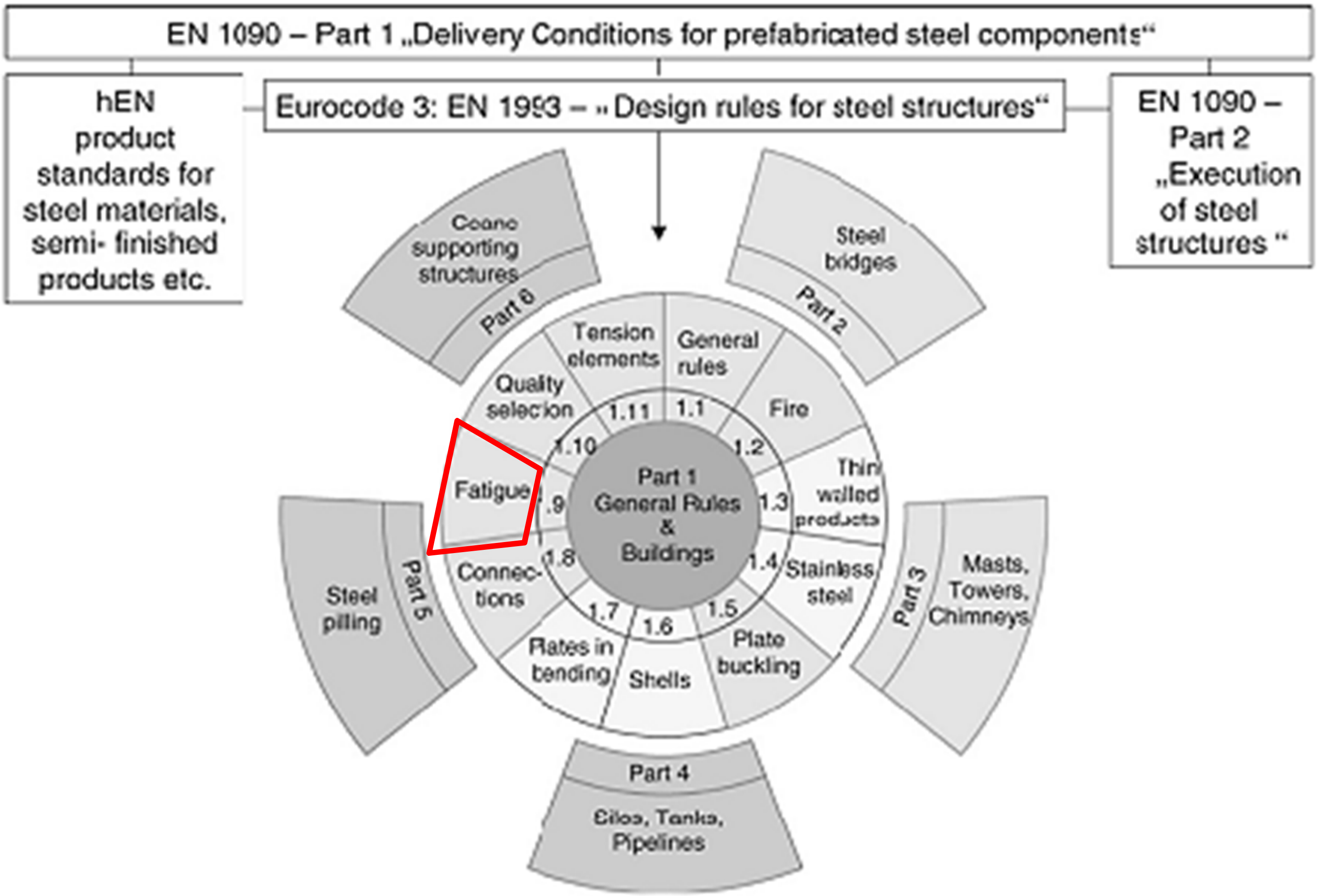
Advanced Design of steel and Composite structures

INTRODUCTION TO FATIGUE

Professor Dan Dubina

Introduction to fatigue

EN 1993 Framerwork



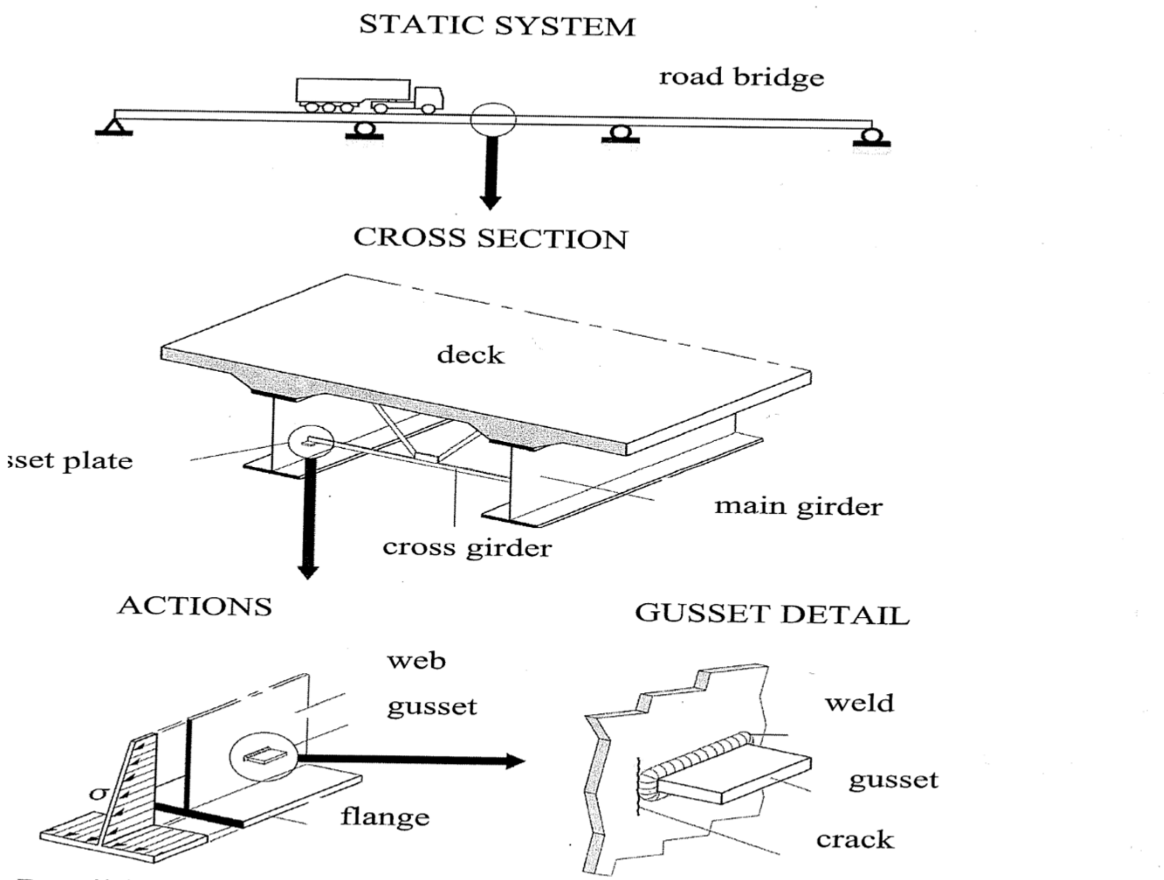
Introduction to Fatigue

- Fatigue may occur when a member is subjected to repeated cyclic loadings (due to action of fluctuating stress, according to the terminology used in the EN 1993-1-9)
- The fatigue phenomenon shows itself in the form of cracks developing at particular locations in the structure.
- Cracks can appear in diverse types of structures such as: planes, boats, bridges, frames ,cranes, overhead cranes, machines parts, turbines, reactors vessels, canal lock doors, offshore platforms, transmission towers, pylons, masts and chimneys
- Structures subjected to repeated cyclic loadings can undergo progressive damage which shows itself by the propagation of cracks. This damage is called *fatigue* and is represented by a loss of resistance with time.

Introduction to Fatigue

- The physical effect of a repeated load on a material is different from the static load.
- Failure always being brittle fracture regardless of whether the material is brittle or ductile.
- Mostly fatigue failure occur at stress well below the static elastic strength of the material.

Introduction to Fatigue



Possible location of a fatigue crack in a road bridge (TGC 10, 2006)

Introduction to Fatigue

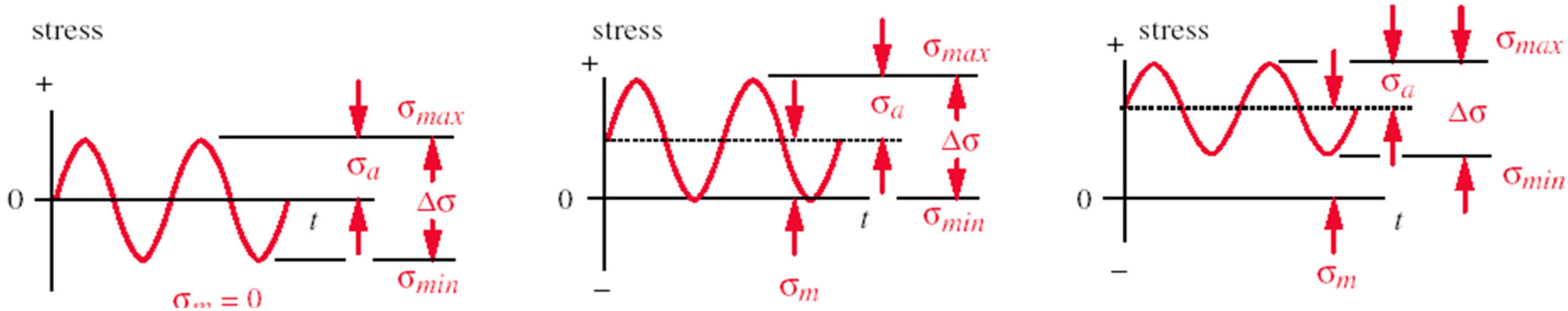
Main parameters influencing fatigue life

The fatigue life of a member or of a structural detail subjected to repeated cyclic loadings is defined as the number of stress cycles it can stand before failure.

Depending upon the member or structural detail geometry, its fabrication or the material used, four main parameters can influence the fatigue strength (or resistance, both used in EN 1993-1-9):

- the stress difference, or as most often called *stress range,*
- the structural detail geometry,
- the material characteristics,
- the environment

