

Chapter 9: Column Analysis and Design

Introduction

Columns are usually considered as vertical structural elements, but they can be positioned in any orientation (e.g. diagonal and horizontal compression elements in a truss).

Columns are used as major elements in trusses, building frames, and sub-structure supports for bridges (e.g. piers).

- Columns support compressive loads from roofs, floors, or bridge decks.
- Columns transmit the vertical forces to the foundations and into the subsoil.

The work of a column is simpler than the work of a beam.

- The loads applied to a column are only axial loads.
- Loads on columns are typically applied at the ends of the member, producing axial compressive stresses.
- However, on occasion the loads acting on a column can include axial forces, transverse forces, and bending moments (e.g. beam-columns).

Columns are defined by the length between support ends.

- Short columns (e.g. footing piers).
- Long columns (e.g. bridge and freeway piers).

Virtually every common construction material is used for column construction.

- Steel, timber, concrete (reinforced and pre-stressed), and masonry (brick, block, and stone).

The selection of a particular material may be made based on the following.

- Strength (material) properties (e.g. steel vs. wood).
- Appearance (circular, square, or I-beam).
- Accommodate the connection of other members.
- Local production capabilities (i.e. the shape of the cross section).

Columns are major structural components that significantly affect the building's overall performance and stability.

- Columns are designed with larger safety factors than other structural components.

- Failure of a joist or beam may be localized and may not severely affect the building's integrity (e.g. there is redundancy with girders and beams, but not with columns).
- Failure of a strategic column may be catastrophic for a large area of the structure.
- Failure may be due to overstressed, loss of section (deterioration), accident/sabotage (terrorism).

Safety factors for columns are used to account for the following.

- Material irregularities (e.g. out of straightness).
- Support fixity at the column ends.
- Construction inaccuracies (e.g. out of plumbness).
- Workmanship.
- Unavoidable eccentric (off-axis) loading.

9.1 Short and Long Columns - Modes of Failure

Column slenderness and length greatly influence a column's ability to carry load.

- Very short, stout columns fail by crushing due to material failure.
 - Failure occurs once the stress exceeds the elastic (yield point) limit of the material.
- Long, slender columns fail by buckling - a function of the column's dimensions and its modulus of elasticity.
 - *Buckling* is the sudden uncontrolled lateral displacement of a column at which point no additional load can be supported.
 - Failure occurs at a lower stress level than the column's material strength due to buckling (i.e. lateral instability).

Short columns

Short columns fail by crushing at very high stress levels that are above the elastic limit of the column material.

Compressive stress for short columns is based on the basic stress equation developed at the beginning of Chapter 5.

- If the load and column size (i.e. cross-sectional area) are known, the compressive stress may be computed as

$$f_a = P_{\text{actual}}/A \leq F_a$$

where

f_a = actual compressive stress (psi or ksi)

A = cross-sectional area of the column (in²)

P_{actual} = actual load on the column (pounds or kips)

F_a = allowable compressive stress per code (psi or ksi)

- This stress equation can be rewritten into a design form to determine the required short column size when the load and allowable material strength are known.

$$A_{\text{required}} = P_{\text{actual}}/F_a$$

where

A_{required} = minimum cross-sectional area of the column

Long Columns - Euler Buckling

Long columns fail by buckling at stress levels that are below the elastic limit of the column material.

- Very short column lengths require extremely large loads to cause the member to buckle.
- Large loads result in high stresses that cause crushing rather than buckling.

Buckling in long, slender columns is due to the following.

- Eccentricities in loading.
- Irregularities in the column material.

Buckling can be avoided (theoretically) if the loads were applied absolutely axially, the column material was totally homogeneous with no imperfections, and construction was true and plumb.

A Swiss mathematician named Leonhard Euler (1707 - 1783) was the first to investigate the buckling behavior of slender columns within the elastic limit of the column's material.

- Euler's equation shows the relationship between the load that causes buckling of a (pinned end) column and the material and stiffness properties of the column.

The critical buckling load can be determined by the following equation.

$$P_{\text{critical}} = \pi^2 EI_{\text{min}}/L^2$$

where

P_{critical} = critical axial load that causes buckling in the column (pounds or kips)

E = modulus of elasticity of the column material (psi or ksi)

I_{min} = smallest moment of inertia of the column cross-section (in^2)
(Most sections have I_x and I_y ; angles have I_x , I_y and I_z .)

L = column length between pinned ends (inches)

- As the column length increases, the critical load rapidly decreases (since it is proportional to L^2), approaching zero as a limit.
- The critical load at buckling is referred to as *Euler's critical buckling load*.

Euler's equation is valid only for long, slender columns that fail due to buckling.

- Euler's equation contains no safety factors.
- Euler's equation results in compressive stresses developed in columns that are well below the elastic limit of the material.

Slenderness Ratios

The radius of gyration is a geometric property of a cross section that was first introduced in Chapter 6.

$$I = Ar^2 \quad \text{and} \quad r = (I/A)^{1/2}$$

where

r = radius of gyration of the column cross section (in)

I = least (minimum) moment of inertia (in^4)

A = cross-sectional area of the column (in^2)

The radius of gyration is geometric property that is used in the analysis and design of columns.

Using the radius of gyration, the critical stress developed in a long column at buckling can be expressed by the following equation.

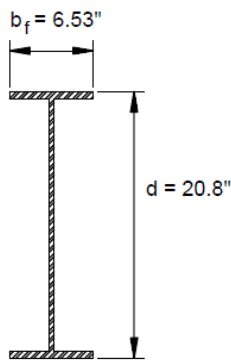
$$f_{\text{critical}} = P_{\text{critical}}/A = \pi^2 EI_{\text{min}}/AL^2 = \pi^2 E(Ar^2)/AL^2 = \pi^2 E/(L/r)^2$$

The term " L/r " is known as the *slenderness ratio*.

- A higher slenderness ratio means a lower critical stress that will cause buckling.
- Conversely, a lower slenderness ratio results in a higher critical stress (but still within the elastic range of the material).

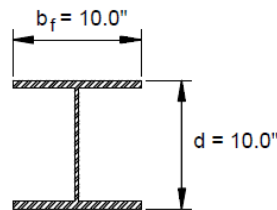
Column sections with large r -values are more resistant to buckling.

- Compare the difference in r_{\min} values and slenderness ratios for the three column cross sections shown below.
 - All three cross sections have relatively equal cross-sectional areas but very different radii of gyration about the critical buckling axis.
 - All three columns are assumed to be 15 feet in length and pin-connected at both ends.



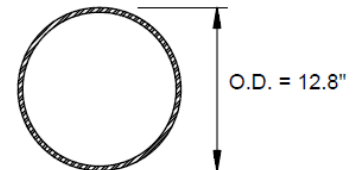
W 21 x 50
 $A = 14.7 \text{ in}^2$
 $r_x = 8.18''$
 $r_y = 1.30''$

$$\frac{L}{r} = \frac{15 (12'')}{1.30} = 138.5$$



W 10 x 49
 $A = 14.4 \text{ in}^2$
 $r_x = 4.35''$
 $r_y = 2.54''$

$$\frac{L}{r} = \frac{15 (12'')}{2.54} = 70.9$$



12" Dia. Std. Pipe
 $A = 13.6 \text{ in}^2$
 $r_x = 4.39''$
 $r_y = 4.39''$

$$\frac{L}{r} = \frac{15 (12'')}{4.39} = 41.1$$

Comparison of steel cross sections with equivalent areas

The most efficient column sections for axial loads are those with almost equal r_x and r_y values.

- Circular pipe sections and square tubes are the most effective shapes since the radii of gyration about both axes are the same ($r_x = r_y$).
- Circular pipe sections and square tubes are often used as columns for light to moderate loads.

Wide-flange shapes may be preferred despite the structural advantages of closed cross-sectional shapes (like tubes and pipes).

- The practical considerations of wide-flange shapes include the following.
 - Wide-flange sections support heavy loads.
 - Wide-flange sections accommodate beam connections.

Special wide-flange sections are specifically manufactured to provide relatively symmetrical columns (r_x/r_y ratios approaching 1.0) with large load-carrying capability.

- Most of these column sections (generally W8, W10, W12, and W14) have depth and flange widths approximately equal (i.e. a "boxy" configuration).

9.2 End Support Conditions and Lateral Bracing

Previously, each column was assumed to have pinned ends in which the member ends were free to rotate (but not translate) in any direction at their ends.

- When the column buckles, it will do so in one smooth curve.
- The length of this curve is referred to as the *effective length*.

In practice, a column may not be pinned at the ends.

- The column length free to buckle is greatly influenced by its end support conditions.
- The load-carrying capacity of a column is affected by the end support conditions.
 - Restraining the ends of a column with a fixed support increases the load-carrying capacity of a column.
 - Allowing translation as well as rotation (i.e. free end) at one end of a column generally reduces its load-carrying capacity.

Column design formulas generally assume a condition in which both ends are pinned.

