7-a

$$\Rightarrow \quad \lambda' - \lambda = 4\pi \lambda_e \sin^2 \theta/2.$$

where $\lambda_e = \frac{\hbar}{mc} \approx 3.86 \times 10^{-11}$ cm

is the Compton wavelength of the electron.

One can observe that the change in wavelength $\Delta\lambda = \lambda' - \lambda$ depends on on the angle of scattering angle and not on the frequency. This is what happens in particle collisions establishing the fact that radiation behaves like a beam of particles.

WAVE ASPECT OF PARTICLES:

a). de-Broglie's Hypothesis: Matter Waves

A Photon of frequency ω has the momentum

$$p = |\underline{p}| = \frac{E}{c} = \frac{\hbar\omega}{c}.$$

or interms of wavelength, it is

$$p = \hbar k = \frac{2\pi\hbar}{\lambda}, \quad k = \frac{2\pi}{\lambda}$$

or $$\lambda = \frac{2\pi\hbar}{p} = \frac{h}{p}.$$

which is photon wavelength. It was suggested by de-Broglie that the formula

$$\boxed{\lambda = \frac{H}{p} = \frac{H}{mv}} \quad \text{——— (1)}$$

applies to material particles as well as photons. For a particle of mass "m", moving with velocity "v", the associated wavelength is given by equ (1). The wavelength $\lambda = \frac{H}{mv}$ is known as de Broglie wavelength of matter waves. The greater the momentum of the particles, the shorter is its wavelength. The phase velocity of matter waves is

8-a

$$V_p = \frac{\omega}{K} = \frac{\hbar\omega}{\hbar K} = \frac{E}{p} = \frac{mc^2}{mV} = \frac{c^2}{V}$$

Since $c > V$, the phase velocity of matter waves is alway greater than that of light which does not make sense. The experimentally measurable velocity is the group velocity

$$V_g = \frac{d\omega}{dk} = \frac{\hbar\, d\omega}{\hbar\, dk} = \frac{dE}{dp}$$

Now

$$E^2 = p^2c^2 + m^2c^4$$

$$2E\frac{dE}{dp} = 2c^2p$$

then

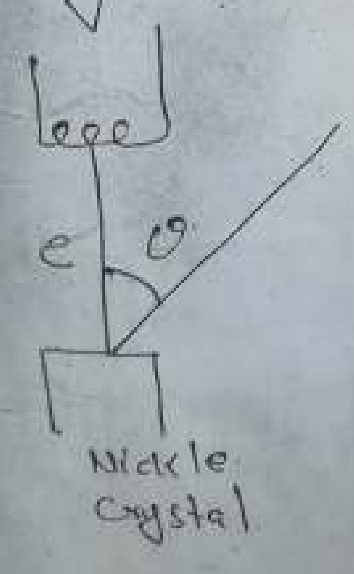
$$V_g = \frac{dE}{dp} = \frac{pc^2}{E} = \frac{mVc^2}{mc^2} = V$$

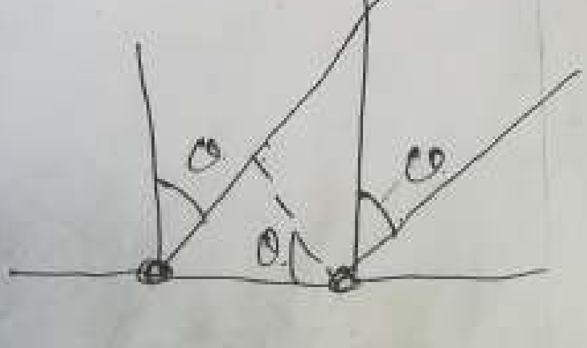
The matter wave group associated with a moving particle travel with the same velocity as that of the particle.

Experimental Confirmation of de-Broglie's Hypothesis:

Davisson-Germer Experiment:

Davisson and Germer used X-ray diffraction method with electrons to study their ~~wave~~ wave behaviour. An electron beam is scattered at the surface of nickle crystal and as a result diffraction patterns are obtained. The diffraction patterns are exactly similar to those obtained in X-ray diffraction.