Types of Fatigue Loading



Fully Reversed

Repeated

Fluctuating

$$\Delta\sigma = \sigma_{\max} - \sigma_{\min}$$ stress range

$$\sigma_a = \frac{\Delta\sigma}{2}$$ alternating component

$$\sigma_m = \frac{\sigma_{\max} + \sigma_{\min}}{2}$$ mean component

amplitude ratio $$A = \frac{\sigma_a}{\sigma_m}$$

stress ratio $$R = \frac{\sigma_{\min}}{\sigma_{\max}}$$

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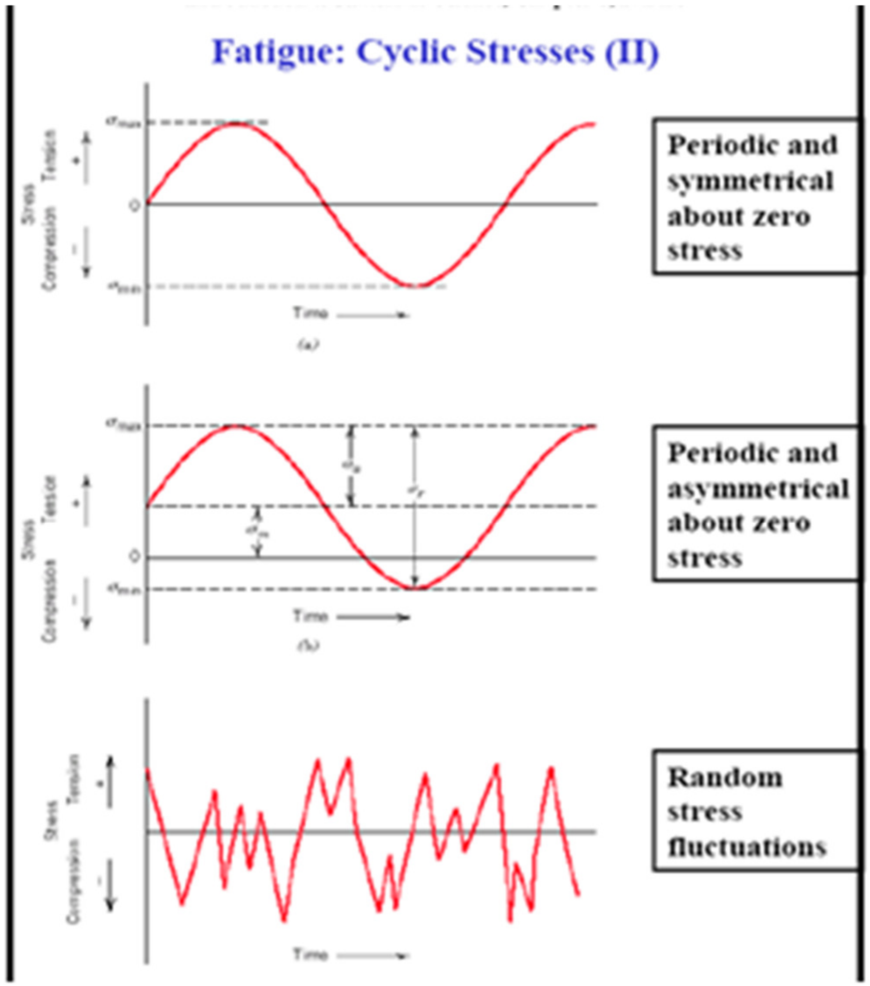


Fatigue: Failure under fluctuating stress

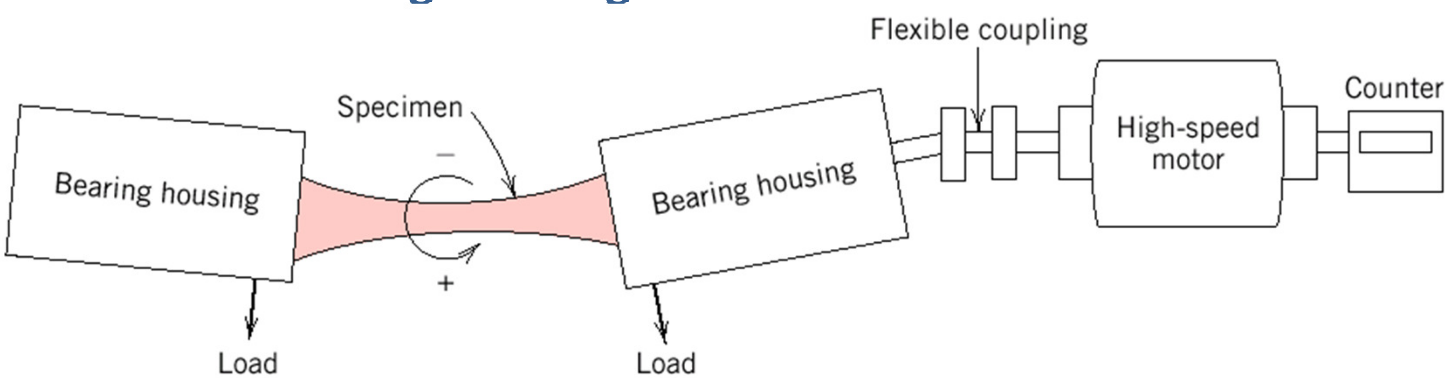
- Under fluctuating / cyclic stresses, failure can occur at lower loads than under a static load.
- 90% of all failures of metallic structures (bridges, aircraft, machine components, etc.)
- Fatigue failure is brittle-like –

  even in normally ductile materials.

  Thus sudden and catastrophic!

Fatigue: S—N curves I

Rotating-bending test → S-N curves



S (stress) vs. N (number of cycles to failure)

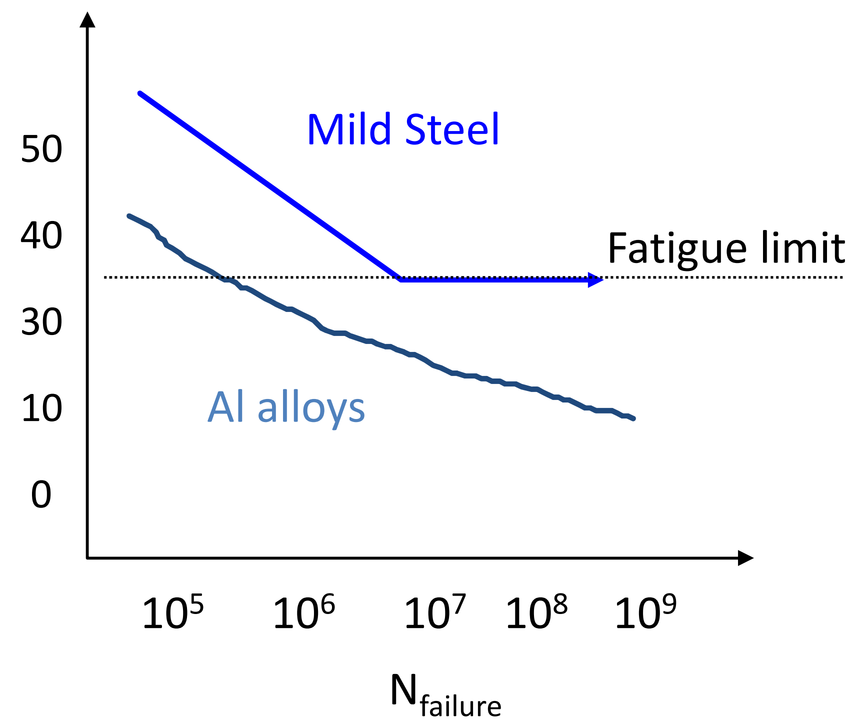
- **Low cycle fatigue: small no. of cycles (N < 10^5) high loads**
  - **plastic and elastic deformation**
- **High cycle fatigue: large # of cycles low loads**
  - **elastic deformation (N > 10^5)**

Introduction to Fatigue

High Cycle Fatigue

S-N Curves

- **Apply controlled $\Delta\sigma$**
  $\sigma_{applied} < \sim {}^{2}/_{3}\ \sigma_{yield}$
- **Stress is elastic on gross scale.**
- **Locally the metal deforms plastically.**



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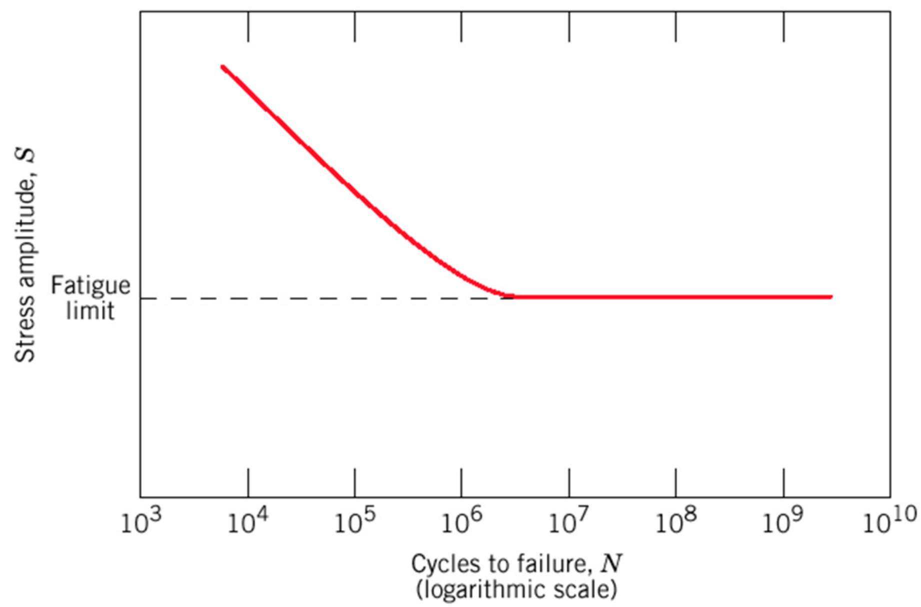
Fatigue: S—N curves II



- **Fatigue limit (some Fe and Ti alloys)**

S—N curve becomes horizontal at large N

- **Stress amplitude below which the material never fails, no matter how large the number of cycles is (Endurance Limit)**

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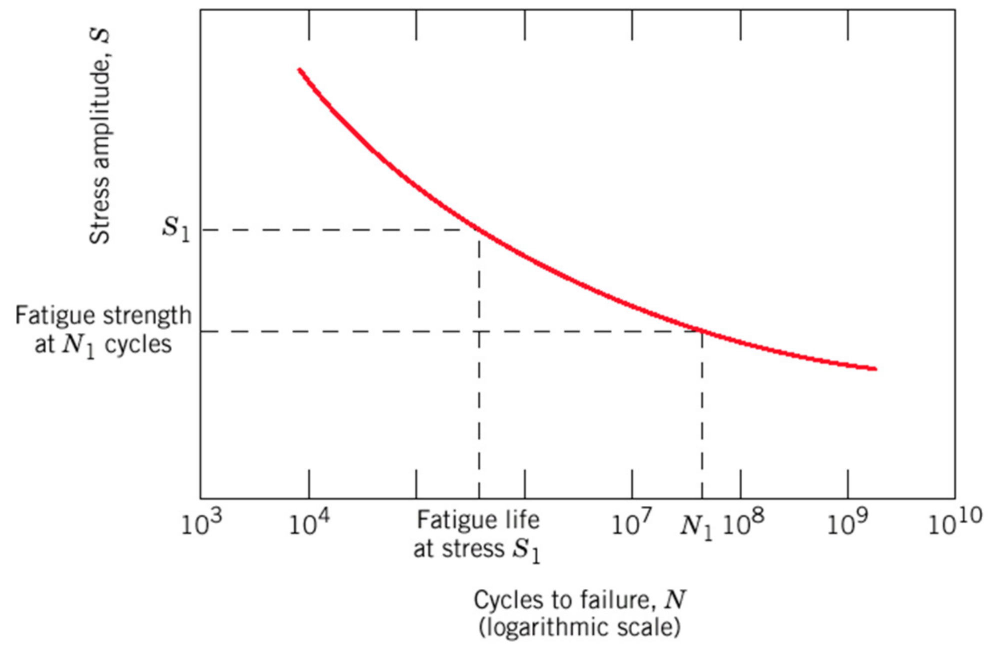
Fatigue: S—N curves III



Most alloys : S decreases with N.