- When other conditions exist, the load-carrying capacity is increased or decreased and the allowable compressive stress is increased or decreased.
- A factor K is used as a multiplier for converting the actual column length to an effective buckling length based on end conditions.

The American Institute of Steel Construction (AISC) provides recommended effective length factors when ideal conditions are approximated.

- The six cases are presented as follows.

Case A: Both ends are pinned.

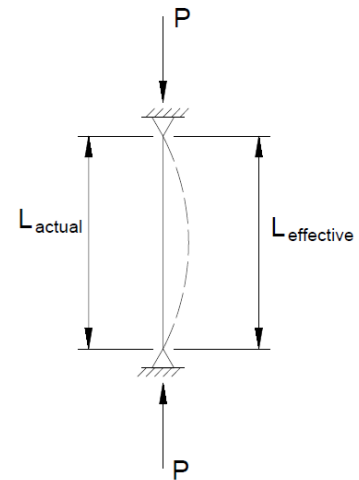
The structure is adequately braced against lateral forces (e.g. wind and earthquake forces).

Theoretical K-value: $K = 1.0$

Effective length: $L_e = L$

$$P_{\text{critical}} = \pi^2 EI_{\text{min}} / L^2$$

Examples: Timber column nailed top and bottom;
steel column with simple clip angle connection
top and bottom.



Case B: Both ends are fixed.

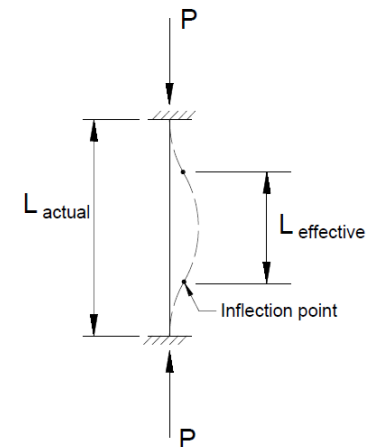
The structure is adequately braced against lateral forces (e.g. wind and earthquake forces).

Theoretical K-value: $K = 0.5$

Effective length: $L_e = 0.5 L$

$$P_{\text{critical}} = \pi^2 EI_{\text{min}} / (0.5L)^2 = 4\pi^2 EI_{\text{min}} / L^2$$

Examples: Concrete column rigidly
(monolithically cast) connected to large beams
top and bottom; steel column rigidly connected
(welded) to large steel beams top and bottom.



Case C: One end is pinned and one end is fixed.

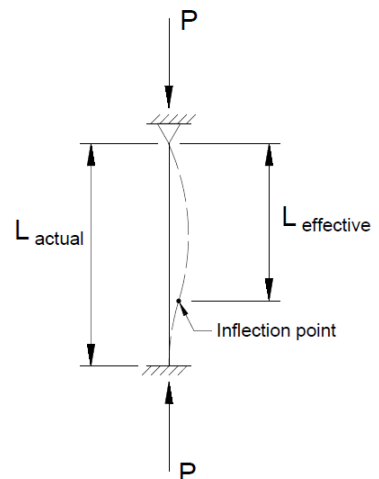
The structure is adequately braced against lateral forces (e.g. wind and earthquake forces).

Theoretical K-value: $K = 0.7$

Effective length: $L_e = 0.707 L$

$$P_{\text{critical}} = \pi^2 EI_{\text{min}} / (0.707L)^2 = 2\pi^2 EI_{\text{min}} / L^2$$

Examples: Concrete column rigidly connected to
concrete slab at the base and attached to light-
gauge roofing at the top.



Case D: One end is free and one end is fixed.

Lateral translation is possible.

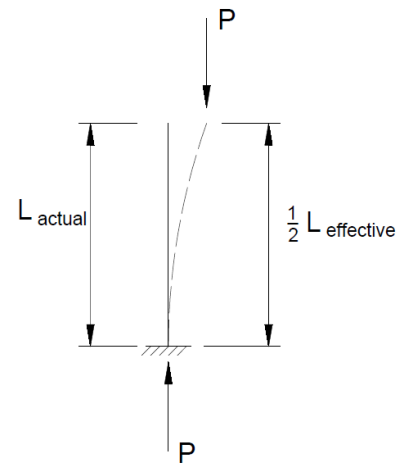
- An eccentric column load is developed.

Theoretical K-value: $K = 2.0$

Effective length: $L_e = 2.0 L$

$$P_{\text{critical}} = \pi^2 EI_{\text{min}} / (2L)^2 = \pi^2 EI_{\text{min}} / 4L^2$$

Examples: Water tank mounted on a simple pipe column; flagpole.



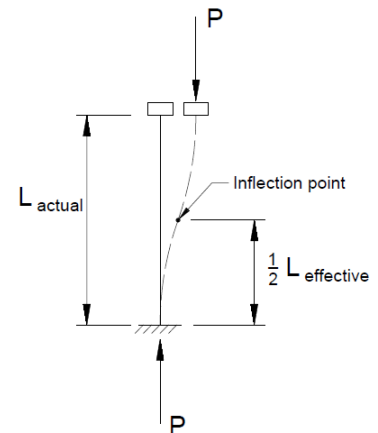
Case E: Both ends are fixed with some lateral translation.

Theoretical K-value: $K = 1.0$

Effective length: $L_e = 1.0 L$

$$P_{\text{critical}} = \pi^2 EI_{\text{min}} / L^2$$

Examples: Flexible column attached to a rigid beam and supported by a fixed base.



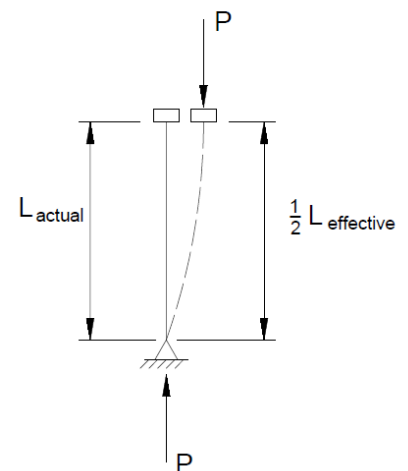
Case F: The base is pinned and the top is fixed with some lateral translation.

Theoretical K-value: $K = 2.0$

Effective length: $L_e = 2.0 L$

$$P_{\text{critical}} = \pi^2 EI_{\text{min}} / (2L)^2 = \pi^2 EI_{\text{min}} / 4L^2$$

Examples: Steel column with a rigid connection to a beam above and a simple pin connection at the base.



Intermediate Bracing

End connections affect the buckling capacity of a column.

- Fixed connections are an obvious solution to minimizing column sizes; however, the cost associated with achieving rigid connections is high and such connections are difficult to make.
- Timber columns are generally assumed as pinned-connected because the material strength generally precludes the construction of true rigid joints.

Lateral bracing about the weak axis can increase the strength and stability of a column by reducing the effective length of the column.

- Examples of such lateral bracing include the following.
 - Infill wall panels.
 - Window and door headers.
 - Supports for curtain walls.

Bracing provided in one plane does not provide resistance to buckling in the perpendicular plane.

- Columns must be checked in both directions to determine the *critical slenderness ratio* to be used in the analysis or design.

Example Problems - Short and Long Columns - Modes of Failure

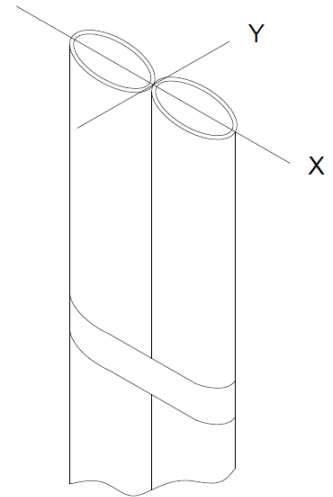
Problem 9.2 (p. 455)

Given: Two 3-1/2" standard steel pipe sections are strapped together to form a pin-connected column.

$$L = 24'$$

$$E = 29,000 \text{ ksi}$$

Find: The critical axial load when buckling occurs.



Solution

Moment of inertia for 3-1/2" standard pipe
(ref. Table A5 of the textbook)

$$I = 4.79 \text{ in}^4$$

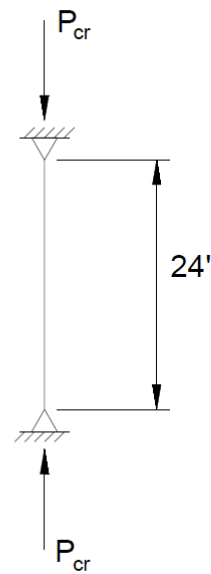
Applicable equation: $P_{\text{critical}} = \pi^2 EI_{\text{min}} / L^2$

$$I_x = I_{\text{min}} = 2 (4.79) = 9.58 \text{ in}^4$$

$$L = 24'$$

$$\begin{aligned} P_{\text{critical}} &= \pi^2 EI_{\text{min}} / L^2 \\ &= \pi^2 (29,000)(9.58) / (24 \times 12' /')^2 \end{aligned}$$

$$P_{\text{critical}} = 33.1 \text{ kips}$$



Problem 9.4 (p. 455)

Given: An 8" diameter timber pole fixed in a large concrete footing at grade and pinned at the top.

$$E = 1.0 \times 10^6 \text{ psi}$$

Find: Maximum height of the pole to support a 25 kip load.