9-a

The diffraction maxima appears if

condition (Bragg relation).

$$n\lambda = d\sin\theta$$

is satisfied. If the accelerating vo

for an electron is V, then its e

is

$$E = eV$$ (e being cha of an elect

The de Broglie wavelength of an

is $$\lambda = \frac{h}{p}.$$

for $$E = p^2/2m \Rightarrow p = \sqrt{2mE}$$

we get the wavelength.

$$\lambda = \frac{h}{\sqrt{2mE}}$$

So the condition for electron diff

maxima is

$$\frac{nh}{\sqrt{2mE}} = d\sin\theta$$

by putting the value of

$$E = eV$$

one gets.

$$\boxed{\frac{nh}{d\sqrt{2me}} = \sqrt{V}\,\sin\theta}$$

This condition is confirmed by the experiment.

* Classical and Quantum View of Particles and Waves:

In Classical Physics particles and waves are treated separately and show completely different behaviour than that of in Quantum physics.

A typical wave is described by a complex wavefunction $\psi(x,t)$, which is

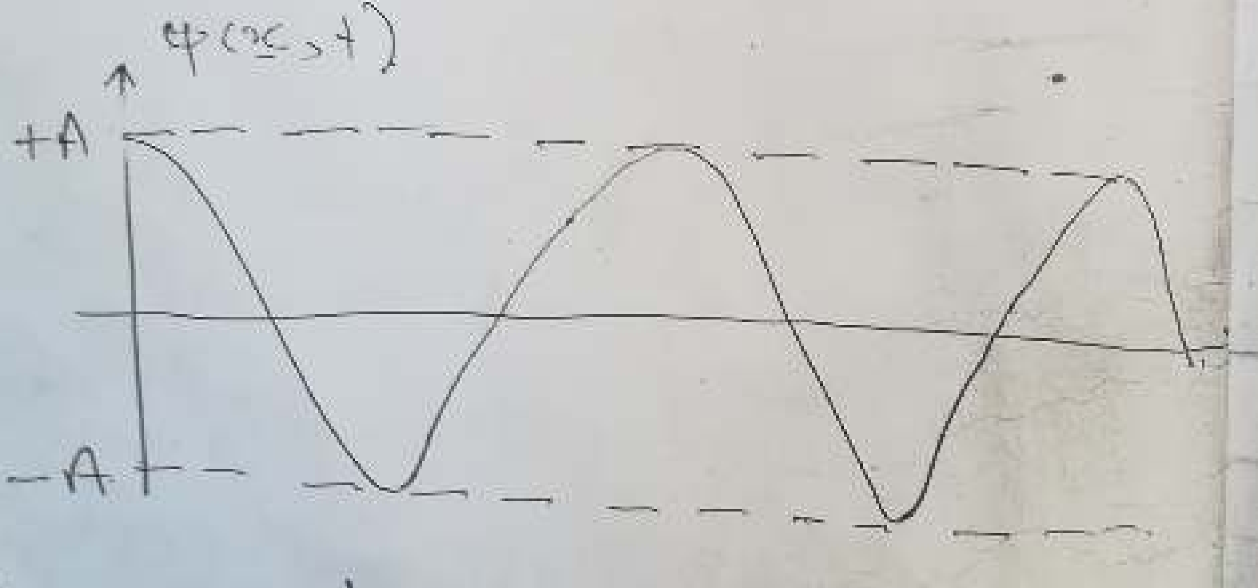
10-A

$$\psi(\underline{x},t) = A \exp[-i(\omega t - \underline{K}\cdot\underline{x})]$$

where A is a +ve constant call[ed] amplitude of vibration and $\omega = 2\pi\nu$ is the angular frequency. $\underline{K} = \frac{2\pi}{\lambda}$ is the wave number. The wavelen[gth] λ is related to ν by

$$\lambda \nu = c \quad \text{(velocity of Light)}$$

The graphical representation of wave is.