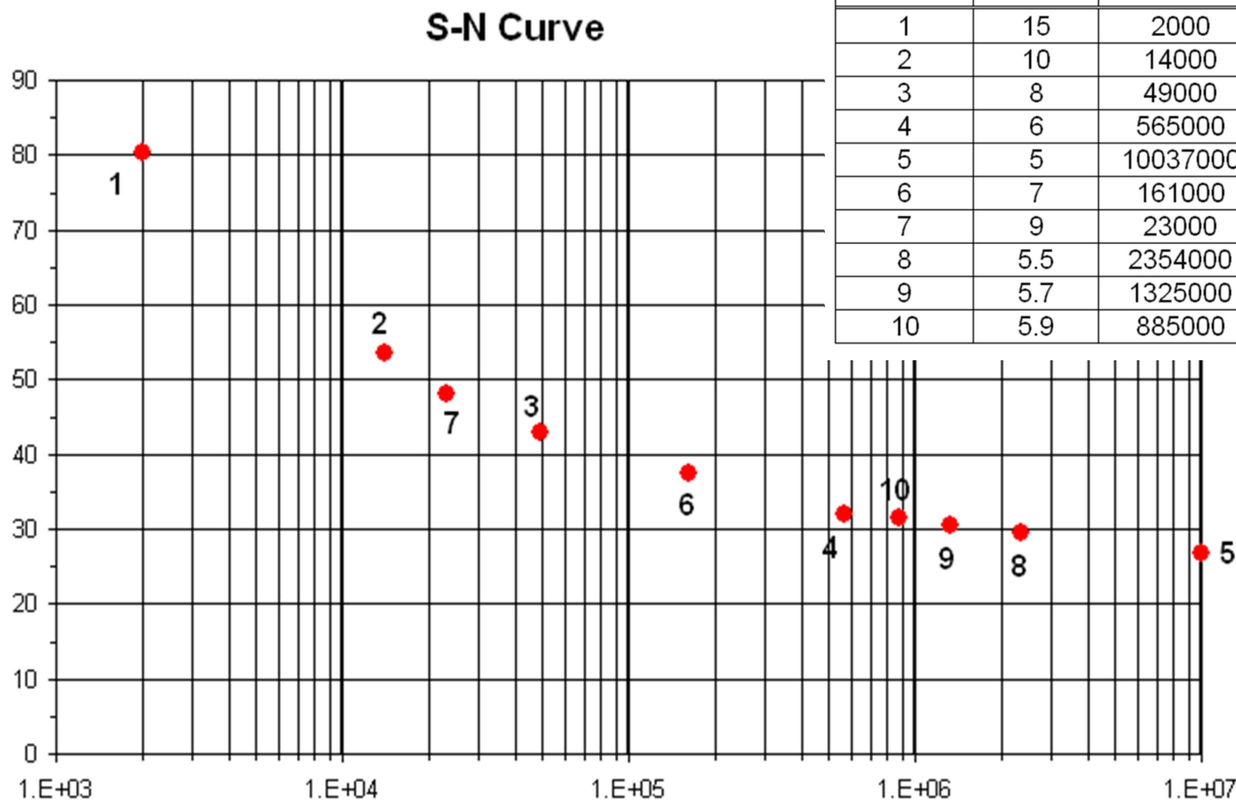
- **Fatigue strength: Stress at which fracture occurs after specified number of cycles (e.g. 10^7)(Endurance Strength)**
- **Fatigue life: Number of cycles to fail at specified stress level**

Introduction to Fatigue

Fatigue Testing

16 ga. Cold Rolled Steel (.058" thick)

Specimen	Load (lbs)	Life (cycles)	Stress (ksi)
1	15	2000	80.3
2	10	14000	53.5
3	8	49000	42.8
4	6	565000	32.1
5	5	10037000	26.8
6	7	161000	37.5
7	9	23000	48.2
8	5.5	2354000	29.4
9	5.7	1325000	30.5
10	5.9	885000	31.6



Number of Cycles to Failure, N

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Fatigue Strength

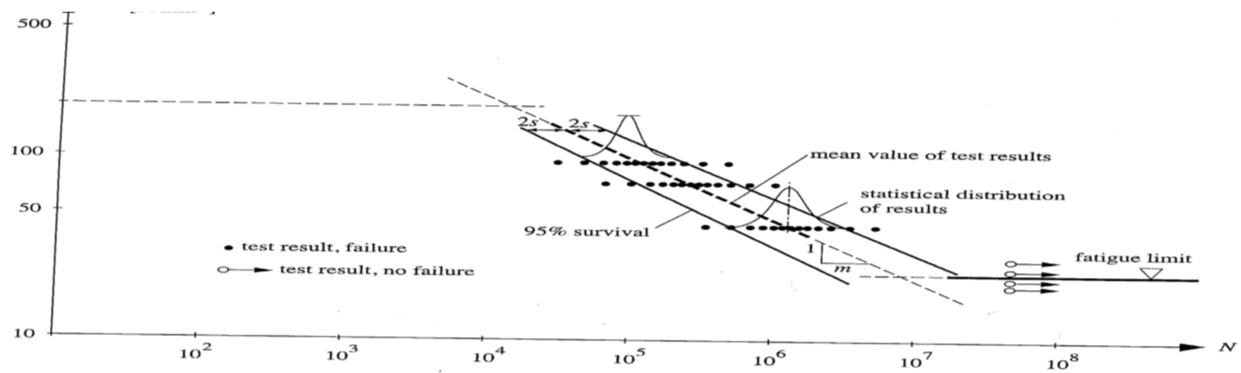

Figure 1.3 – Fatigue test results of structural steel members, plotted in double logarithm scale, carried out under constant amplitude loading (TGC 10, 2006)

The fact is that the scatter of the test results is less at high ranges and larger at low stress ranges, see for example Schijve (2001). By using a logarithmic scale for both axes, the mean value of the test results for a given structural detail can be expressed, in the range between 10^4 cycles and $5 \cdot 10^6$ to 10^7 cycles, by a straight line with the following expression:

$$N = C \cdot \Delta\sigma^{-m} \tag{1.2}$$

where

- N number of cycles of stress range $\Delta\sigma$,
- C constant representing the influence of the structural detail,
- $\Delta\sigma$ constant amplitude stress range,
- m slope coefficient of the mean test results line.

The expression represents a straight line when using logarithmic scales:

$$\log N = \log C - m \cdot \log(\Delta\sigma) \tag{1.3}$$

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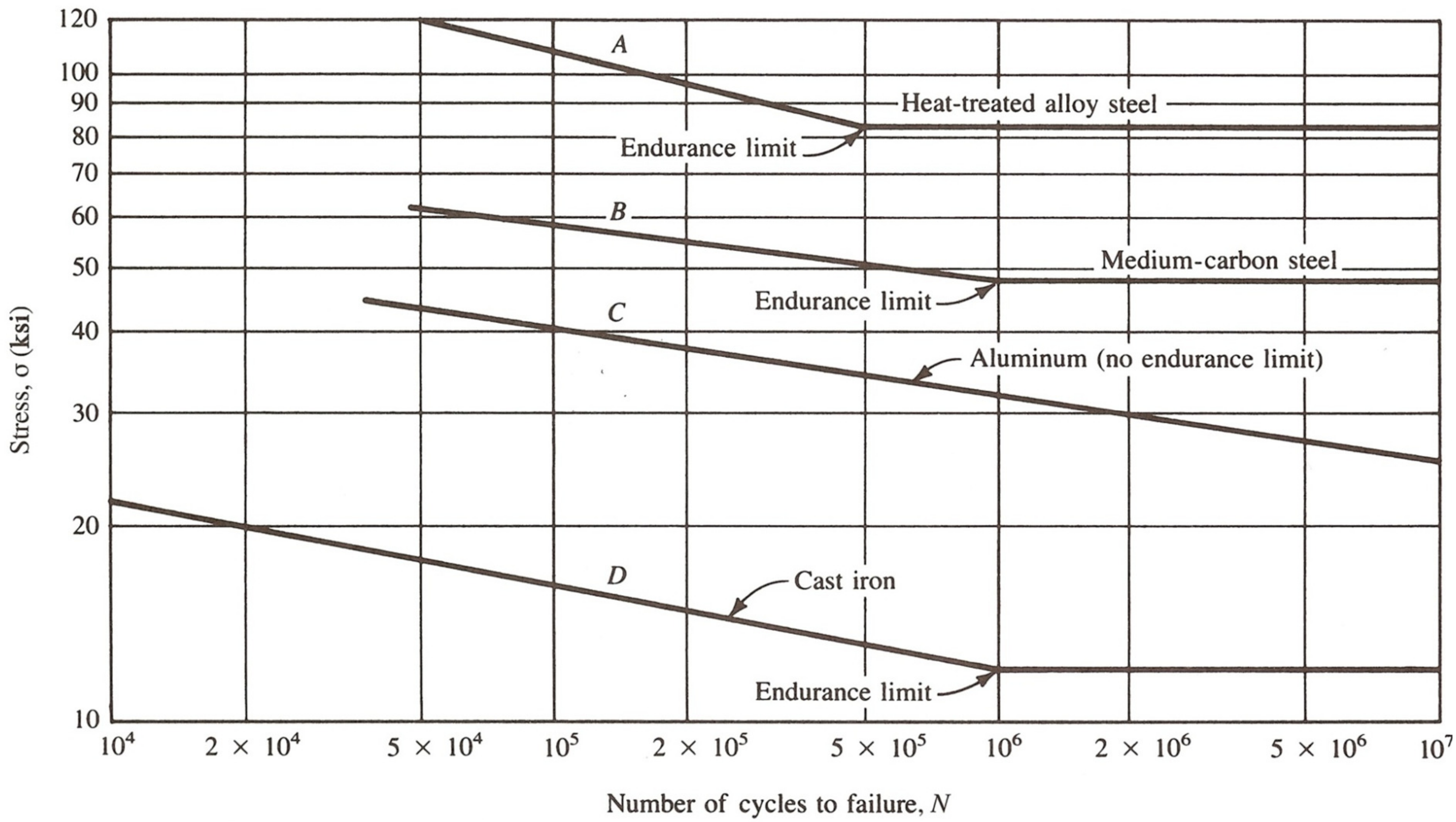
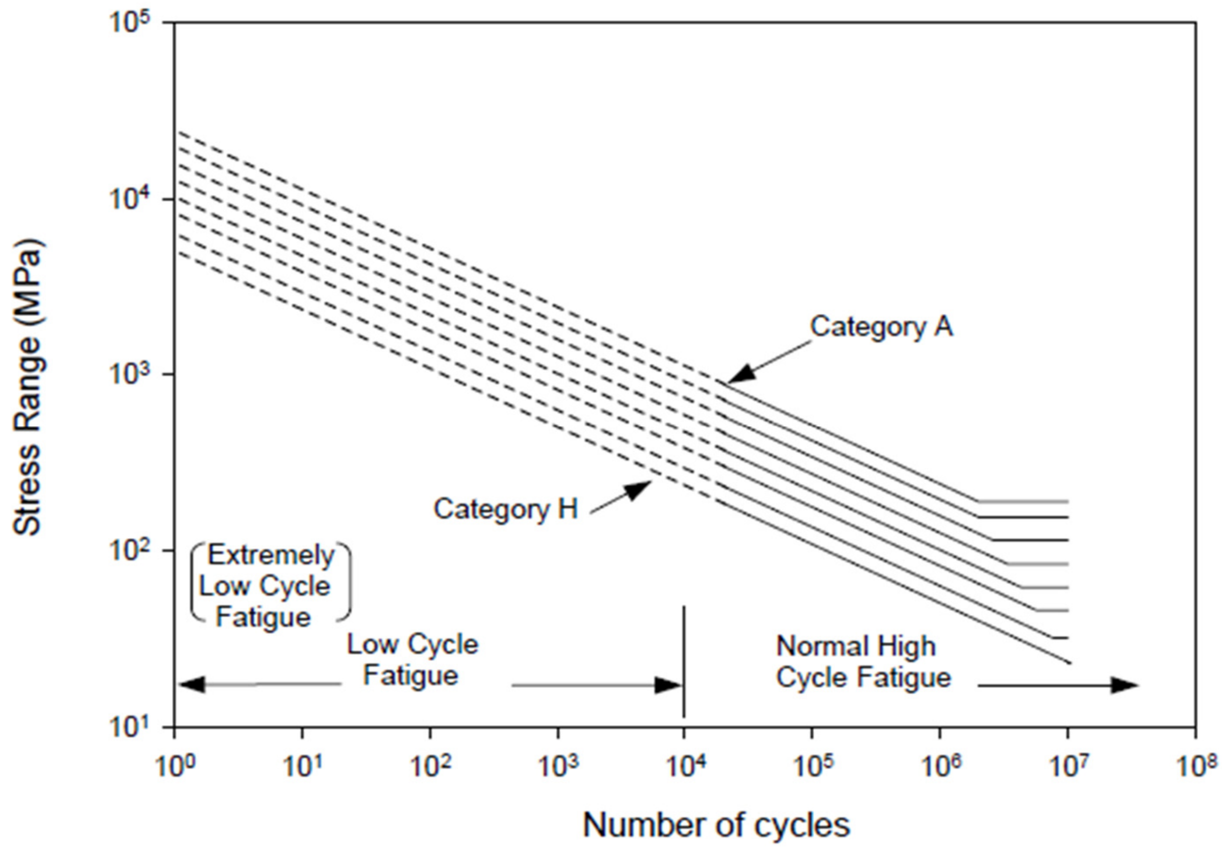
Representative Endurance Strengths



