc Re_x \phi + Gr Re_x \theta) \sin(\Omega/2) = 0 \end{aligned} \quad (23)$$

$$\begin{aligned} \partial^2 \theta / \partial \eta^2 + Pr(f + ((1-m)/(1+m))\xi \partial f / \partial \xi) \partial \theta / \partial \eta \\ - Pr S_1 \theta - (2Pr/(1+m)\theta \partial f / \partial \eta \\ - (((1-m)/(1+m))\xi \partial \theta / \partial \xi) \partial f / \partial \eta = 0 \end{aligned} \quad (24)$$

$$\begin{aligned} \partial^2 \phi / \partial \eta^2 + Sc f \partial \phi / \partial \eta - (2Sc/(1+m)) Re_x \gamma \phi \\ + Sc((1-m)/(1+m))(\partial \phi / \partial \eta \xi \partial f / \partial \xi \\ - \partial f / \partial \eta \xi \partial \phi / \partial \xi) - (2Sc/(1+m)) \partial f / \partial \eta \phi = 0 \end{aligned} \quad (25)$$

$$\begin{aligned} \eta = 0: \partial f / \partial \eta = 0, \quad (1+m)f/2 + ((1-m)/2)\xi \partial f / \partial \xi = S, \\ \theta = 1, \quad \phi = 1 \\ \eta \rightarrow \infty: \partial f / \partial \eta = 1, \quad \theta = 0, \quad \phi = 0 \end{aligned} \quad (26)$$

where S is the suction parameter if $S > 0$ and injection if $S < 0$ and $\xi = kx^{(1-m)/2}$ is the dimensionless distance along the wedge ($\xi > 0$).

In this system of equations $f(\xi, \eta)$ is the dimensionless stream function; $\theta(\xi, \eta)$ be the dimensionless temperature; $\phi(\xi, \eta)$ be the dimensionless concentration; Pr , the Prandtl number, Re_x , Reynolds number etc., which are defined in (9)–(18). The parameter ξ indicates the dimensionless distance along the wedge ($\xi > 0$). It is obvious that to retain the ξ -derivative terms, it is necessary to employ a numerical scheme suitable for partial differential equations for the solution. In addition, owing to the coupling between adjacent stream wise location through the ξ -derivatives, a locally autonomous solution, at any given stream wise location can not be obtained. In such a case, an implicit marching numerical solution scheme is usually applied proceeding the solution in the ξ -direction, i.e., calculating unknown profiles at ξ_{i+1} when the same profiles at ξ_i are known. The process starts at $\xi = 0$ and the solution proceeds from ξ_i to ξ_{i+1} but such a procedure is time consuming.

However, when the terms involving $\partial f / \partial \xi$, $\partial \theta / \partial \xi$ and $\partial \phi / \partial \xi$ and their η derivatives are deleted, the resulting system of equations resemble, in effect, a system of ordinary differential equations for the functions f , θ and ϕ with ξ as a parameter and the computational task is

simplified. Furthermore, a locally autonomous solution for any given ξ can be obtained because the stream wise coupling is severed. So, following the lines of [6], a recent numerical solution scheme is utilized for obtaining the solution of the problem. Now, due to the above mentioned factors, Eqs. (23)–(25) are changed to

$$\begin{aligned} f''' + ff'' + (2m/(1+m))(1 - (f')^2) + (2/(1+m)) \\ \times (Gc Re_x \phi + Gr Re_x \theta) \sin(\Omega/2) = 0 \end{aligned} \quad (27)$$

$$\theta'' + Pr f \theta' - (2Pr/(1+m))f' \theta - S_1 Pr \theta = 0 \quad (28)$$

$$\begin{aligned} \phi'' + Sc f \phi' - (2Sc/(1+m))f' \phi - (2Sc/(1+m)) Re_x \gamma \phi \\ = 0 \end{aligned} \quad (29)$$

with boundary conditions

$$\begin{aligned} \eta = 0: f(0) = (2/(1+m))S, \quad f'(0) = 0, \\ \theta(0) = 1, \quad \phi(0) = 1 \\ \eta \rightarrow \infty: f'(\infty) = 1, \quad \theta(\infty) = 0, \quad \phi(\infty) = 0 \end{aligned} \quad (30)$$

Eqs. (27)–(29) with boundary conditions (30) are integrated using R.K. Gill method. Velocity, temperature and concentration are studied for different values of chemical reaction, heat source and suction or injection at the wall of wedge.