The argument of exponential

$$\phi(\underline{x},t) = \omega t - \underline{K}\cdot\underline{x}$$

is called phase. At a fixed time the points with same phase ϕ define a plane called the wavefront. The velocity with which this plane moves is called the phase velocity. A wave whose wavefront is a plane is called a plane wave. For a constant phase the velocity is obtained as

$$\frac{d\phi}{dt} = 0$$

or

$$\omega - \underline{k} \cdot \frac{dx}{dt} = 0$$

$$\omega - \underline{k} \cdot \underline{v}_\phi = 0$$

Hence, we get the magnitude of the phase velocity

$$|v_\phi| = v_\phi = \frac{\omega}{|\underline{k}|} = \frac{\omega}{k}$$

11-a

A single travelling wave conveys no information and can not be used for practicle purposes. There are modulat called wave groups or wavepackets, which carry energy and can be used to send signals. A wavepacket is the result of superposing waves of slightly diff wavelengths. The velocity with which the wavepacket travels in space is called the group velocity and it is the only velocity which can be experimentally measured.

Let us suppose that a wavepacket is formed by superposition of waves in 1-dimenison (x-directio

$$\psi_1(x,t) = A\,e^{-i(\omega t - kx)}$$

$$\psi_2(x,t) = A\,e^{-i[(\omega+d\omega)t - (k+dk)x]}$$

For simplicity, we take only the real part

so

$$\psi_1(x,t) = A\cos(\omega t - kx)$$

$$\psi_2(x,t) = A\cos[(\omega+d\omega)t - (k+dk)x]$$

Now, total wavefn. is.

$$\psi(x,t) = \psi_1(x,t) + \psi_2(x,t).$$

$$\psi = A\cos(\omega t - kx) + A\cos[(\omega+d\omega)t - (k+dk)x]$$

$$= 2A\cos\tfrac{1}{2}[(2\omega+d\omega)t - (2k+dk)x].\cos\tfrac{1}{2}(d\omega t - dkx)$$

where we have used

$$\cos\theta_1 + \cos\theta_2 = 2\cos\tfrac{1}{2}(\theta_1+\theta_2)\cos\tfrac{1}{2}(\theta_1-\theta_2)$$

$$\cos(-\theta) = \cos\theta$$

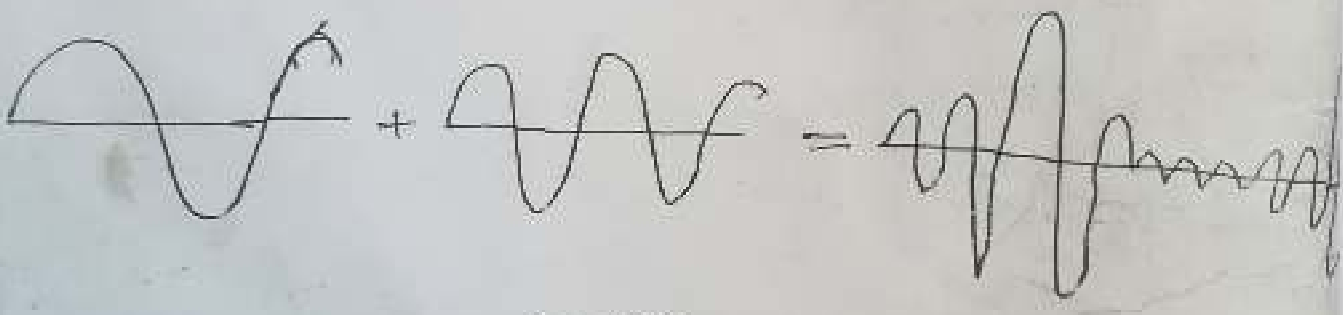
Since $d\omega$ and dk are small as Comp
to ω and k respectively, we make
approximation