Fatigue classification



Introduction to Fatigue

Methods to assess fatigue resistance

Stress-based approach (Linear elastic)

- Nominal stress approach
- Structural stress approach
- Notch stress approaches

→ High cycle fatigue
Endurance limit
Failure criterion: fracture

Strain-based approach (Elastoplastic)

- Local strain approach

→ Low and high cycle fatigue
Endurance limit
Failure criterion: initial crack

- Crack propagation

→ Linear elastic or elastoplastic
Fracture mechanics
Failure criterion: crack of defined length, fracture

Endurance limit is the stress level that a material can survive for an infinite number of load cycles; ***Endurance strength*** is the stress level that a material can survive for a given number of load cycles

Low Cycle Fatigue

- **Apply controlled amounts of $\Delta\varepsilon_{total}$**

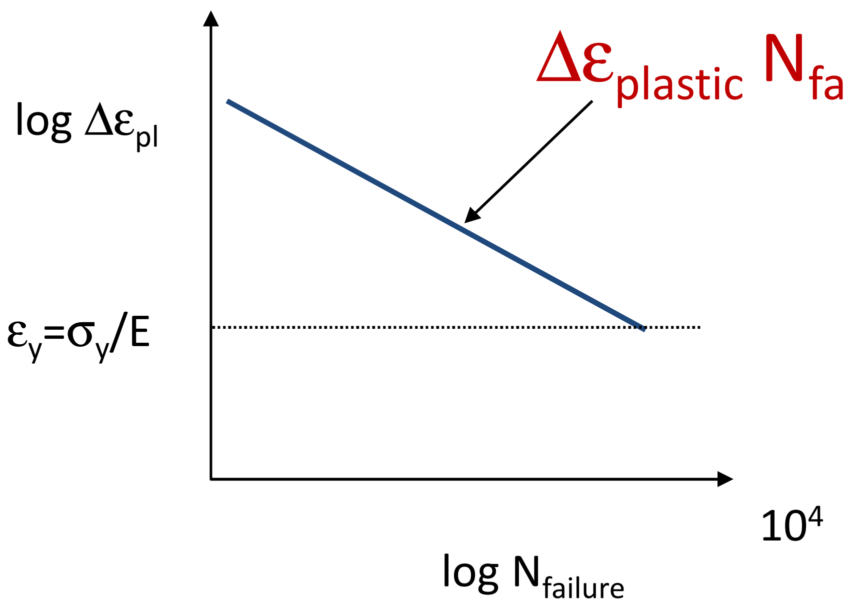
  $$\Delta\varepsilon_{total} = \Delta\varepsilon_{elastic} + \Delta\varepsilon_{plastic}$$

- **Empirical Observations and Rules**
  - Coffin-Manson Law
  - Miner's Rule

Low Cycle Fatigue

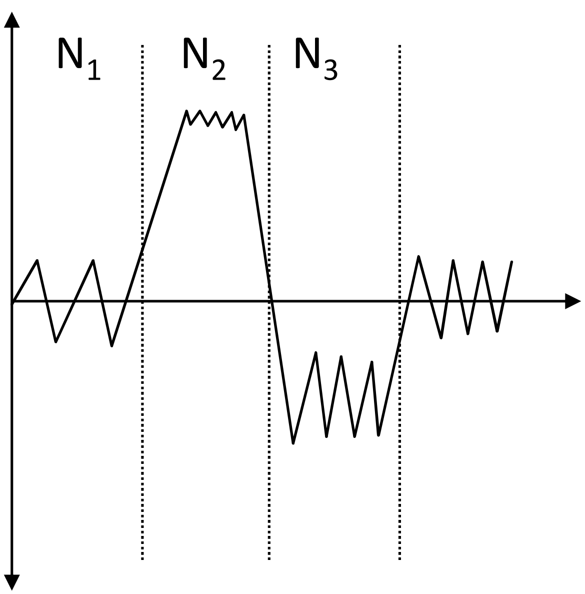
Coffin Manson Law

$$\Delta\varepsilon_{plastic}\, N_{failure}^{1/2} = \text{Const.}$$



Low Cicle Fatigue : Miner's Rule

Rule of Accumulative damage:



$$\sum \frac{N_i}{N_{failure\ @\ i}} = 1$$

Fraction of life time @ i

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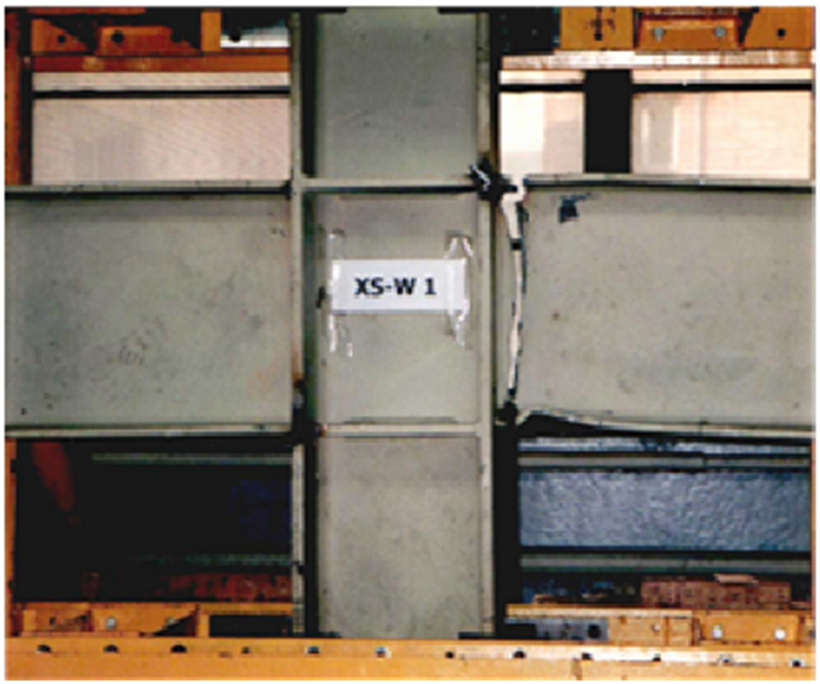
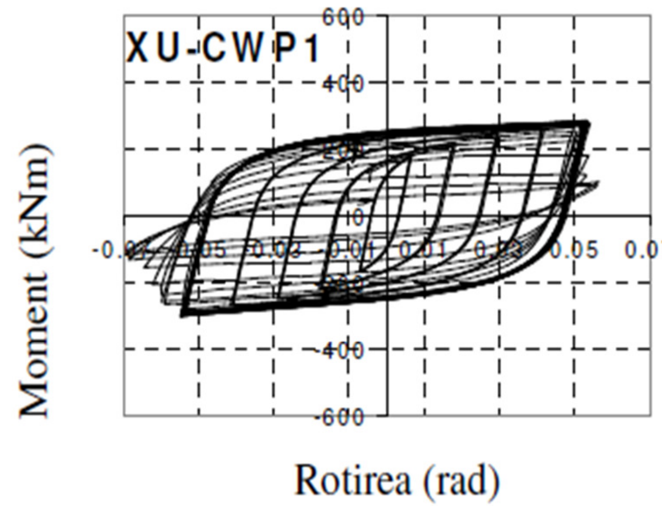
Low cycle fatigue damages





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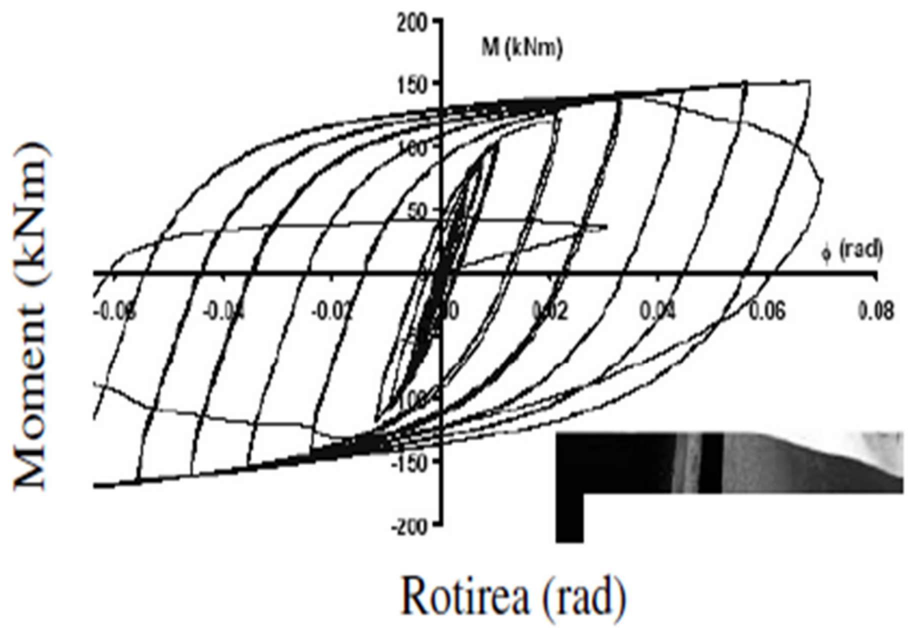

The Fatigue Process

- Crack initiation
  - early development of damage
- Stage I crack growth
  - deepening of initial crack on shear planes
- Stage II crack growth
  - growth of well defined crack on planes normal to maximum tensile stress
- Ultimate Failure

Fatigue: Crack initiation+ propagation (I)

Three stages:

1. crack initiation in the areas of stress concentration (near stress raisers, inclusions, exsiting vracks)
2. incremental crack propagation
3. rapid crack propagation after crack reaches critical size

The total number of cycles to failure is the sum of cycles at the first and the second stages:

$$N_f = N_i + N_p$$

N_f : Number of cycles to failure
N_i : Number of cycles for crack initiation
N_p : Number of cycles for crack propagation

High cycle fatigue (low loads): N_i is relatively high. With increasing stress level, N_i decreases and N_p dominates

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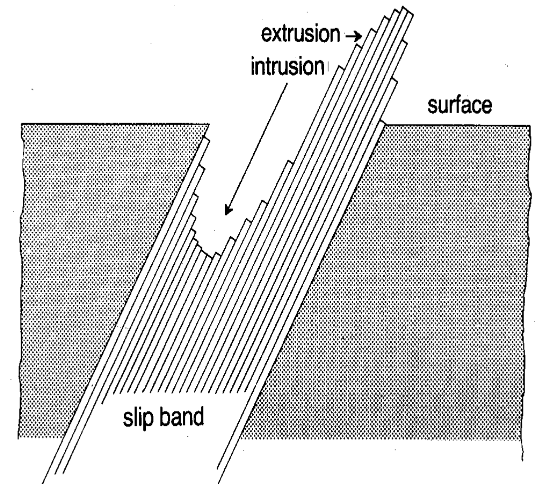
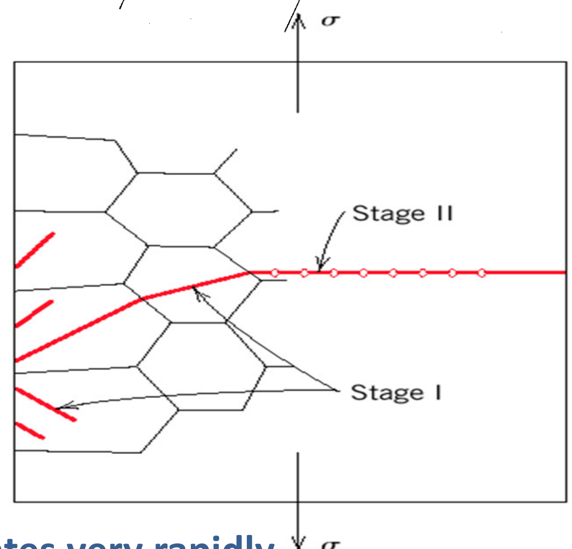
Fatigue: Crack initiation and propagation (II)