Solution

Applicable equation: $P_{\text{critical}} = 2\pi^2 EI_{\text{min}}/L^2$

• Case C: One end pinned and one end fixed.

$$K = 0.7$$

$$L_e = 0.707 L$$

$$P_{\text{critical}} = \pi^2 EI_{\text{min}}/(0.707 L)^2 = 2\pi^2 EI_{\text{min}}/L^2$$

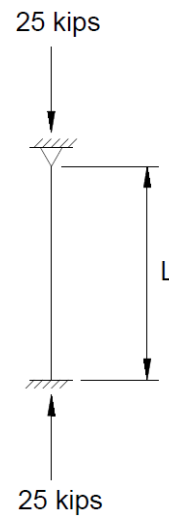
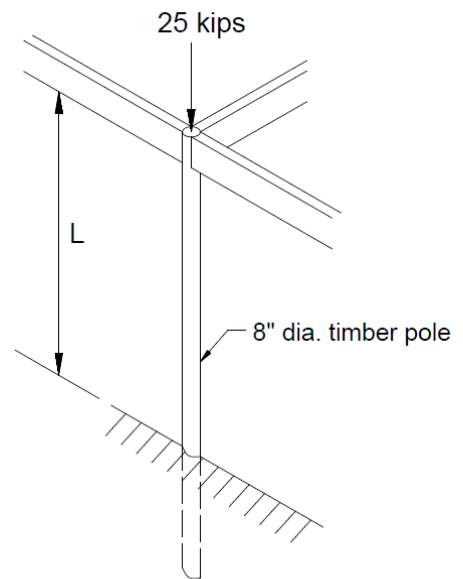
$$I = \pi d^4/64 = \pi (8)^4/64 = 201.1 \text{ in}^4$$

$$P_{\text{critical}} = 25 \text{ kips (25,000 lb)}$$

$$25,000 = 2\pi^2 (1.0 \times 10^6)(201.1 \text{ in}^4)/L^2$$

$$L^2 = 2\pi^2 (1,000,000)(201.1)/25,000 = 158,780 \text{ in}^2$$

$$L = 398.5" (33.2')$$



9.3 Axially Loaded Steel Columns

The discussion so far has been limited to short columns that crush and long slender columns that buckle.

- Somewhere in between these two extremes lies a zone where a "short" column transitions into a "long" column.

Short columns: Steel (A36) columns with slenderness ratios $\ell/r \leq 40$ are defined as "short columns".

- The mode of failure is crushing.

Long columns: Steel columns with a slenderness ratio of $120 \leq K\ell/r \leq 200$ are defined as "long columns."

- The mode of failure is buckling.
- For structural steel columns: $K\ell/r \leq 200$

Intermediate columns: Steel columns with a slenderness ratio of $40 \leq K\ell/r \leq 120$ are defined as "intermediate columns."

- The mode of failure is a combination of crushing (yielding) and buckling.

Column classification is based on slenderness.

- Figure 9.14 (*Column classification based on slenderness*, p. 456 of the textbook) compares Euler's equation with the American Institute of Steel Construction (AISC) equations.
 - The initially flat portion of the curve (in the short-column range, that is, $K\ell/r < 40$) indicates material yielding with no buckling taking place.
 - On the far right end of the curve ($K\ell/r > 120$), the compressive stresses are relatively low and buckling is the mode of failure.
 - In the intermediate-column range ($40 < K\ell/r < 120$), failure has aspects of both yielding and buckling.

The load-carrying ability of intermediate-length columns is influenced by both the strength and elastic properties of the column material.

- *Empirical design formulas*, based on extensive testing and research, have been developed to cover the design of columns within the limits of each column category.

Sine 1961, the American Institute of Steel Construction (AISC) has adopted a set of column design formulas that incorporate the use of a variable factor of safety, depending on slenderness, for determining allowable compressive stress.

- The AISC formulas recognize only two slenderness categories: short/intermediate and long (ref. Figure 9.15, p. 457 of the textbook).
 - Long steel (A36) columns are defined as those having a $K\ell/r$ exceeding a value called $C_c = 126.1$.
 - Long steel columns for high strength steel ($F_y = 50$ ksi) are defined as those having a $K\ell/r$ exceeding a value called $C_c = 107.0$.

The C_c value represents the theoretical demarcation line between inelastic (i.e. short/intermediate columns) and elastic (i.e. long columns) behavior.

- $C_c = (2\pi^2 E / F_y)^{1/2}$

Allowable compressive stress (AISC)

- Short/intermediate columns: The AISC allowable compressive stress (F_a) for short/intermediate columns ($K\ell/r < C_c$) is computed using AISC Eq. E2-1 (ref. p. 458 of the textbook).
- Long columns: When axially loaded compression members have a slenderness ratio $K\ell/r > C_c$, the allowable compressive stress is computed using AISC Eq. E2-2 (ref. p. 458 of the textbook).

where (for both equations)

$K\ell/r$ = the largest effective slenderness ratio of any unbraced length of column

F_a = allowable compressive stress (psi or ksi)

The two preceding equations represent actual design equations that can be used to analyze or design steel columns.

- These equations appear rather intimidating, especially equation E2-1.
- The AISC Manual of Steel Construction has developed a table for $K\ell/r$ from 1 to 200 with the representative allowable stress F_a .
- No computations using E2-1 and E2-2 are necessary since the equations have been used in generating these tables (Tables 9.1 and 9.2, pp. 461 - 462 of the textbook).

Pinned end supports are often assumed for design purposes and provide a conservative result.

- Pinned ends are assumed even if the ends of steel columns are typically restrained to some degree at the bottom by being welded to a base plate, which in turn is anchor-bolted to a concrete footing.

- Steel pipe columns generally have plates welded at each end, and then bolted to other parts of the structure.
 - Such restraints vary greatly and are difficult to evaluate.
- Designers rarely take advantage of the restraint to increase the allowable stress, which therefore adds to the factor of safety of the design.

On the other hand, tests have indicated that, in the case of fixed-end conditions, the "theoretical" $K = 0.5$ values are somewhat *non-conservative* when designing steel columns.

- Since true joint fixity is rarely possible, the AISC recommends the use of *recommended K-values* (listed in Figure 9.16, p. 459 of the textbook).

Following are examples of recommended design values for K when ideal conditions are approximated (ref. Figure 9.16, p. 459 of the textbook).

<u>Condition</u>	<u>Theoretical K value</u>	<u>Recommended K value</u>
Pinned-pinned	1.0	1.0
Fixed-fixed	0.5	0.65
Fixed-pinned	0.7	0.8

Analysis of Steel Columns

Column analysis means finding the allowable compressive stress F_a on a given column or its allowable load capacity P .

- A simple analysis procedure is outlined below.

Given: Column length, support conditions, grade of steel (F_y), applied load, and column size.

Required: Check the load capacity of the column.
(In other words, is $P_{\text{actual}} < P_{\text{allowable}}$?)

Procedure:

- Calculate the slenderness ratio $K\ell/r_{\min}$; the largest $K\ell/r$ governs.
- Enter the appropriate AISC Table
(Table 9.1 for $F_y = 36$ ksi or Table 9.2 for $F_y = 50$ ksi)
- Pick out the respective F_a
- Compute $P_{\text{allowable}}$: $P_{\text{allowable}} = F_a \times A$
where

A = cross sectional area of the column (in^2)

F_a = allowable compressive stress (ksi)

e. Check the column adequacy.

If $P_{\text{actual}} < P_{\text{allowable}}$ then the column is OK.

If $P_{\text{actual}} > P_{\text{allowable}}$ then the column is overstressed.

Example Problems - Axially Loaded Steel Columns

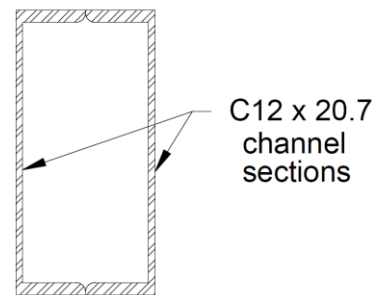
Problem 9.8 (p. 466)

Given: Two C12 x 20.7 channel sections welded together to form a closed box section.

$$F_y = 36 \text{ ksi}$$

$$L = 20'$$

Top and bottom are pinned.



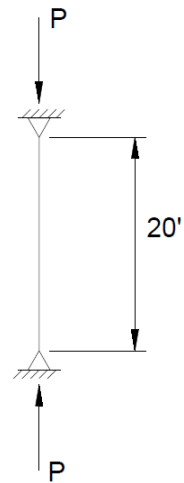
Find: Allowable axial load P_a .