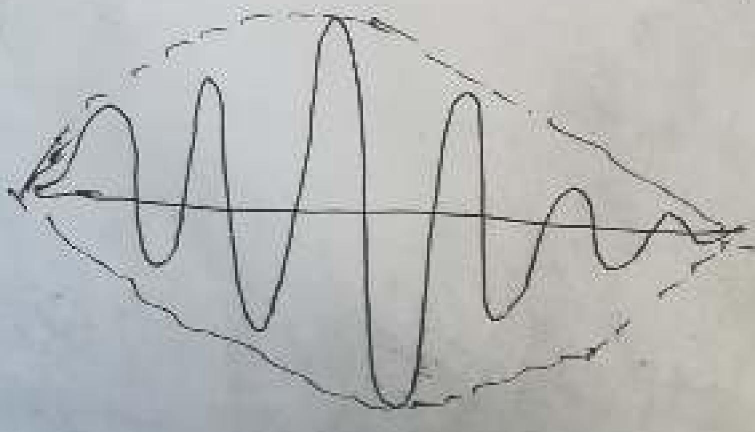
$$2\omega + d\omega \simeq 2\omega$$
$$2k + dk \simeq 2k$$

then

$$\psi = 2A\cos(\omega t - kx)\underbrace{\cos\frac{1}{2}(d\omega t - dkx)}_{\text{Modulation}}$$

So the wave group is





So the group velocity of the wave group is

$$v_g = \frac{d\omega}{dk}$$

In general a wavepacket can be represented by

$$\psi(x,t) = \int_{-\infty}^{\infty} A(k)\, e^{-i[\omega(k)t - kx]}\, dk$$

Amplitude.
A: the wavepacket

Heisenberg's Uncertainty Principle:

The classical concepts of position and momentum are independent of each other and can be determined exactly at the same time. At the atomic and nuclear scale, position and momentum become complementary properties of a system, so the quantum theory does not admit the possibility of their exact determination

Simultaneously. The uncertainty Principle of quantum mechanics can be stated as:

"It is impossible to know the exact position and exact momentum of a particle at the same time."

The amount of uncertainty in position and momentum is given by the following relations:

$$\Delta x \Delta P_x \gtrsim \hbar$$
$$\Delta y \Delta P_y \gtrsim \hbar$$
$$\Delta z \Delta P_z \gtrsim \hbar$$

These relations simply mean that the simultaneous determination of position and momentum in quantum physics is not exactly possible. Since $\hbar$ is very small, so the uncertainty principle is important only at the atomic and nuclear scales. If the position of a particle is exactly known i.e $\Delta x = 0$, then there is a total uncertainty regarding the momentum of the particle

i.e $\Delta p_x = \infty$.

Measurement of Position of an Electron:

In order to measure the position of an $\bar{e}$, we observe it by shining light on it and the scattered light carries the information about the position of the electron. When a photon of momentum $2\pi\hbar/\lambda$ hits the electron, it disturbs its position and momentum. The uncertainty in momentum of the electron is of the order of momentum of the photon i.e

$$\Delta p \sim 2\pi\hbar/\lambda$$

The uncertainty in the position of the electron is of the order of the wavelength of light i.e.

$$\Delta x \sim \lambda$$

14-a

The product of uncertainties give:

$$\Delta x \Delta p \sim 2\pi\hbar/\lambda \cdot \lambda$$

$$\Delta x \Delta p \sim 2\pi\hbar$$

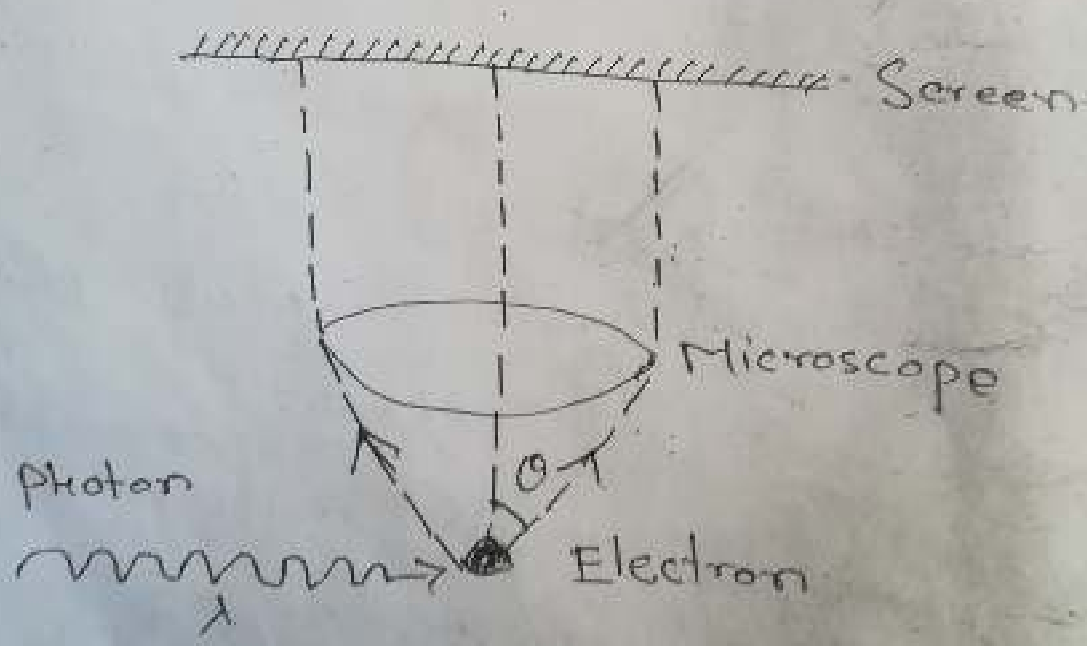
or

$$\boxed{\Delta x \Delta p \gtrsim \hbar}$$

A more precise description is given by Heisenberg microscope.

Heisenberg Microscope:

Consider an experiment illustrated in the figure:



The purpose of this experiment is to measure the position of an electron. From the theory of microscope, it is known that the position of a particle under observation can be determined with an accuracy of

$$\Delta x \approx \frac{\lambda}{\sin\theta} \longrightarrow \text{Classically}$$

This is the resolving power of the microscope i.e. the smallest distance determined by the microscope.

When the ~~position~~ photon hits the $\bar{e}$, the electron suffers a recoil momentum of the order of h/λ. The uncertainty in momentum of the electron in x-direction has to lie in the range.

$$\Delta P_x \approx \frac{h}{\lambda} \sin\theta.$$

The product of Δx and ΔP_x is

$$\Delta x \Delta p_x \sim h = 2\pi\hbar$$

$$\boxed{\Delta x \Delta p_x \gtrsim \hbar}.$$

The Energy-Time Uncertainty:

The energy of a free particle is $E = \frac{p^2}{2m}$, so that

$$\Delta E = \frac{p \Delta p}{m}$$

The uncertainty relation of positi[on] and momentum can be re-expresse[d] as

$$\Delta p \cdot \Delta x \gtrsim \hbar$$

$$\frac{p}{m} \Delta p \cdot \frac{m}{p} \Delta x \gtrsim \hbar$$

$$\Delta E \cdot \frac{m}{mv} \Delta x \gtrsim \hbar$$

$$\Delta E \frac{\Delta x}{v} \gtrsim \hbar.$$

$$\boxed{\Delta E \cdot \Delta t \gtrsim \hbar}$$

This is the energy-time uncertain[ty] relation.

ΔE can be interpreted as the difference between energies at times separated by Δt.