- **Crack initiation: Quality of surface and sites of stress concentration (microcracks, scratches, indents, interior corners, dislocation slip steps, etc.).**
- **Alternate stresses-> slip bands -> surface rumpling**



- **Crack propagation**
- **I: Slow propagation along crystal planes with high resolved shear stress. Involves a few grains.**
  Flat fracture surface
- **II: Fast propagation perpendicular to applied stress.**
  Crack grows by repetitive blunting and sharpening process at crack tip. Rough fracture surface
- **Crack eventually reaches critical dimension and propagates very rapidly**

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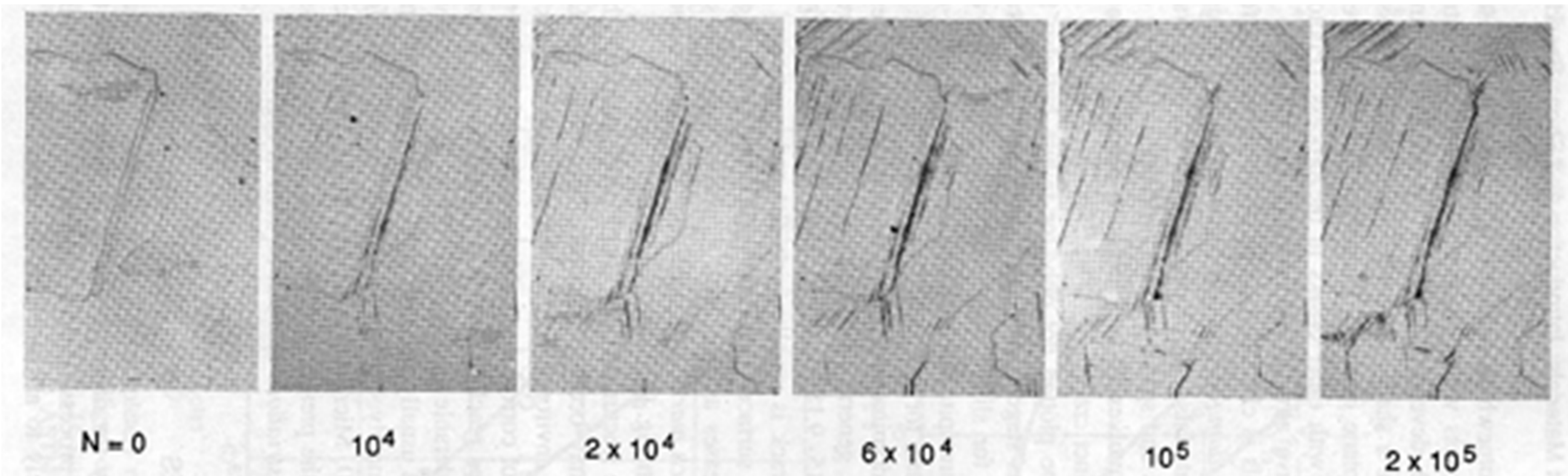
Crack initiation and progress with number of cycles



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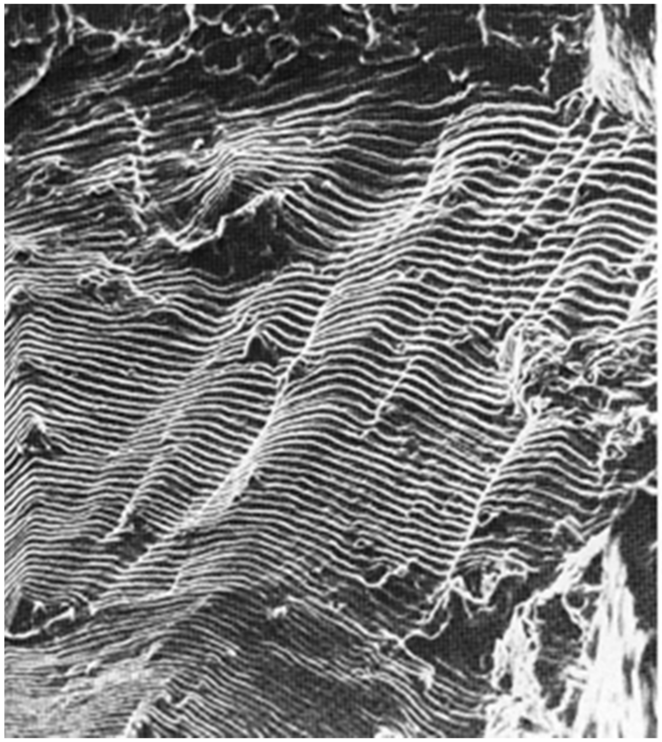
Crack development



Striation indicating steps in crack advancement.

Fracture surfaces

Initiation site

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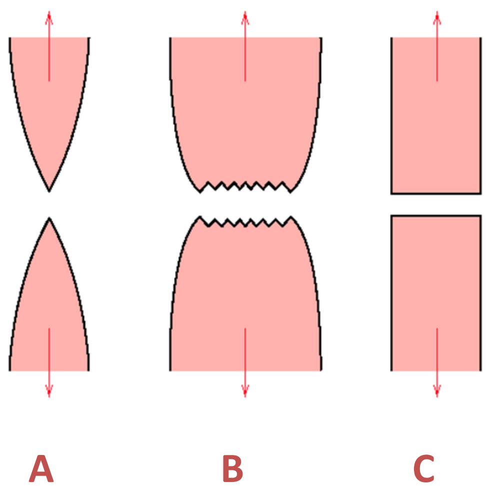
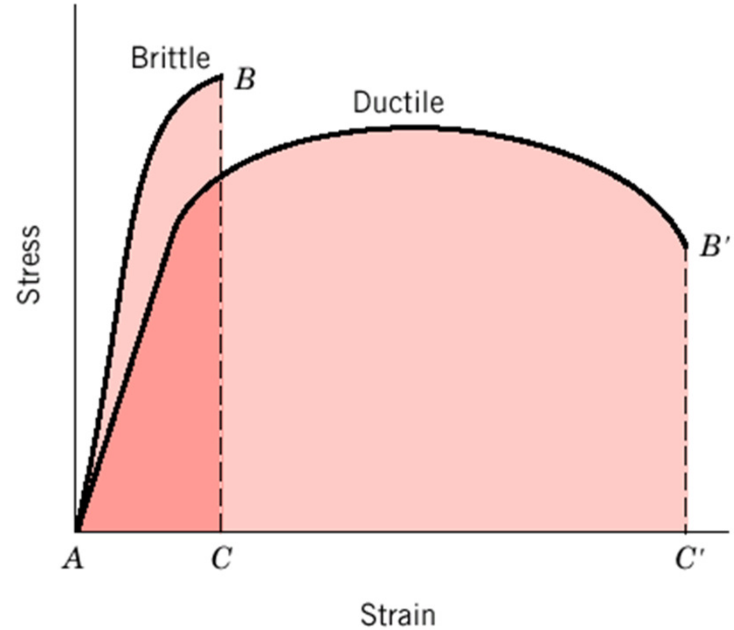
Brittle vs. Ductile Fracture

- **Ductile materials - extensive plastic deformation and energy absorption ("toughness") before fracture**
- **Brittle materials - little plastic deformation and low energy absorption before fracture**





A. **Moderately ductile fracture** typical for metals

A. **Very ductile:** soft metals (e.g. Pb, Au) at room T, polymers, glasses at high T

B. **Brittle fracture:** ceramics, cold metals,

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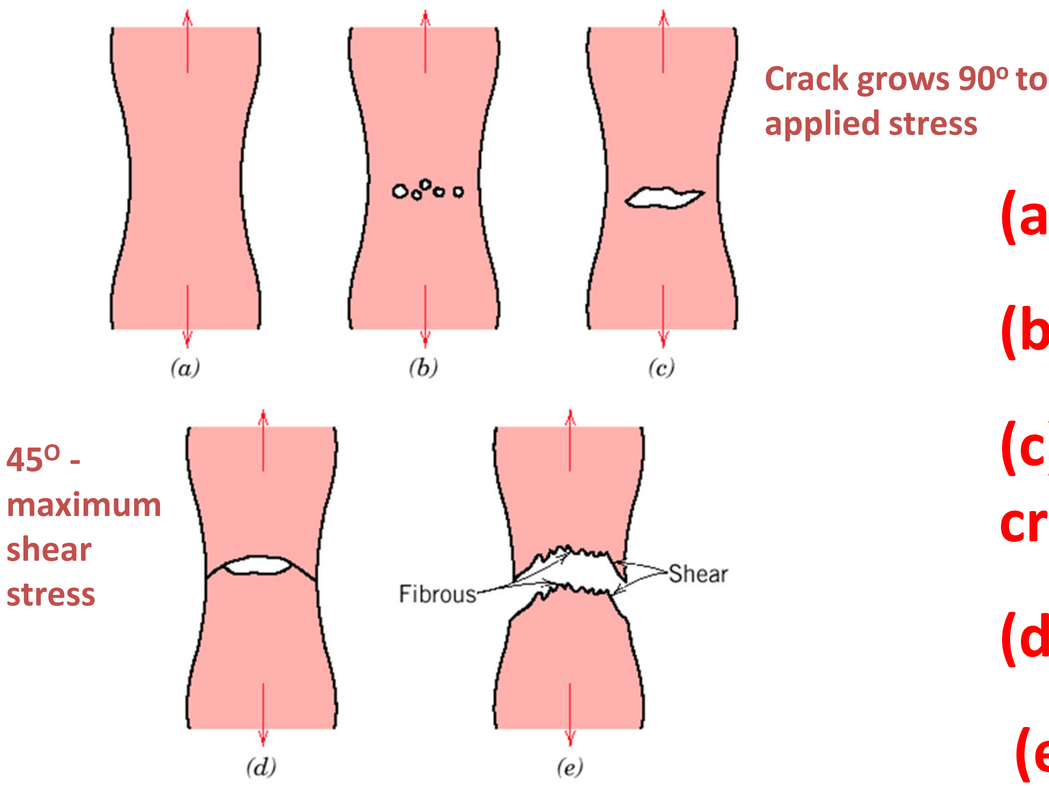
Ductile Fracture (Dislocation Mediated