Solution

Find the following geometric properties for the channel section (C12 x 20.7) from Table A4 (p. 573 of the textbook).

$$A = 6.09 \text{ in}^2, b_f = 2.942", I_x = 129 \text{ in}^4, r_x = 4.61"$$

$$I_y = 3.88 \text{ in}^4, r_y = 0.799", \bar{x} = 0.698"$$



Calculate the moment of inertia and radius of gyration for each axis.

$$A = 2 (6.09) = 12.18 \text{ in}^2$$

$$I_x = 2 (129) = 258 \text{ in}^4$$

$$r_x = (I_x/A)^{1/2} = (258/12.18)^{1/2} = 4.61" \text{ (same as } r_x \text{ for a single channel)}$$

$$I_y = 2 [3.88 + (6.09)(2.942 - 0.698)^2] = 2 (34.55) = 69.10 \text{ in}^4$$

$$r_y = (I_y/A)^{1/2} = (69.10/12.18)^{1/2} = 2.38"$$

Calculate the slenderness ratios ($K = 1.0$).

$$KL/r_x = (1.0) 20 (12"/')/4.61 = 52.1$$

$$KL/r_y = (1.0) 20 (12"/')/2.38 = 100.8 \quad \text{Governs}$$

Find F_a (i.e. the allowable compressive stress).

- Use a slenderness ratio $KL/r_y = 100.8$ (ref, Table 9.1, p. 461 of the textbook).

$$F_a = 12.88 \text{ ksi (by interpolation)}$$

Calculate the allowable axial load.

$$P_{\text{allow}} = F_a (A) = 12.88 (12.18) = 156.9 \text{ k}$$

$$P_{\text{allow}} = 156.9 \text{ k}$$

$\frac{KL}{r_y}$	F_a
100	12.98
100.8	
101	12.85

$$1.0 \left[0.8 \left[\begin{array}{cc} 100 & 12.98 \\ 100.8 & \\ 101 & 12.85 \end{array} \right] x \right] 0.13$$

$$\frac{x}{0.13} = \frac{0.8}{1.0}$$

$$x = \frac{0.8}{1.0} (0.13) = 0.104$$

$$F_a = 12.98 - 0.10 = 12.88$$

Compare P_{allow} with P_{critical} calculated using the Euler equation.

- Recall that P_{critical} has no factor of safety.

$$P_{\text{critical}} = \pi^2 EI_{\text{min}} / L^2$$

$$I_y = I_{\text{min}} = 69.10 \text{ in}^4$$

$$E = 29,000 \text{ ksi}$$

$$P_{\text{critical}} = \pi^2 (29,000)(69.10) / (20 \times 12'')^2 = 343.2 \text{ kips}$$

$$P_{\text{critical}} = 343.2 \text{ kips}$$

Problem 9.10 (p. 466)

Given: 5" dia standard steel pipe column (A36 steel) with an applied load of 60 kips. Top is pinned and the base is fixed.

Find: Maximum allowable height of column.

Solution

From Table A5 (p. 607 of the textbook) find the following geometric properties for the 5" dia. standard steel pipe section.

$$A = 4.30 \text{ in}^2, r = 1.88''$$

Calculate the compressive stress in the column.

$$F_a = P_a/A = 60/4.30 = 13.95 \text{ ksi}$$

Determine the slenderness ratio (ref. Table 9.1, p. 486 of the textbook).

$$KL/r = 92.15 \text{ for } F_a = 13.95 \text{ ksi} \\ \text{(by interpolating)}$$

Select the recommended design value K.

- K = 0.80 Case (b): Pinned-fixed (ref. Figure 9.16, p. 459 of the textbook).

$$KL/r = 0.8 (L)/1.88 = 92.15$$

$$L = 92.15 (1.88)/0.8 = 216.6'' (18.0')$$

Thus, maximum height is L = 18.0'



	$\frac{KL}{r_y}$	F_a	
	92	13.97] 0.02
	93	13.84	
1.0 [x			0.13

$$\frac{x}{1.0} = \frac{0.02}{0.13}$$

$$x = \frac{0.02}{0.13} (1.0) = 0.15$$

$$\frac{KL}{r} = 92.0 + 0.15 = 92.15$$

Design of Steel Columns

The design of axially loaded steel columns involves the selection of an appropriate column size.

- Accomplished by using specialized column tables such as those contained in the American Institute of Steel Construction's *Manual of Steel Construction - Allowable Stress Design*, Ninth Edition.
- Structural design varies from analysis in that there are several possible answers to a problem (e.g. different sizes, different shapes).

The selection of a column size is dependent on the following.

1. Strength and safety requirements.
2. Architectural issues (e.g. shape, appearance) and construction issues (e.g. beam connections) may influence the final selection.

Since the AISC Column Design Tables are not available (that would require the purchase of the AISC manual), steel column design will involve an iterative trial-and-error process.

A few guidelines for selecting a column cross section follow.

- Efficient column cross sections for axial loads often use circular or "boxier" wide-flange members.
- Along with spatial and construction concerns, relative maximum or minimum sizes may be specified by the architect, thus limiting the choices.
- Smaller scale steel structures may use 8" and 10" nominal size wide-flange columns, while larger buildings with heavier loads will often use 12" and 14" nominal sizes.
 - These sections are the "boxier" or square sizes, with the depth and flange width of the cross section being approximately equal.

The trial-and-error procedure may be outlined as follows.

Given: Column length, support conditions, grade of steel (F_y), applied load (P_{actual}).

Required: Column size to safely support the load.

Procedure:

- a. Guess at a size.
 - For a smaller scale building, maybe try a square W8 or W10 in the middle of the weight grouping.
 - A similar trial using larger sections is appropriate for heavier loads.
- b. Once the trial size has been selected, cross-sectional properties are known.
 - Compute the critical slenderness ratio, taking into account the end conditions and intermediate bracing.
- c. Using the larger $K\ell/r$ value, enter Table 9.1 (for $F_y = 36$ ksi, p. 461 of the textbook) or Table 9.2 (for $F_y = 50$ ksi, p. 462 of the textbook) and obtain the respective F_a (i.e. the allowable compressive stress).
- d. Calculate the $P_{allowable} = F_a \times A$ of the trial section.
- e. Compare P_{actual} to $P_{allowable}$.
 - If $P_{actual} > P_{allowable}$, then the column is overstressed and a larger section should be selected next.
 - If the trial section is too strong (i.e. P_{actual} much less than $P_{allowable}$), try again with a smaller column size.

One way to check the relative efficiency of the cross section is to examine its percent of stress level.

$$\text{Percent of stress} = P_{actual}/P_{allowable} \times 100\%$$

A percent of stress in the 90 - 100 percent level is very efficient.

- f. Repeat this process until an adequate but efficient section is obtained.

Note: Steps (b) through (e) are essentially the procedure used previously in the analysis of steel columns.

Alternative trial and error procedures:

1. Assume a design stress (F_a).
 - Divide the column load by the compression stress for an estimated column area (i.e. $A = P/F_a$).
 - Select a trial column section with approximately that area.
 - Then, continue the procedure following steps b through f, as outlined above.

2. Assume a slenderness ratio KL/r .

- Determine the corresponding allowable compressive stress F_a from Table 9.1 (for $F_y = 36$ ksi, p. 461 of the textbook) or Table 9.2 (for $F_y = 50$ ksi, p. 462 of the textbook).
- Divide the column load by the compressive stress for an estimated column area (i.e. $A = P/F_a$).
- Select a trial column section with approximately that area.
- Then, continue the procedure following steps b through f, as outlined above.

The following suggestions allow an inexperienced designer to make good initial assumptions for the slenderness ratio KL/r .

- The slenderness ratio KL/r for an average column of 10 to 15-foot length will generally fall between 40 and 60.
- If the column is longer than 15 feet, assume the slenderness ratio to be a little higher than 40 to 60.
- If the column carries a very heavy load (e.g. 750 - 1000 kips), a larger column will be required and the designer may estimate a smaller value for the slenderness ratio.
- For lightly loaded bracing members, the designer may estimate high slenderness ratios of 100 or more.

Example Problem - Design of Steel Columns

Problem 9.15 (p. 473)

Given: Six-story building.

$$\begin{aligned}\text{Tributary floor area} &= 20' \times 25' \\ &= 500 \text{ SF}\end{aligned}$$

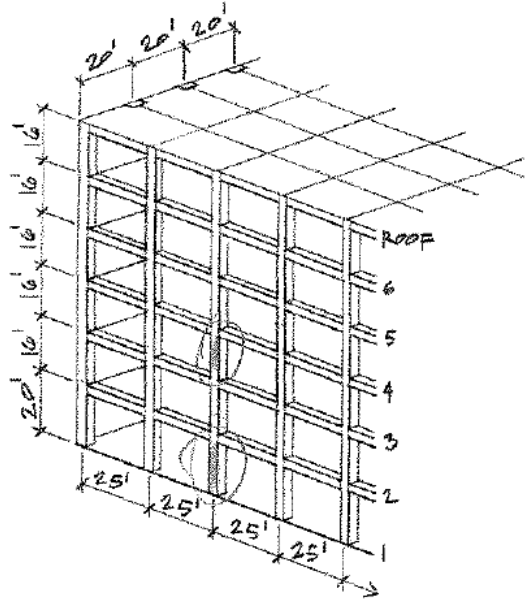
$$\begin{aligned}\text{Roof load:} & \quad 80 \text{ psf (dead load)} \\ & \quad 40 \text{ psf (snow load)}\end{aligned}$$

$$\begin{aligned}\text{Floor load:} & \quad 125 \text{ psf (live load)} \\ & \quad 100 \text{ psf (dead load)}\end{aligned}$$

$$L_{\text{first floor}} = 20'$$

$$L_{\text{third floor}} = 16'$$

$$F_y = 36 \text{ ksi}$$



Find: Design a typical interior third-floor column and first-floor column using the most economical W12* section at each level.