3. Results and discussion

In order to get a clear insight of the physical problem, the velocity, temperature and concentration have been discussed by assigning numerical values to the parameters encountered in the problem. To be realistic, the values of Schmidt number (Sc) are chosen for hydrogen ($Sc = 0.22$), water vapour ($Sc = 0.62$) and ammonia ($Sc = 0.78$) at temperature 25°C and one atmospheric pressure. The values of Prandtl number is chosen to be $Pr = 0.71$ which represents air at temperature 20°C and one atmospheric pressure. The effect of buoyancy is significant for $Pr = 0.71$ (air) due to the lower density of air that makes it more sensitive to the buoyancy forces. Grashof number for heat transfer is chosen to be $Gr = 1.0$ and modified Grashof number for mass transfer, $Gc = 3.0$ corresponding to cooling ($Gr > 0$) of the plate. Reynolds number $Re_x = 3.0$, heat source parameter (absorption) $S_1 = 1.0$, the study of heat absorption ($S_1 > 0$) effects in moving fluids is important in view of several physical problems, such as fluids undergoing exothermic or endothermic chemical reaction and the chemical reaction parameter is chosen to be $\gamma = 0.5, 1.0, 3.0$ and 4.0 . Numerical results are displayed with the help of graphical illustrations. In the absence of mass transfer and magnetic effects, the results have been compared with that of previous work [6] and it is found that they are in good agreement. The

numerical results obtained are illustrated by means of Figs. 2–10.

Effects of the Schmidt number with uniform chemical reaction at the wall of the wedge over the velocity, temperature and concentration are shown through Figs. 2–4.

Fig. 2 depicts the dimensionless velocity profiles $f'(\eta)$ for different values of Schmidt number (Sc). Due to the uniform suction and heat source, it is observed that the velocity component of the fluid along the wall of the wedge increases with increase of Schmidt number. On the contrary, the dimensionless temperature $\theta(\eta)$ and concentration $\phi(\eta)$ of the fluid reduce with increase of Schmidt number and these are shown in the Figs. 3 and 4, respectively. So, it is also observed that the temperature and concentration of the fluid gradually changes from higher value to the lower value only when the diffusive effect D is smaller than kinematic viscosity. All these physical behavior are due to the combined effects of suction at the wall of the wedge and chemical reaction.

The effects of chemical reaction over velocity, temperature and concentration of the fluid along the wall of the wedge are shown through Figs. 5–7.

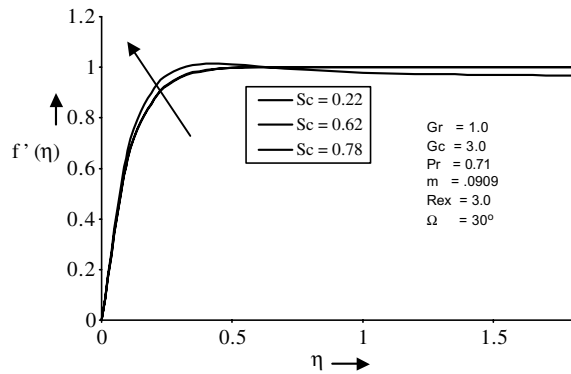


Fig. 2. Effects of Schmidt number over the velocity profiles.

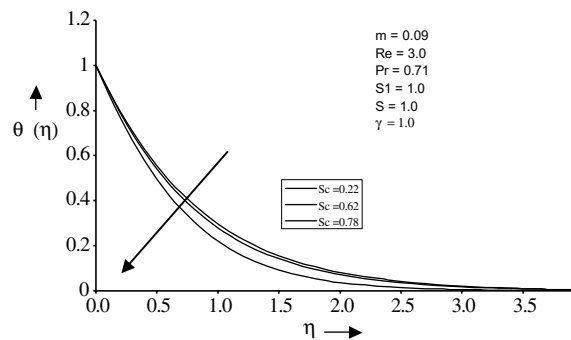


Fig. 3. Effects of Schmidt number over the temperature profiles.