(a) Necking,

(b) (b) Cavity Formation,

(c) Cavities coalesce → form crack

(d) Crack propagation,

(e) Fracture

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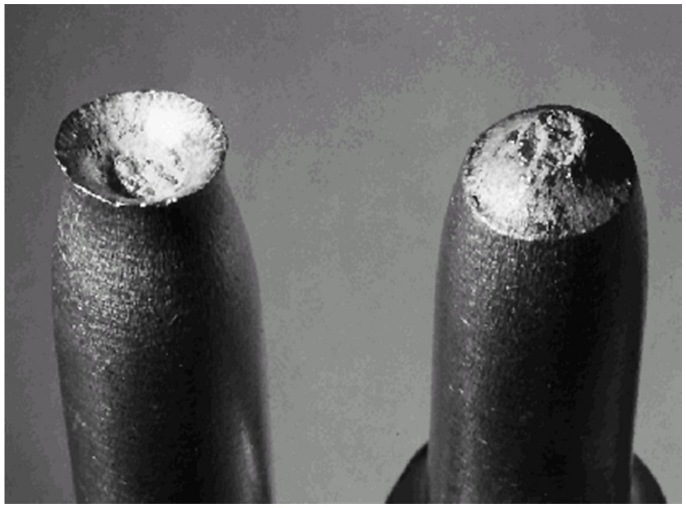
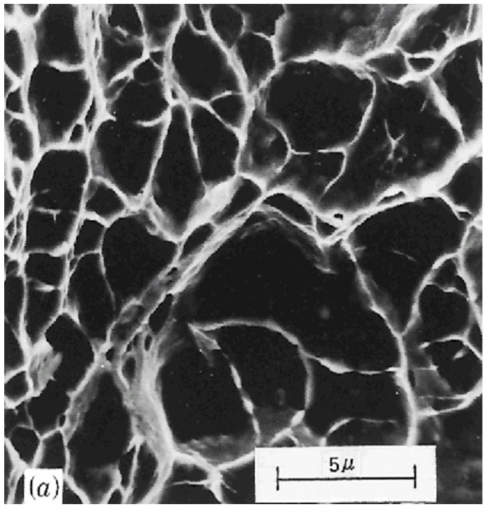
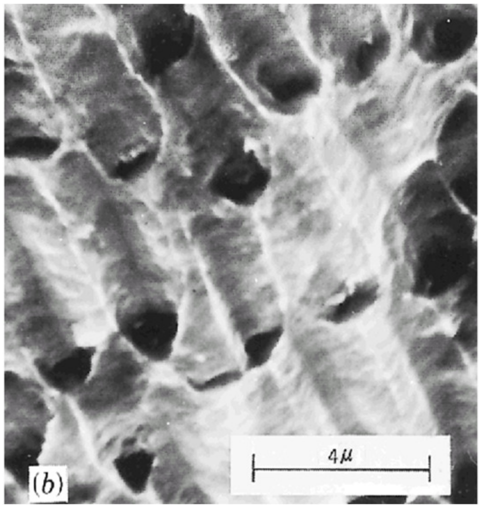
Ductile Fracture (Dislocation Mediated



Cup-and-cone fracture in Al)





Scanning Electron Microscopy. Spherical "dimples" → micro-cavities that initiate crack formation

(*University of Virginia , Dept. of materials Science Engineering*)

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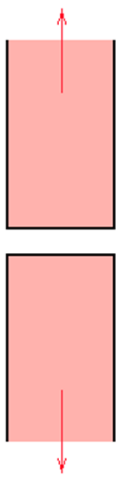
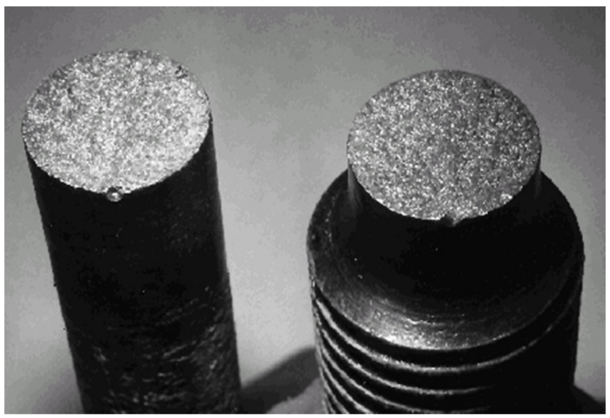
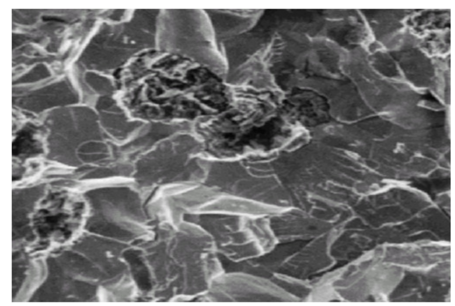
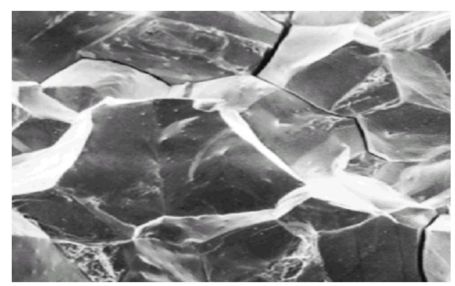
Brittle Fracture (Low Dislocation Mobility





- **Crack propagation is fast**
- **Propagates nearly perpendicular to direction of applied stress**
- **Often propagates by cleavage - breaking of atomic bonds along specific crystallographic planes**
- **No appreciable plastic deformation**





A.Transgranular fracture: Cracks pass through grains. Fracture surface: faceted texture because of different orientation of cleavage planes in grains.

B.Intergranular fracture: Crack propagation is along grain boundaries (grain boundaries are weakened/ embrittled by impurity segregation etc.)

(*University of Virginia , Dept. of materials Science Engineering* **)**

Stress Concentration

Fracture strength of a **brittle solid:**

- related to **cohesive forces** between atoms.
- Theoretical strength: ~E/10
- Experimental strength ~ E/100 - E/10,000

Difference due to:

Stress concentration at microscopic flaws

Stress amplified at tips of micro-cracks etc., called **stress raisers**

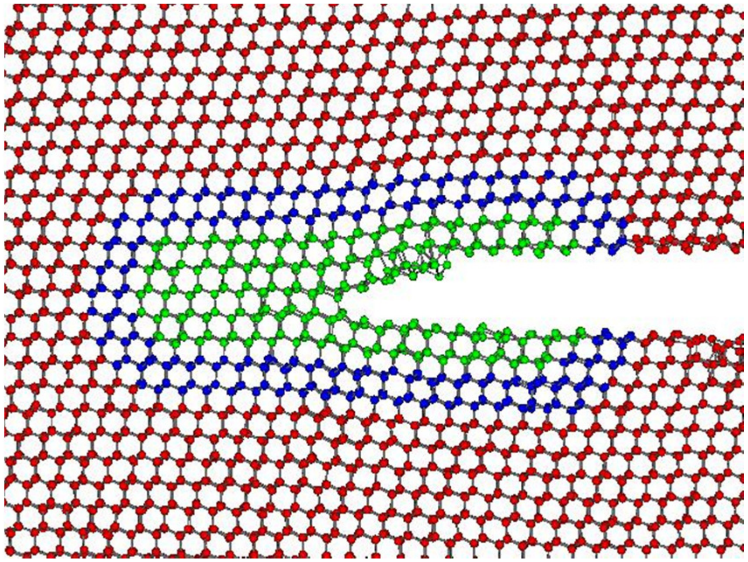


Figure by
N. Bernstein &
D. Hess, NRL

(*University of Virginia , Dept. of materials Science Engineering)*

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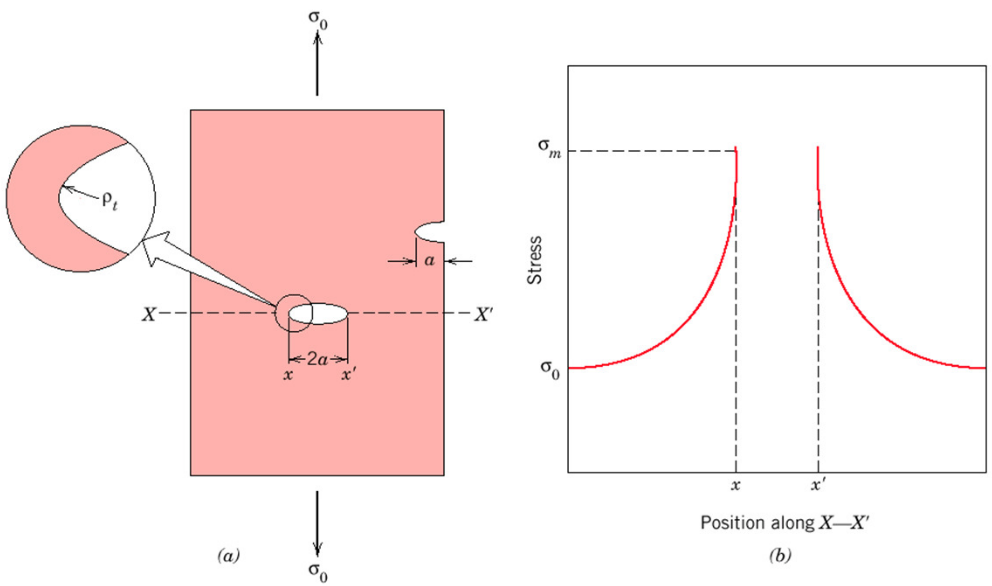
Stress Concentration



Crack perpendicular to applied stress: maximum stress near crack tip

$$\rightarrow \qquad \sigma_m \approx 2\sigma_0 \left(\frac{a}{\rho_t}\right)^{1/2}$$

σ_0 = applied stress; a = half-length of crack;

ρ_t = radius of curvature of crack tip.

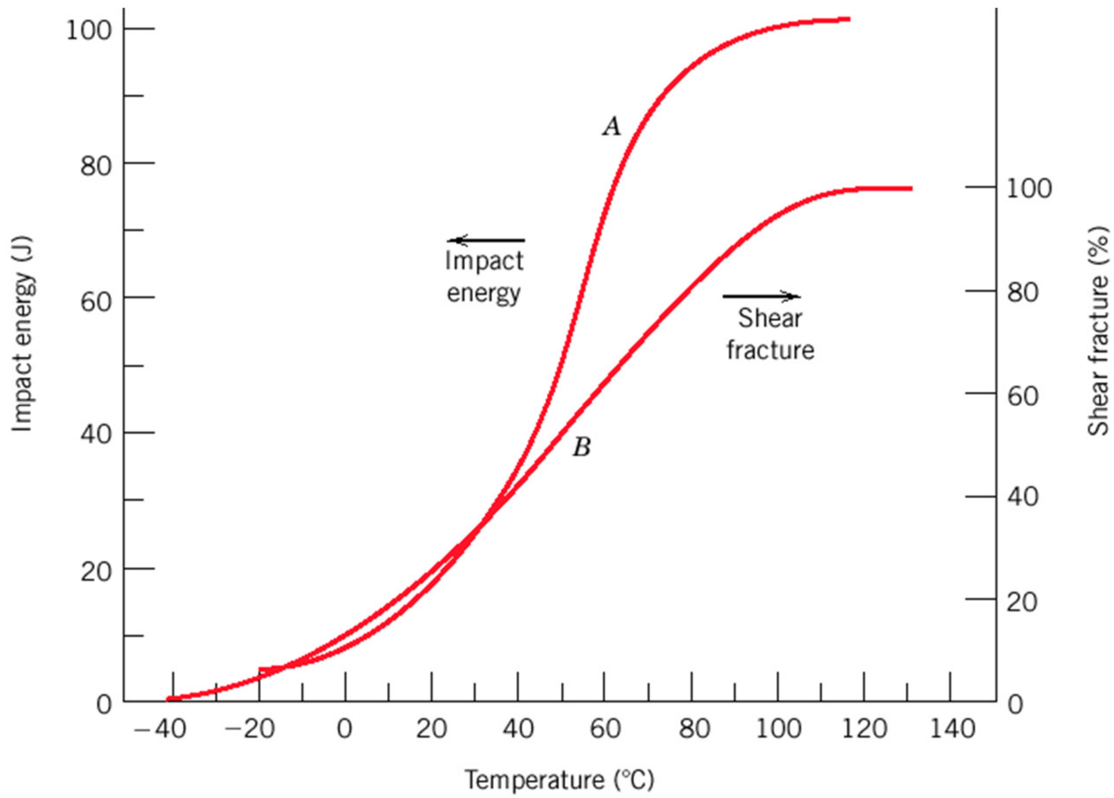
Stress concentration factor

$$\rightarrow \qquad K_t = \frac{\sigma_m}{\sigma_0} \approx 2\left(\frac{a}{\rho_t}\right)^{1/2}$$

Introduction to Fatigue

Ductile-to-Brittle Transition

- **As temperature decreases a ductile material can become brittle**



Introduction to Fatigue

Low temperatures can severely embrittle steels.

Ex. The Liberty ships. Produced during the WWII were the first all-welded ships. A significant number of ships failed by catastrophic fracture. Fatigue cracks nucleated at the corners of square hatches and propagated rapidly by brittle fracture.