(*Note: There are several possible solutions using W10 and W14; specifying a W12 limits the number of solutions.)

Solution

Third floor column

Determine load on third-floor column.

$$P_{\text{actual}} = P_{\text{roof}} + P_{\text{floors}} \text{ (4}^{\text{th}}, \text{5}^{\text{th}}, \text{and 6}^{\text{th}} \text{ floor loadings)}$$

$$\begin{aligned}P_{\text{actual}} &= 500 \text{ SF (80 psf + 40 psf)} + 3 [500 \text{ SF (125 psf + 100 psf)}] \\ &= 60,000 + 3 (112,500) = 397,500 \text{ pounds}\end{aligned}$$

$$P_{\text{actual}} = 397.5 \text{ k (third floor column)}$$

First try

a. Select W12 section for third floor column.

$$\text{Try W12} \times 40 \text{ (} A = 11.8 \text{ in}^2, d = 11.94\text{'}, r_x = 5.13\text{'}, r_y = 1.93\text{'})$$

b. Compute the critical slenderness ratio ($K = 1.0$).

$$KL/r_{\text{min}} = 1.0 (16)(12\text{'})/1.93 = 99.5$$

- c. Using Table 9.1 (p. 461 of the textbook), find F_a (i.e. the allowable compressive stress) using $KL/r_{min} = 99.5$.

$$F_a = 13.04 \text{ ksi (by interpolating)}$$

- d. Calculate the allowable axial load.

$$P_{allowable} = F_a (A) = 13.04 (11.8) = 153.87 \text{ k}$$

- e. Compare P_{actual} to $P_{allowable}$.

$$P_{allowable} = 153.87 \text{ k} < 397.5 \text{ k} \quad \text{NG}$$

$$1.0 \left[0.5 \left[\begin{array}{cc} \frac{KL}{r} & F_a \\ 99 & 13.10 \\ 99.5 & \\ 100 & 12.98 \end{array} \right] x \right] 0.12$$

$$\frac{x}{0.12} = \frac{0.5}{1.0}$$

$$x = \frac{0.5}{1.0} (0.12) = 0.06$$

$$F_a = 13.10 - 0.06 = 13.04$$

Second try

- a. Select W12 section for third floor column.

$$\text{Try W12} \times 96 (A = 28.2 \text{ in}^2, d = 12.71", r_x = 5.44", r_y = 3.09")$$

- b. Compute the critical slenderness ratio ($K = 1.0$).

$$KL/r_{min} = 1.0 (16)(12"/)/3.09 = 62.1$$

- c. Using Table 9.1 (p. 461 of the textbook), find F_a (i.e. the allowable compressive stress) using $KL/r_{min} = 62.1$.

$$F_a = 17.23 \text{ ksi (by interpolating)}$$

- d. Calculate the allowable axial load.

$$P_{allowable} = F_a (A) = 17.23 (28.2) = 485.9 \text{ k}$$

- e. Compare P_{actual} to $P_{allowable}$.

$$P_{allowable} = 485.9 \text{ k} > 397.5 \text{ k} \quad \text{OK (but over-designed)}$$

$$1.0 \left[0.1 \left[\begin{array}{cc} \frac{KL}{r} & F_a \\ 62 & 17.24 \\ 62.1 & \\ 63 & 17.14 \end{array} \right] x \right] 0.10$$

$$\frac{x}{0.10} = \frac{0.1}{1.0}$$

$$x = \frac{0.1}{1.0} (0.10) = 0.01$$

$$F_a = 17.24 - 0.01 = 17.23$$

Third try

- a. Select W12 section for third floor column.

$$\text{Try W12} \times 72 (A = 21.1 \text{ in}^2, d = 12.25", r_x = 5.31", r_y = 3.04")$$

- b. Compute the critical slenderness ratio ($K = 1.0$).

$$KL/r_{min} = 1.0 (16)(12"/)/3.04 = 63.2$$

- c. Using Table 9.1 (p. 461 of the textbook), find F_a (i.e. the allowable compressive stress) using $KL/r_{min} = 63.2$.

$$F_a = 17.12 \text{ ksi (by interpolating)}$$

- d. Calculate the allowable axial load.

$$P_{allowable} = F_a (A) = 17.12 (21.1) = 361.2 \text{ k}$$

- e. Compare P_{actual} to $P_{allowable}$.

$$P_{allowable} = 361.2 \text{ k} < 397.5 \text{ k} \quad \text{NG}$$

$$1.0 \left[0.2 \left[\begin{array}{cc} \frac{KL}{r} & F_a \\ 63 & 17.14 \\ 63.2 & \\ 64 & 17.04 \end{array} \right] x \right] 0.10$$

$$\frac{x}{0.10} = \frac{0.2}{1.0}$$

$$x = \frac{0.2}{1.0} (0.10) = 0.02$$

$$F_a = 17.14 - 0.02 = 17.12$$

Fourth try

- a. Select W12 section for third floor column.

$$\text{Try W12} \times 79 (A = 23.2 \text{ in}^2, d = 12.38", r_x = 5.34", r_y = 3.05")$$

- b. Compute the critical slenderness ratio ($K = 1.0$).

$$KL/r_{min} = 1.0 (16)(12"/)/3.05 = 62.95$$

- c. Using Table 9.1 (p. 461 of the textbook), find F_a (i.e. the allowable compressive stress) using $KL/r_{min} = 62.95$.

$$F_a = 17.14 \text{ ksi (by interpolating)}$$

- d. Calculate the allowable axial load.

$$P_{allowable} = F_a (A) = 17.14 (23.2) = 397.6 \text{ k}$$

- e. Compare P_{actual} to $P_{allowable}$.

$$P_{allowable} = 397.6 \text{ k} > 397.5 \text{ k} \quad \text{OK}$$

$$1.0 \left[0.95 \left[\begin{array}{cc} \frac{KL}{r} & F_a \\ 62 & 17.24 \\ 62.95 & \\ 63 & 17.14 \end{array} \right] x \right] 0.10$$

$$\frac{x}{0.10} = \frac{0.95}{1.0}$$

$$x = \frac{0.95}{1.0} (0.10) = 0.095$$

$$F_a = 17.24 - 0.10 = 17.14$$

- f. Check efficiency.

$$\text{Efficiency} = P_{actual}/P_{allowable} \times 100\% = 397.5/397.6 \times 100\% = 99.97\%$$

Select: W12 x 79

First floor column

Determine load on first-floor column.

$$P_{\text{actual}} = P_{\text{roof}} + P_{\text{floors}} \text{ (2}^{\text{nd}}, 3^{\text{rd}}, 4^{\text{th}}, 5^{\text{th}}, \text{ and } 6^{\text{th}} \text{ floor loadings)}$$

$$P_{\text{actual}} = 500 \text{ SF (80 psf + 40 psf)} + 5 [500 \text{ SF (125 psf + 100 psf)}] \\ = 60,000 + 5(112,500) = 622,500 \text{ pounds}$$

$$P_{\text{actual}} = 622.5 \text{ k (first floor column)}$$

Recall from previous work: W12 x 96 $P_a = 485.9$ for 16' column. So a heavier column will be needed for a longer column - 20' - carrying a greater P_{actual} .