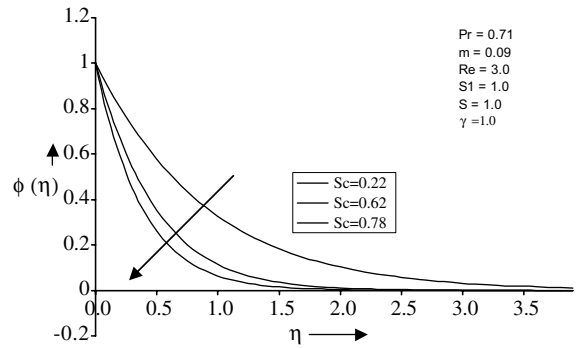


Fig. 4. Effects of Schmidt number over the concentration profiles.

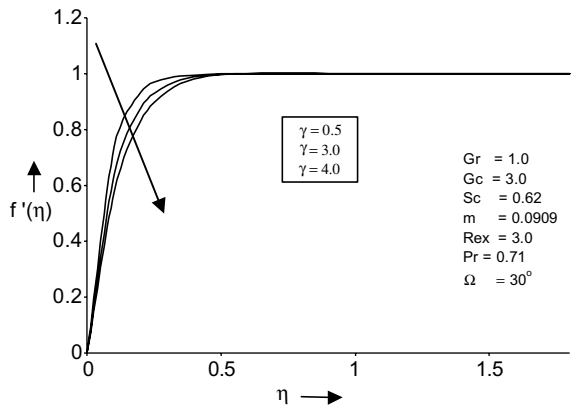


Fig. 5. Effects of chemical reaction over the velocity profiles.

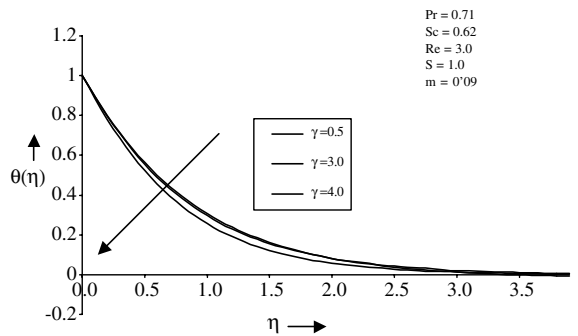


Fig. 6. Effects of chemical reaction over the temperature profiles.

Fig. 5 represents the dimensionless velocity profiles $f'(\eta)$ for different values of the chemical reaction parameter. For uniform suction and heat source, an increase in chemical reaction, leads to fall in the velocity, temperature distribution and concentration of the fluid along the wall of the wedge and these are shown in the Figs. 5–7, respectively. So, in the case of suction, the chemical reaction decelerates the fluid motion, temperature distri-

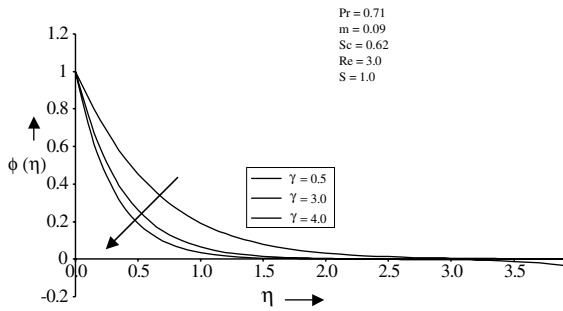


Fig. 7. Effects of chemical reaction over the concentration profiles.

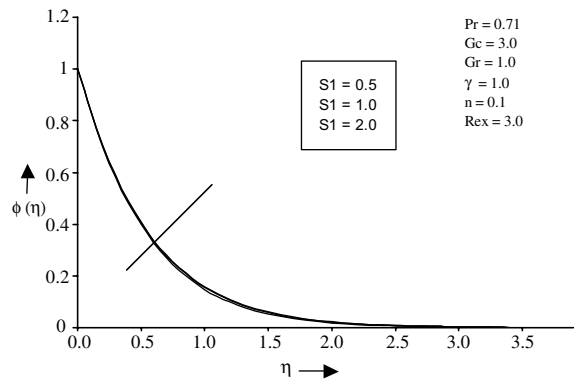


Fig. 10. Effects of heat source over the concentration profiles.