Introduction to Fatigue

Factors that affect fatigue life :

Application; Environment; Loads: Types of Stresses; Material; confidence

- Magnitude of stress
- Quality of the surface

Solutions:

- **Polish surface**
- **Introduce compressive stresses (compensate for applied tensile stresses) into surface layer.**

  Shot Peening -- fire small shot into surface

  High-tech - ion implantation, laser peening.

- **Case Hardening:** Steel - create C- or N- rich outer layer by atomic diffusion from surface

  Harder outer layer introduces compressive stresses

- **Optimize geometry**

  Avoid internal corners, notches etc.

Environmental effects

- Thermal Fatigue. Thermal cycling causes expansion and contraction, hence thermal stress.
  Solutions:
  - change design!
  - use materials with low thermal expansion coefficients
- Corrosion fatigue. Chemical reactions induce pits which act as stress raisers. Corrosion also enhances crack propagation.
  Solutions:
  - decrease corrosiveness of medium
  - add protective surface coating
  - add residual compressive stresses

Design checking against fatigue

- Fatigue strength

    The quantitative relationship between the stress range and number of stress cycles to fatigue failure, used for the fatigue assessment of a particular category of structural detail

- Deratail category

    The numerical designation given to a particular detail for a given direction of stress fluctuation, in order to indicate which fatigue strength curve is applicable for the fatigue assessment (The detail category number indicates the reference fatigue strength $\Delta\sigma C$ in N/mm^2

- Constant amplitude fatigue limit

    The limiting direct or shear stress range value below which no fatigue damage will occur in tests under constant amplitude stress conditions. Under variable amplitude conditions all stress ranges have to be below this limit for no fatigue damage to occur

Design checking against fatigue

Cut-of-limit

Limit below which stress ranges of the design spectrum do not contribute to the calculated cumulative damamage

Endurance

The life to failure expressed in cycles, under the action of a constant amplitude stress history.

Reference fatigue strength

The life to failure expressed in cycles, under the action of a constant amplitude stress history

Introduction to Fatigue : Design according to EN 1993-1-9

(1) The fatigue strength for nominal stresses is represented by a series of (log $\Delta\sigma_R$) – (log N) curves and (log $\Delta\tau_R$) – (log N) curves (S-N-curves), which correspond to typical detail categories. Each detail category is designated by a number which represents, in N/mm², the reference value $\Delta\sigma_C$ and $\Delta\tau_C$ for the fatigue strength at 2 million cycles.

(2) For constant amplitude nominal stresses as shown in Figure 7.1 and Figure 7.2 fatigue strengths can be obtained as follows:

$$\Delta\sigma_R^m\, N_R = \Delta\sigma_C^m\, 2\times10^6 \quad \text{with } m = 3 \text{ for } N \le 5\times10^6 \text{, see Figure 7.1}$$

$$\Delta\tau_R^m\, N_R = \Delta\tau_C^m\, 2\times10^6 \quad \text{with } m = 5 \text{ for } N \le 10^8 \text{, see Figure 7.2}$$

$$\Delta\sigma_D = \left(\frac{2}{5}\right)^{1/3} \Delta\sigma_C = 0{,}737\Delta\sigma_C \quad \text{is the constant amplitude fatigue limit, see Figure 7.1, and}$$

$$\Delta\tau_L = \left(\frac{2}{100}\right)^{1/5} \Delta\tau_C = 0{,}457\Delta\tau_C \quad \text{is the cut off limit, see Figure 7.2.}$$

(3) For nominal stress spectra with stress ranges above and below the constant amplitude fatigue limit $\Delta\sigma_D$ the fatigue strength should be based on the extended fatigue strength curves as follows:

$$\Delta\sigma_R^m \, N_R = \Delta\sigma_C^m \, 2\times10^6 \quad \text{with } m = 3 \text{ for } N \le 5\times10^6$$

$$\Delta\sigma_R^m \, N_R = \Delta\sigma_D^m \, 5\times10^6 \quad \text{with } m = 5 \text{ for } 5\times10^6 \le N_R \le 10^8$$

$$\Delta\sigma_L = \left(\frac{5}{100}\right)^{1/5} \Delta\sigma_D = 0{,}549\Delta\sigma_D \quad \text{is the cut off limit, see Figure 7.1.}$$

Introduction to Fatigue : Design according to EN 1993-1-9

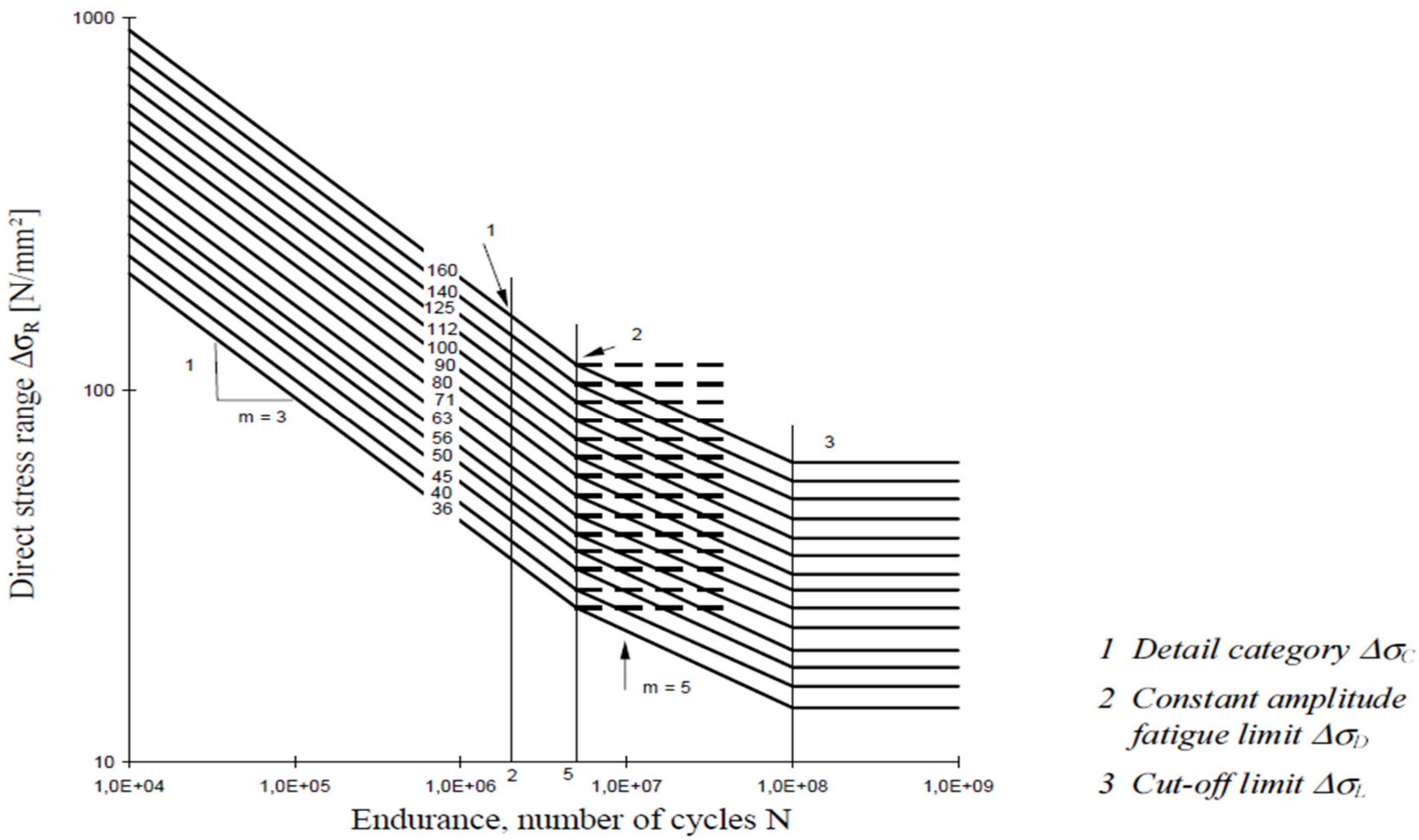


Figure 7.1: Fatigue strength curves for direct stress ranges

Introduction to Fatigue : design according to EN 1993-1-9

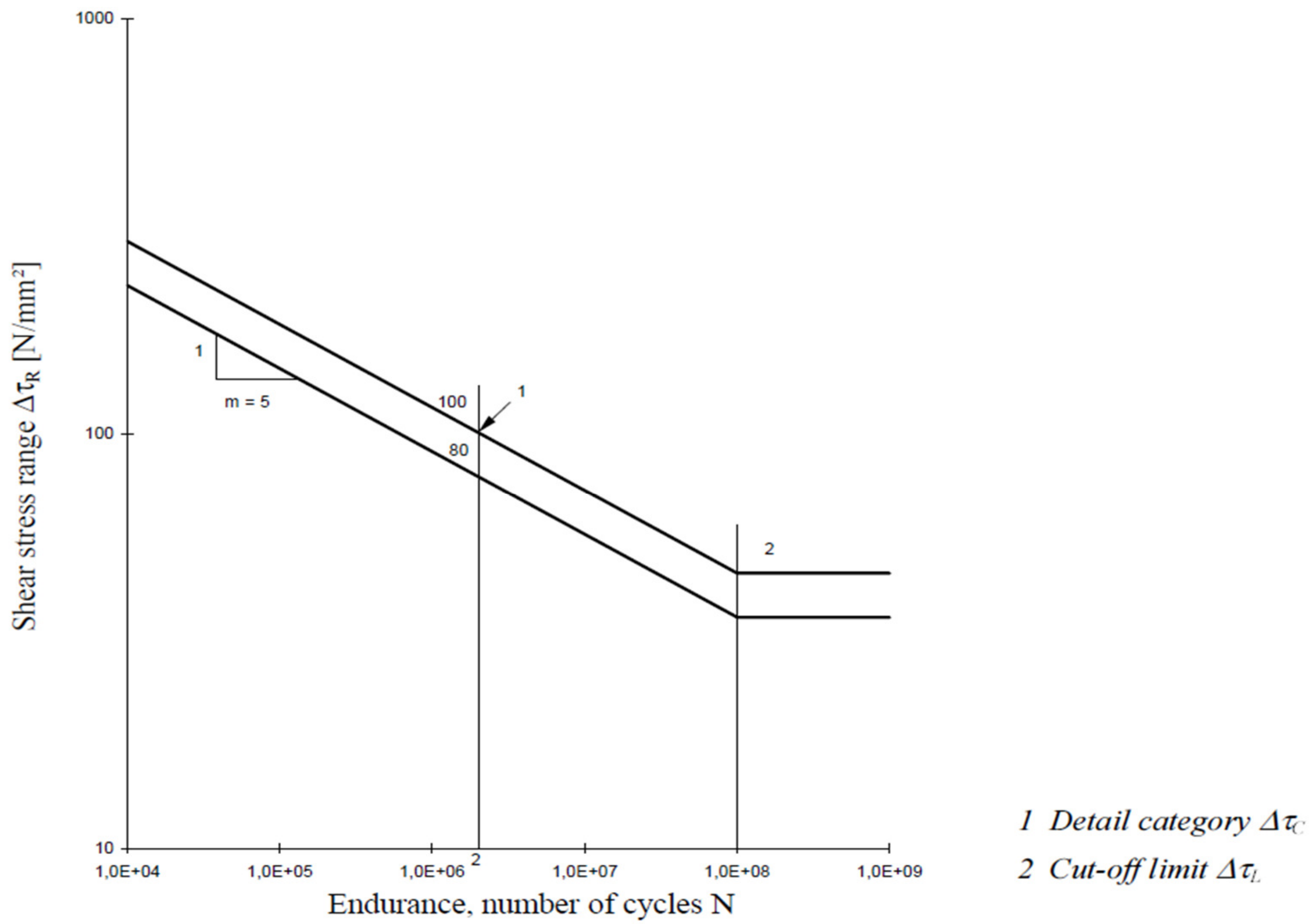


Figure 7.2: Fatigue strength curves for shear stress ranges

Introduction to Fatigue : Design Checking

(1) Nominal, modified nominal or geometric stress ranges due to frequent loads $\psi_1 Q_k$ (see EN 1990) shall not exceed

$$\begin{aligned} &\Delta\sigma \le 1{,}5\, f_y && \text{for direct stress ranges} \\ &\Delta\tau \le 1{,}5\, f_y / \sqrt{3} && \text{for shear stress ranges} \end{aligned} \tag{8.1}$$

(2) It shall be verified that under fatigue loading

$$\frac{\gamma_{Ff}\ \Delta\sigma_{E,2}}{\Delta\sigma_C / \gamma_{Mf}} \le 1{,}0$$

and (8.2)

$$\frac{\gamma_{Ff}\ \Delta\tau_{E,2}}{\Delta\tau_C / \gamma_{Mf}} \le 1{,}0$$

NOTE Table 8.1 to Table 8.9 require stress ranges to be based on principal stresses for some details.