First try

a. Select W12 section for first floor column.

$$\text{Try W12 x 120 (} A = 35.3 \text{ in}^2, d = 13.12\text{"}, r_x = 5.51\text{"}, r_y = 3.13\text{"})$$

b. Compute the critical slenderness ratio ($K = 1.0$).

$$KL/r_{\text{min}} = 1.0 (20)(12\text{'})/3.13 = 76.7$$

c. Using Table 9.1 (p. 461 of the textbook), find F_a (i.e. the allowable compressive stress) using $KL/r_{\text{min}} = 76.7$.

$$F_a = 15.72 \text{ ksi (by interpolating)}$$

d. Calculate the allowable axial load.

$$P_{\text{allowable}} = F_a (A) = 15.72 (35.3) = 554.9 \text{ k}$$

e. Compare P_{actual} to $P_{\text{allowable}}$.

$$P_{\text{allowable}} = 554.9 \text{ k} < 622.5 \text{ k} \quad \text{NG}$$

$$\begin{array}{c} \frac{KL}{r} \qquad F_a \\ \hline \begin{array}{cc} 76 & 15.79 \\ 76.7 & \\ 77 & 15.69 \end{array} \end{array} \times \begin{array}{c} 0.7 \\ 0.10 \end{array} \times \begin{array}{c} 0.10 \\ 1.0 \end{array}$$
$$\frac{x}{0.10} = \frac{0.7}{1.0}$$
$$x = \frac{0.7}{1.0} (0.10) = 0.07$$
$$F_a = 15.79 - 0.07 = 15.72$$

Second try

a. Select W12 section for first floor column.

$$\text{Try W12 x 152 (} A = 44.7 \text{ in}^2, r_x = 5.66\text{"}, r_y = 3.19\text{"})$$

b. Compute the critical slenderness ratio ($K = 1.0$).

$$KL/r_{\text{min}} = 1.0 (20)(12\text{'})/3.19 = 75.2$$

- c. Using Table 9.1 (p. 461 of the textbook), find F_a (i.e. the allowable compressive stress) using $KL/r_{min} = 75.2$.

$$F_a = 15.88 \text{ ksi (by interpolating)}$$

- d. Calculate the allowable axial load.

$$P_{allowable} = F_a (A) = 15.88 (44.7) = 709.8 \text{ k}$$

- e. Compare P_{actual} to $P_{allowable}$.

$$P_{allowable} = 709.8 \text{ k} > 622.5 \text{ k} \quad \text{OK (but over-designed)}$$

$$1.0 \left[0.2 \left[\begin{array}{cc} \frac{KL}{r} & F_a \\ 75 & 15.90 \\ 75.2 & 15.79 \\ 76 & 15.79 \end{array} \right] x \right] 0.11$$

$$\frac{x}{0.11} = \frac{0.2}{1.0}$$

$$x = \frac{0.2}{1.0} (0.11) = 0.022$$

$$F_a = 15.90 - 0.02 = 15.88$$

Third try

- a. Select W12 section for first floor column.

Try W12 x 136 ($A = 39.9 \text{ in}^2$, $d = 13.41''$, $r_x = 5.58''$, $r_y = 3.16''$) - only section remaining between the W12 x 120 and W12 x 152.

- b. Compute the critical slenderness ratio ($K = 1.0$).

$$KL/r_{min} = 1.0 (20)(12''/)/3.16 = 75.9$$

- c. Using Table 9.1 (p. 461 of the textbook), find F_a (i.e. the allowable compressive stress) using $KL/r_{min} = 75.9$.

$$F_a = 15.80 \text{ ksi (by interpolating)}$$

- d. Calculate the allowable axial load.

$$P_{allowable} = F_a (A) = 15.80 (39.9) = 630.4 \text{ k}$$

- e. Compare P_{actual} to $P_{allowable}$.

$$P_{allowable} = 630.4 \text{ k} > 622.5 \text{ k} \quad \text{OK}$$

$$1.0 \left[0.9 \left[\begin{array}{cc} \frac{KL}{r} & F_a \\ 75 & 15.90 \\ 75.9 & 15.79 \\ 76 & 15.79 \end{array} \right] x \right] 0.11$$

$$\frac{x}{0.11} = \frac{0.9}{1.0}$$

$$x = \frac{0.9}{1.0} (0.11) = 0.099$$

$$F_a = 15.90 - 0.10 = 15.80$$

- f. Check efficiency.

$$\text{Efficiency} = P_{actual}/P_{allowable} \times 100\% = 622.5/630.4 \times 100\% = 98.75\%$$

Select: W12 x 136

9.4 Axially Loaded Wood Columns

Wood columns support beams and girders that, in turn, support tributary areas of roof and floor loads.

Other structural members that are in compression are designed using the same methods that are utilized for the design of building columns.

- Bridge piers
- Compression chords of a truss
- Studs in a load-bearing wall

Modes of failure

- Long columns tend to buckle under critical load.
- Short columns fail by the crushing of the wood fibers.

Slenderness ratio

- For wood columns, the ratio of the column length to its width is just as important as the slenderness ratio is for steel columns.
- In wood columns, the slenderness ratio is defined as the laterally unsupported length in inches divided by the least (minimum) dimension of the column.

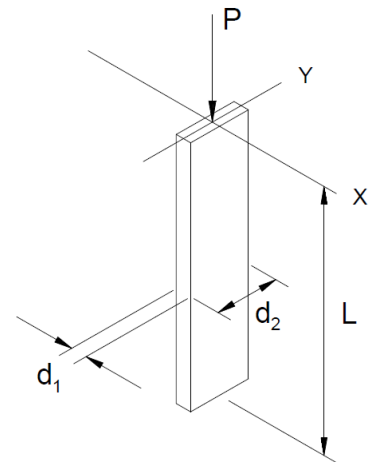
$$\text{Slenderness ratio} = L/d_{\min} = L/d_1$$

where $d_1 < d_2$

- Wood columns are restricted to a maximum slenderness ratio of $\ell_e/d \leq 50$.
 - This slenderness ratio is analogous to the limiting slenderness ratio of $KL/r_{\min} \leq 200$ used for steel columns.

Effective length

- The effective length of steel columns was determined by applying a K factor to the unsupported length of the column to adjust for the end support.
- Similar effective length factors, called K_e in wood columns, are used to adjust for the various end conditions.
- Recommended K_e values (ref. Figure 9.20, p. 474 of the textbook) are identical to those of steel columns.



Typical K_e values

Most wood construction is detailed such that translation (sidesway) is restrained but the ends of the column are free to rotate (i.e. pin connection).

- The K_e value is generally taken as 1.0, and the effective length is equal to the actual unsupported length.
- Even if some fixity exists in the top or bottom connection, it is difficult to evaluate the degree of fixity to assume in design.
- $K_e = 1.0$ is an acceptable assumption that is usually a bit conservative in some connection conditions.

Wood columns

- Wood columns can be solid members or rectangular, round, or other shapes, or spaced columns built up from two or more individual solid members separated by blocking.
- The majority of all wood columns in buildings are solid rectangular sections. The analysis and design methods examined in this section will be limited to these types.

The National Design Specification for Wood Construction (NDS-91) approved a new standard in 1992.

- The new standard incorporated a new method and new equations for the design of wood elements and connections.
- Previous categorizing of wood columns into the short-, intermediate-, or long-column range resulted in three different equations for each respective slenderness range.
- The NDS-91 now utilizes a single equation, providing a continuous curve over the entire range of slenderness ratios.

The compressive stress for an axially loaded wood column of known size is expressed as follows.

$$f_c = P/A \leq F_c'$$

where

f_c = actual compressive stress parallel to grain

P = axial compressive force in the member

A = cross-sectional area of the column

F_c' = allowable compressive stress parallel to grain

The allowable load for an axially loaded wood column is expressed as follows.

$$P_{\text{allow}} = F_c' A \text{ and } P_{\text{actual}} \leq P_{\text{allow}}$$

To obtain the allowable compressive stress F_c' , many adjustments to the tabulated base (reference) compressive stress (F_c) are necessary.

The NDS-91 defines the F_c' as follows.

$$F_c' = F_c (C_D) (C_M) (C_t) (C_F) (C_p)$$

where

F_c' = allowable compressive stress parallel to grain

F_c = reference compressive stress parallel to grain (found in building code tables, NDS tables, and wood design handbooks, such as Table 5.2, p. 278 of the textbook)

C_D = load duration factor (ref. Figure 9.23, p. 477 of the textbook)

C_M = wet service factor (accounts for moisture content in the wood)
= 1.0 for dry service conditions as in most covered structures, dry service condition defined as follows:

Moisture content \leq 19% for sawn lumber

Moisture content \leq 16% for glu-lams

C_t = temperature factor (usually taken as 1.0 for normal temperature conditions)

C_F = size factor (an adjustment based on member sizes used)

C_p = column stability factor (accounts for buckling and is directly affected by the slenderness ratio)

The objective of this course is to analyze and design structural elements in a preliminary way (rather than the full complement of equations and checks performed by a structural engineer).

- Accordingly, the preceding allowable compressive stress equation is simplified as follows.

$$F_c' = F_c^* C_p$$

where

$F_c^* = F_c (C_D) (C_M) (C_t) (C_F) \approx F_c C_D$ (for preliminary column design)

C_p = column stability factor (ref. Table 9.3)

F_c = tabulated compressive stress parallel to grain (found in building code tables, NDS tables, and wood design handbooks, such as Table 5.2, p. 278 of the textbook)

C_D = load duration factor

C_M , C_t , and C_F are all assumed to be equal to 1.0, which is generally the case for a majority of wood columns.

Load duration factor C_D (ref. Figure 9.23, p. 477 of the textbook)

- Wood has a unique structural property in which it can support higher stresses if the applied loads are for a short period of time.
- All tabulated stress values contained in building codes, NDS, or wood design manuals apply to "normal" load duration and dry service conditions.
- The C_D value adjusts tabulated stresses to allowable values based on the duration (time) of loading.
 - a) "Normal" duration is taken as 10 years and $C_D = 1.0$.
 - b) For permanent dead load, $C_D = 0.9$.
 - c) Short-duration loading from wind, earthquake, snow, or impact allows C_D values higher than 1.0 but less than 2.0.