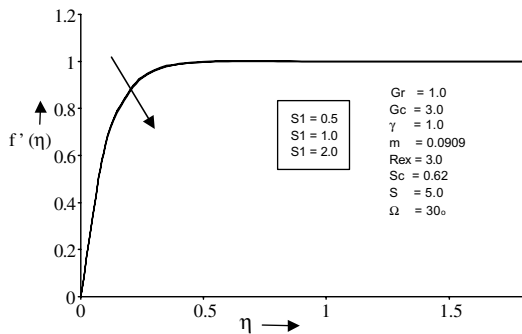


Fig. 8. Influence of heat source over the velocity profiles.

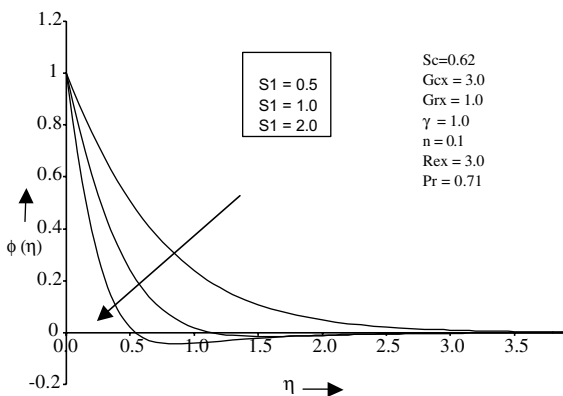


Fig. 9. Effects of heat source over the temperature profiles.

bution and concentration of the fluid along the wall of the wedge. It is observed that the effect of destructive reaction on the velocity, temperature and concentration profiles are much more pronounced than that of the generative reaction.

The effects of heat source over velocity, temperature and concentration of the fluid along the wall of the wedge are shown through Figs. 8–10, respectively.

Fig. 8 represents the dimensionless velocity profiles for different values of heat source parameter. Due to the uniform chemical reaction and in the case of suction,

it is clear that the velocity of the fluid is uniform but slightly decreases with increase of heat source parameter.

Fig. 9 demonstrates the dimensionless temperature profiles for different values of heat source. In the presence of constant chemical reaction parameter and in the case of suction, it is seen that the temperature of the fluid decreases with increase of heat source.

The concentration of the fluid is uniform with increase of heat source and this is shown through Fig. 10.

4. Conclusion

- In the presence of uniform chemical reaction and suction at the wall, it is interesting to note that the fluid flow along the wall of the wedge accelerates fluid motion and the temperature and concentration of the fluid reduce with increase of Schmidt number. All these facts clearly depict the combined effects of chemical reaction and suction at the wall of the wedge.
- Due to the uniform suction and heat source, the increase of chemical reaction decelerates the fluid motion, temperature distribution and concentration of the fluid along the wall of the wedge, which encounter the consumption reactions of the chemical reaction parameter.
- Due to the uniform suction with constant chemical reaction, the velocity and the temperature of the fluid decrease and the concentration of the fluid is uniform with increase of heat source.
- Comparison of velocity profiles shows that the velocity increases near the plate and thereafter remains uniform. It is interesting to note that due to increase in Sc , the concentration decreases at a faster rate in comparison to variation in the parameters in the case of cooling of plate ($Gr > 0$).
- Decrease of the concentration field due to increase in Sc shows that it increases gradually as we replace hydrogen ($Sc = 0.22$) by water vapour ($Sc = 0.62$)

and ammonia ($Sc = 0.78$) in the said sequence. It is also observed that the effect of chemical reaction and Schmidt number are very important in the concentration field.

The analyses of the present investigation of flow through a wedge medium is playing a predominant role in the applications of Science and Engineering. The flow of this kind has enormous importance in technical problems such as flow through packed beds, sedimentation, environmental problem, centrifugal separation of particles, blood rheology and in many transport processes in nature and Engineering devices, nuclear reaction, electronic equipments etc., in which the effect of buoyancy forces on the forced flow, or the effect of forced flow on the buoyant flow is significant. Particularly the findings may be useful for the study of movement of oil or gas and water through the reservoir of an oil or gas field, in the migration of underground water and in the filtration and water purification processes.

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