Physical Interpretation of the Wavefunction

A matter wave is represented by a complex variable quantity $\psi(x,t)$ called the wavefunction. Since, the Schrodinger equation is linear ($\rightarrow y = 2x+1$) and homogeneous ($\hookrightarrow \frac{dy}{dx} = f\left(\frac{y}{x}\right)$) in $\psi(x,t)$ and its space and time derivatives, therefore, the superposition of two solutions would be another solution of the equation and also a constant multiplied with a solution gives another solution.

Let $\psi'(x,t)$ be a solution of the Schrodinger wave equation and

$$\int_{-\infty}^{\infty} |\psi'(x,t)|^2 \, dx = A^2$$

Since $|\psi'(\underline{x},t)|^2$ is a +ve real numb

its integral is also a +ve real num

product of wavefn with itself

A^2. The +ve number "A" is known as

norm of the wavefunction $\psi'(\underline{x},t)$

Let us define another wavefunction

inner product of wavefn

$$\psi(\underline{x},t) = \frac{1}{A}\,\psi'(\underline{x},t)$$

(ψ,ψ). This new wavefunction $\psi(\underline{x},t)$ is

also a solution of the Schrodinger

equation. In this case

$$\int_{-\infty}^{\infty} |\psi(\underline{x},t)|^2 \, d\underline{x} = 1$$

A function having the above proper

is said to be normalized to unity

The normalizable wavefunctions are

those which have finite norm. Since

the norm is defined as the integ

of $|\psi|^2$ over all space, its finite re implies that $|\psi(x,t)|^2$ vanishes at spatial infinities i.e.

$$|\psi(x,t)| \longrightarrow 0$$

as $\quad x \longrightarrow \pm\infty$

This is the boundary condition which holds for all normalizable wavefunctions. The position of a particle can not in general be determined exactly. All one can say is that there is a certain probability that the particle is within any specified region. The wave-function $\psi(x,t)$ gives a measure of the probability of finding a particle at a particular position. Let us denote this probability by $P(x,t)$ and it must have the following properties:

$P(x)\,dx = |\psi(x)|^2 dx$

1) $P(x,t)$ is large when $\psi(x,t)$ is large.

gives the probability of finding the particle b/w x & $x+dx$

17-a

ii) $P(\underline{x},t)$ is +ve definite i.e. $P(x,$

iii) $\int_{-\infty}^{\infty} P(\underline{x},t)\,dx = 1$

i.e. the total probability of find the particle in the region from $-\infty$ to $+\infty$ (whole space) is always

So the probability density is given by

$$P(x,t) = |\psi(\underline{x},t)|^2$$

probability of finding the particle at x.

when

$$\int_{-\infty}^{\infty} |\psi(x,t)|^2\,d\underline{x} = 1$$

is satisfied, the wavefunction $\psi($ is said to be normalized.

For a physically acceptable or well behaved wave function $\psi(\underline{x},t)$, it satisfy the following Conditions:

1- $\psi(x,t)$ must be finite

2- $\psi(x,t)$ " " single-valued

3- $\psi(x,t)$ " " Continuous.

The condition

$$\int_{-\infty}^{\infty} P(x,t)\, dx = 1$$

is satisfied for all t i.e.

$$\frac{d}{dt}\int_{-\infty}^{\infty} P(x,t)\, dx = 0.$$

linear eqn

$y = 2x + 1$
An equation b/w two variables that gives a straight line when plotted on a graph

homogeneous $y = mx$ straight

$f_n(x)\, y^{(n)} + \dots + f_1(x)\, y' + f_0(x)\, y = g(x)$

was called homogeneous if $g(x) = 0$ — ~~nothing~~

If $g(x) \neq 0$ then called Trivial eq non-homogeneous or inhomogeneous

Atomic Transitions:

Rutherford's Atomic Model:

According to Rutherford's atomic model, the orbital motion of an electron round the nucleus represents an accelerated motion, so that the electron should continuously emit electromagnetic radiation and as a result should lose energy. At the end, the electron would spiral into the nucleus. Since the orbital frequency would change continuously, the