The column stability factor C_p multiplied by F_c essentially defines the column curve (equation).

- This equation, originally developed by Ylinen, explains the behavior of wood columns as the interaction of the two modes of failure: buckling and crushing.
- An equation for computing C_p is provided on page 477 of the textbook.
 - Terms in the equation for C_p include the following.

F_{cE} = Euler critical buckling stress for columns

$$F_{cE} = \frac{K_{cE} E'}{(\ell_e/d)^2} \approx \frac{K_{cE} E}{(\ell_e/d)^2}$$

$$F_c^* \approx F_c C_D$$

c = buckling and crushing interaction factor for columns

= 0.8 for sawn lumber columns

= 0.9 for glu-lam columns

- Terms used in calculating F_{cE} include the following.

$K_{cE} = 0.30$ for sawn lumber

= 0.418 for glu-lams

E' = adjusted modulus of elasticity associated with the axis of buckling

$$E' = E (C_M) (C_t) (C_i) \quad \text{for sawn lumber}$$

$$E' = E (C_M) (C_t) \quad \text{for glu-lam columns}$$

E = reference modulus of elasticity associated with the axis of buckling

Table 9.3 (p. 478 of the textbook) is provided to simplify the computations for preliminary column analysis/design.

- This table (similar to Tables 9.1 and 9.2 for steel) was developed by inputting slenderness ratios between 1 and 50 to determine F_{cE} values for sawn and glu-lam members.
- By dividing F_{cE} values by F_c^* and generating ratios of a (F_{cE}/F_c^*), corresponding C_p values can be taken from the table.
- Table 9.3 eliminates the necessity of laborious computations for C_p using the equation developed by Ylinen.

Analysis of Wood Columns

A simple procedure can be adopted for checking the adequacy or capacity of wooden columns.

- This methodology is for approximate analysis and assumes the simplifications discussed previously.

Given: Column size, column length, grade and species of lumber, and end conditions.

Required: The allowable capacity of a column or the adequacy of a given column.

Procedure:

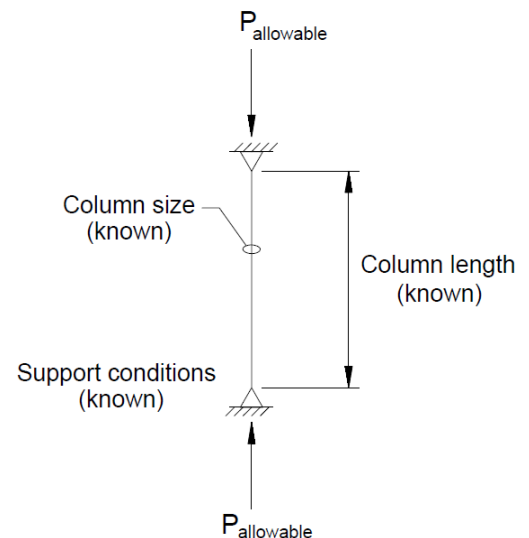
- Calculate the (ℓ_e/d_{min}) .
- Compute F_{cE} (i.e. Euler's critical buckling stress).

$$F_{cE} = \frac{K_{cE} E}{(\ell_e/d)^2}$$

$$K_{cE} = 0.3 \text{ for sawn lumber}$$

$$K_{cE} = 0.418 \text{ for glu-lams}$$

E = reference modulus of elasticity (ref. Table 5.2, p. 278 of the textbook)



c. Compute $F_c^* \approx F_c C_D$

F_c = reference compressive stress parallel to grain (found in building code tables, NDS tables, and wood design handbooks, such as Table 5.2, p. 278 of the textbook)

C_D = load duration factor (ref. p. 477 of the textbook)

d. Calculate the ratio: F_{cE}/F_c^*

e. Enter Table 9.3 (p. 478); obtain respective C_p .

f. Calculate the allowable compressive stress F_c' .

$$F_c' = F_c^* C_p$$

g. Calculate the allowable load and compare the allowable load with the actual load.

$$P_{\text{allowable}} = F_c' \times A \leq P_{\text{actual}}$$

where

A = cross-sectional area of column

Example Problem - Analysis of Wood Columns

Problem 9.16 (p. 485)

Given: 6x6 S4S Southern Pine Dense No. 1 column.

Assume 7-day duration roof live load.

Pin connections top and bottom.

$$L = 14' - 0''$$

$$F_c = 975 \text{ psi (Table 5.2, p. 278 of the textbook)}$$

$$E = 1.6 \times 10^6 \text{ psi (Table 5.2, p. 278 of the textbook)}$$

Find: Capacity of column.

Solution

Section properties for the 6 x 6 S4S column.

Actual dimensions: 5.5" x 5.5"

$$A = 30.25 \text{ in}^2 \text{ (Table A1-b, p. 567 of the textbook)}$$

a. Calculate the slenderness ratio ℓ_e/d_{\min} .

$$\ell_e/d_{\min} = 14(12''/')/5.5 = 30.55$$

b. Compute F_{cE} (i.e. Euler's critical buckling stress).

$$F_{cE} = K_{cE} E / (\ell_e/d)^2 = 0.30(1.6 \times 10^6) / (30.55)^2 = 514.3$$

where $K_{cE} = 0.30$ for visually graded lumber (p. 502)

c. Compute $F_c^* \approx F_c C_D$ ($C_M, C_t, C_F = 1.0$)

$$F_c^* = 975 (1.25) = 1,218.75$$

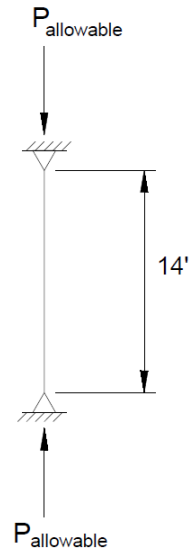
where $C_D = 1.25$ for 7-day roof live load (Figure 9.23, p. 477)

d. Calculate the ratio F_{cE}/F_c^*

$$F_{cE}/F_c^* = 514.3/1,218.75 = 0.422$$

e. Enter Table 9.3 (p. 478) and obtain C_p .

$C_p = 0.377$ (for sawn lumber, by interpolating)



$$0.01 \left[\begin{array}{c} 0.002 \left[\begin{array}{cc} \frac{F_{cE}}{F_c^*} & C_p \\ 0.42 & 0.375 \\ 0.422 & \\ 0.43 & 0.383 \end{array} \right] \times \end{array} \right] 0.008$$

$$\frac{x}{0.008} = \frac{0.002}{0.01}$$

$$x = \frac{0.002}{0.01} (0.008) = 0.0016$$

$$C_p = 0.375 + 0.002 = 0.377$$

f. Calculate the allowable compressive stress F_c' .

$$F_c' = F_c * C_p = 1,218.75 (0.377)$$

$$F_c' = 459.5 \text{ psi}$$

g. Calculate the allowable load (i.e. the capacity of the column).

$$P_{\text{allowable}} = F_c' \times A = 459.5 (30.25) = 13,900 \text{ lb}$$

$$P_{\text{allowable}} = 13.9 \text{ k}$$

Problem 9.18 (p. 485)

Given: 6-3/4" x 10-1/2" glu-lam column

Lateral bracing about the weak axis at mid height.

Pin connections top and bottom.

$$A = 70.88 \text{ in}^2$$

$$F_c = 1,650 \text{ psi (Table 5.2, p. 278 of the textbook)}$$

$$E = 1.8 \times 10^6 \text{ psi (Table 5.2, p. 278 of the textbook)}$$

Find: Load capacity of the column.

Solution

- a. Calculate the slenderness ratios ℓ_e/d for each axis of bending and determine the critical value.

$$(\ell_e/d)_x = 22(12''/)/10.5 = 25.14 \quad \text{Governs}$$

$$(\ell_e/d)_y = 11(12''/)/6.75 = 19.56$$

- b. Compute F_{cE} (i.e. Euler's critical buckling stress).

$$F_{cE} = K_{cE} E / (\ell_e/d)^2 = 0.418(1.8 \times 10^6) / (25.14)^2 = 1,190.5$$

where $K_{cE} = 0.418$ for glu-lams (p. 477)

- c. Compute $F_c^* \approx F_c C_D$ ($C_M, C_t, C_F = 1.0$)

$$F_c^* = 1,650 (1.00) = 1,650$$

where $C_D = 1.00$ for 10-year design (Figure 9.23, p. 477)

- d. Calculate the ratio F_{cE}/F_c^*

$$F_{cE}/F_c^* = 1,190.5/1,650 = 0.722$$

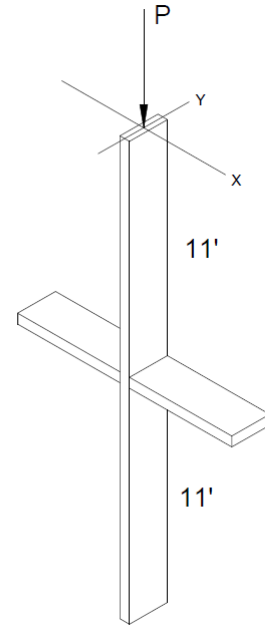
- e. Enter Table 9.3 (p. 478) and obtain C_p .

$C_p = 0.620$ (for glu-lam, by interpolating)

- f. Calculate the allowable compressive stress F_c' .

$$F_c' = F_c^* C_p = 1,650 (0.620)$$

$$F_c' = 1,023.0 \text{ psi}$$



$$0.01 \left[\begin{array}{c} 0.002 \left[\begin{array}{cc} \frac{F_{cE}}{F_c^*} & C_p \\ 0.72 & 0.619 \\ 0.722 & \\ 0.73 & 0.626 \end{array} \right] x \end{array} \right] 0.007$$

$$\frac{x}{0.007} = \frac{0.002}{0.01}$$

$$x = \frac{0.002}{0.01} (0.007) = 0.0014$$

$$C_p = 0.619 + 0.001 = 0.620$$

g. Calculate the allowable load (i.e. the capacity of the column).

$$P_{\text{allowable}} = F_c' \times A = 1,023.0 (70.88) = 72,510 \text{ lb}$$

$$P_{\text{allowable}} = 72.5 \text{ k}$$

Design of Wood Columns

Column design in wood is a trial-and-error process.

- Start by making an estimate on size (try out your intuition) and check out the adequacy or inadequacy by following the analysis procedure given in the previous section.
- Axially loaded wood columns without mid height bracing are generally square in cross section, or in some cases just slightly rectangular.
- Fortunately, there are fewer possible wood sections to choose from compared with the wide array of sizes available in steel.

One design procedure using the trial-and-error method could be:

Given: Column length, column load, grade and species of lumber to be used, and end conditions.

Required: An economical column size.

Procedure:

- a. Guess at a trial size.
 - Try and select a square or almost square cross section unless the column's weak axis is braced.
- b. Follow the same steps used in the analysis procedure in the previous section.
- c. If $P_{\text{allowable}} \geq P_{\text{actual}}$, then OK.
- d. If $P_{\text{allowable}} \leq P_{\text{actual}}$, pick a larger size and cycle through the analysis procedure again.

9.5 Columns Subjected to Combined Loading or Eccentricity

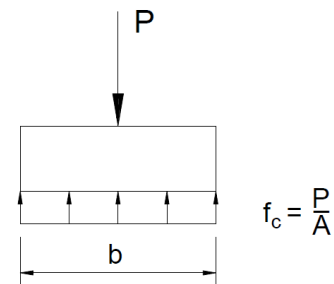
Previous sections have assumed that the compression members are subjected to concentric loading (loads acting through the centroid of the column section).

- In practice, however, concentric loading is rarely the case.

Many columns are subjected to bending in combination with axial compression loads.

- A load may not act through the centroid of a column cross section due to
 - Non-uniform bearing.
 - Misalignment of the framing.
 - The crookedness of a member.
- Compression members carrying bending moment due to eccentricity or side loading in addition to compression are referred to as *beam columns*.

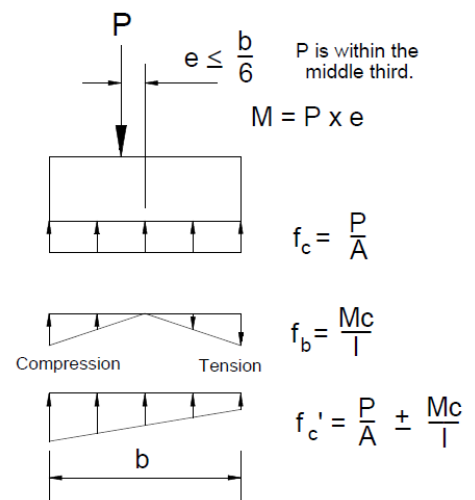
Axially loaded columns have a relatively uniform distribution of stress over the cross-sectional area.



Axially loaded - uniform compressive stress

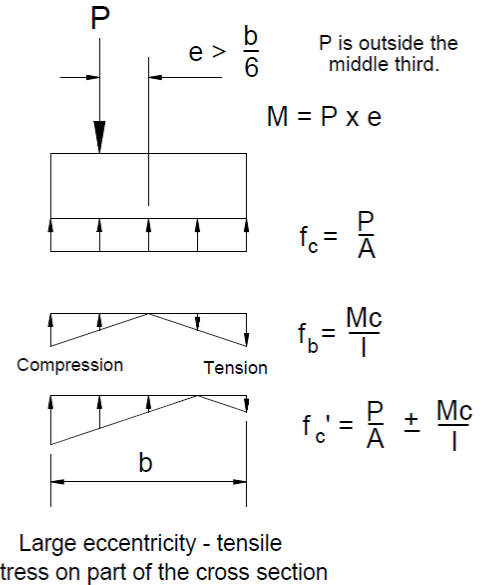
Bending stress, which involves tension and compression stresses, must be added algebraically to the compressive stress due to the axial load.

- Small eccentricities (e.g. flexible beam connected to a rigid column) alter the stress distribution.
 - The cross section remains in compression, although non-uniform as shown at the right.



Small eccentricity - linearly varying stress (all compression)

- If large eccentricities (e.g. rigid beam connected to a less rigid column) exist, tensile stresses may develop over part of the cross section, as shown in the figure at the right.



Tensile stress

- The tension stresses that developed in masonry construction of the past were formerly of great concern.
- Tension stresses are of little significance for the building systems and materials used today in contemporary buildings.
- Timber, steel, pre-stressed concrete, and reinforced concrete all possess good tension capability.

Beam columns are evaluated using an *interaction equation* that incorporates the bending stress with the compressive stress.

- The general interaction equation is expressed mathematically as follows.

$$f_a/F_a + f_b/F_b \leq 1.0$$

where

$f_a = P/A$ (the actual compressive - axial - stress)

$F_a =$ allowable compressive stress (based on Kl/r for steel or l_e/d for timber)

$f_b = Mc/I = M/S$ (actual bending stress)

$M = P \times e$ for eccentrically loaded members

$M =$ bending moment due to side load or rigid frame action

$e =$ eccentricity

$F_b =$ allowable bending stress

If a member is subjected to axial compression and bending about both the x- and y-axes, the interaction formula is adapted to incorporate the biaxial bending.

- Therefore, the most generalized form of the equation is expressed mathematically as follows.

$$f_a/F_a + f_{bx}/F_{bx} + f_{by}/F_{by} \leq 1.0 \text{ (for biaxial bending)}$$

where

$f_{bx} = M/S_x =$ actual bending stress about the x-axis

$f_{by} = M/S_y =$ actual bending stress about the y-axis

An interaction curve, shown as Figure 9.33 (p. 490 of the textbook), illustrates the theoretical combining of the axial compressive and bending stresses.

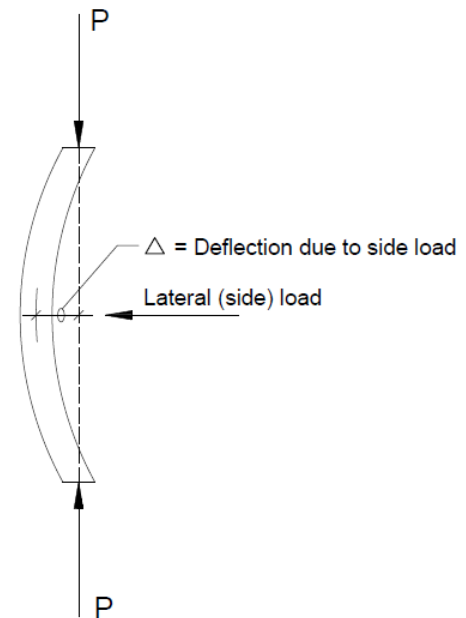
P- Δ effect

Bending moments in columns may result from the following.

- Lateral forces
- Applied moments
- Eccentricity of the end loads

These bending moments can cause a member to deflect laterally, resulting in additional bending moment due to the P - Δ effect.

- The lateral displacement generates an eccentricity for the load P.
- The eccentricity results in the creation of additional moment at the mid-height of the column equal to $P \times \Delta$ (known as a *second-order bending moment*).
- Slender columns are particularly sensitive to this P- Δ effect and must be accounted for in the interaction equation.



P - Δ effect on a slender column

The AISC (steel) and NDS (timber) manuals have introduced a magnification factor to incorporate the P- Δ effect.

- A generalized interaction equation for both steel and wood is expressed mathematically as follows.

$$f_a/F_a + [f_b \times (\text{Magnification Factor})]/F_b \leq 1.0$$

- The actual analysis/design equations for steel and wood are noted in the textbook (pp. 490 - 491).

Analyzing and designing beam columns using the AISC and NDS equations are more appropriately done in follow-up courses dealing specifically with steel and wood design.