(3) Unless otherwise stated in the fatigue strength categories in Table 8.8 and Table 8.9, in the case of combined stress ranges $\Delta\sigma_{E,2}$ and $\Delta\tau_{E,2}$ it shall be verified that:

$$\left(\frac{\gamma_{Ff}\ \Delta\sigma_{E,2}}{\Delta\sigma_C / \gamma_{Mf}}\right)^3 + \left(\frac{\gamma_{Ff}\ \Delta\tau_{E,2}}{\Delta\tau_C / \gamma_{Mf}}\right)^5 \le 1{,}0 \tag{8.3}$$

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$$\Delta\sigma_{Ed}(\gamma_{Ff}Q_k) = \sigma_{Ed,max}(\gamma_{Ff}Q_k) - \sigma_{Ed,min}(\gamma_{Ff}Q_k)$$

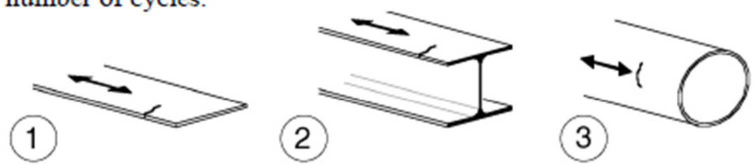
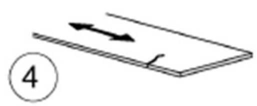
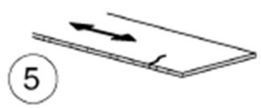
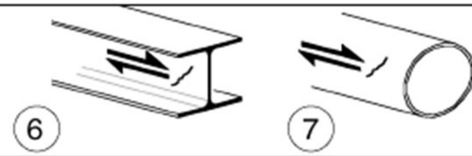
$$\Delta\tau_{Ed}(\gamma_{Ff}Q_k) = \tau_{Ed,max}(\gamma_{Ff}Q_k) - \tau_{Ed,min}(\gamma_{Ff}Q_k)$$

Table 3.1: Recommended values for partial factors for fatigue strength

Assessment method	Consequence of failure	
	Low consequence	High consequence
Damage tolerant	1,00	1,15
Safe life	1,15	1,35

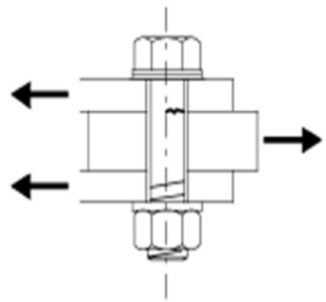
Introduction to Fatigue : Detail Categories (EN 1993-1-9)

Table 8.1: Plain members and mechanically fastened joints

Detail category	Constructional detail	Description	Requirements
160	NOTE The fatigue strength curve associated with category 160 is the highest. No detail can reach a better fatigue strength at any number of cycles. 	<u>Rolled and extruded products:</u> 1) Plates and flats; 2) Rolled sections; 3) Seamless hollow sections, either rectangular or circular.	<u>Details 1) to 3):</u> Sharp edges, surface and rolling flaws to be improved by grinding until removed and smooth transition achieved.
140		<u>Sheared or gas cut plates:</u> 4) Machine gas cut or sheared material with subsequent dressing. 5) Material with machine gas cut edges having shallow and regular drag lines or manual gas cut material, subsequently dressed to remove all edge discontinuities. Machine gas cut with cut quality according to EN 1090.	4) All visible signs of edge discontinuities to be removed. The cut areas are to be machined or ground and all burrs to be removed. Any machinery scratches for example from grinding operations, can only be parallel to the stresses. <u>Details 4) and 5):</u> - Re-entrant corners to be improved by grinding (slope ≤ ¼) or evaluated using the appropriate stress concentration factors. - No repair by weld refill.
125			
100 m = 5		6) and 7) Rolled and extruded products as in details 1), 2), 3)	<u>Details 6) and 7):</u> Δτ calculated from: $\tau = \frac{V\,S(t)}{I\,t}$

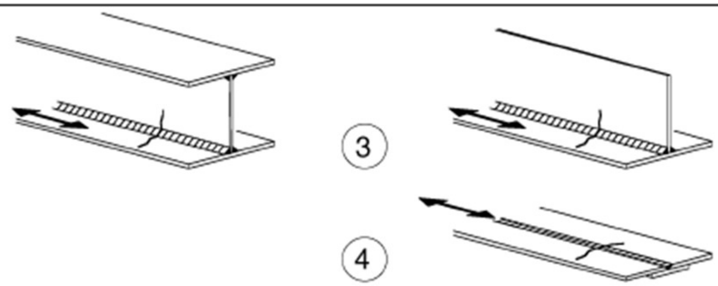
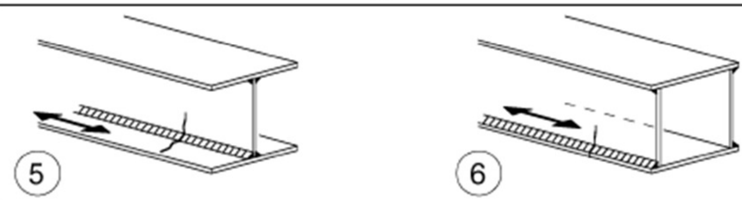
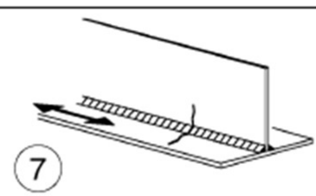
Introduction to Fatigue : Detail Categories (EN 1993-1-9)

Table 8.1 (continued): Plain members and mechanically fastened joints

Detail category	Constructional detail	Description	Requirements
100 m=5	15 	Bolts in single or double shear Thread not in the shear plane 15) -Fitted bolts -normal bolts without load reversal (bolts of grade 5.6, 8.8 or 10.9)	15) $\Delta\tau$ calculated on the shank area of the bolt.

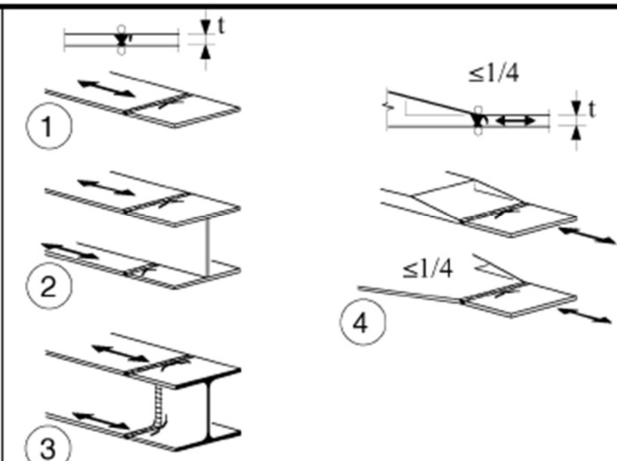
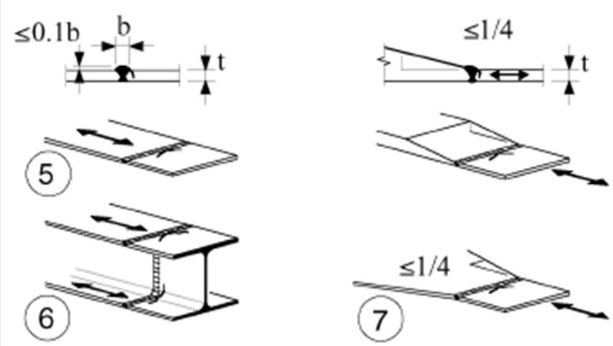
Introduction to Fatigue : Detail Categories (EN 1993-1-9)

Table 8.2: Welded built-up sections

Detail category	Constructional detail	Description	Requirements
125		Continuous longitudinal welds: 1) Automatic butt welds carried out from both sides. 2) Automatic fillet welds. Cover plate ends to be checked using detail 6) or 7) in Table 8.5.	Details 1) and 2): No stop/start position is permitted except when the repair is performed by a specialist and inspection is carried out to verify the proper execution of the repair.
112		3) Automatic fillet or butt weld carried out from both sides but containing stop/start positions. 4) Automatic butt welds made from one side only, with a continuous backing bar, but without stop/start positions.	4) When this detail contains stop/start positions category 100 to be used.
100		5) Manual fillet or butt weld. 6) Manual or automatic butt welds carried out from one side only, particularly for box girders	5), 6) A very good fit between the flange and web plates is essential. The web edge to be prepared such that the root face is adequate for the achievement of regular root penetration without break-out.
100		7) Repaired automatic or manual fillet or butt welds for categories 1) to 6).	7) Improvement by grinding performed by specialist to remove all visible signs and adequate verification can restore the original category.

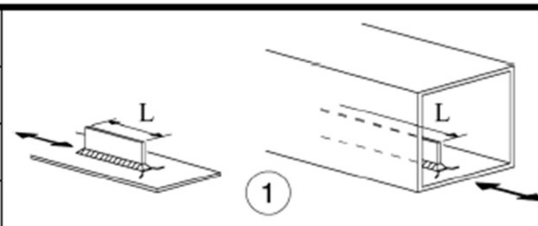
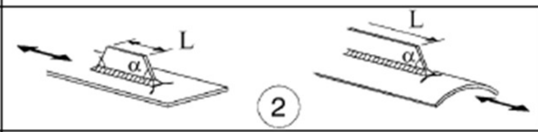
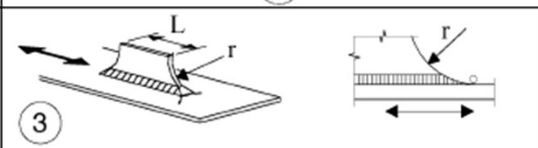
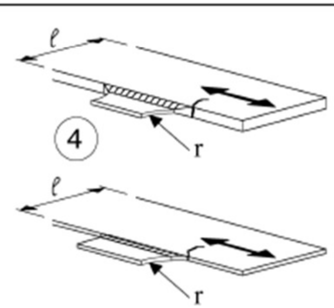
Introduction to Fatigue : Detail Categories (EN 1993-1-9)

Table 8.3: Transverse butt welds

Detail category	Constructional detail		Description	Requirements
112	size effect for t>25mm: $k_s=(25/t)^{0,2}$		<u>Without backing bar:</u> 1) Transverse splices in plates and flats. 2) Flange and web splices in plate girders before assembly. 3) Full cross-section butt welds of rolled sections without cope holes. 4) Transverse splices in plates or flats tapered in width or in thickness, with a slope ≤ ¼.	- All welds ground flush to plate surface parallel to direction of the arrow. - Weld run-on and run-off pieces to be used and subsequently removed, plate edges to be ground flush in direction of stress. - Welded from both sides; checked by NDT. <u>Detail 3):</u> Applies only to joints of rolled sections, cut and rewelded.
90	size effect for t>25mm: $k_s=(25/t)^{0,2}$		5) Transverse splices in plates or flats. 6) Full cross-section butt welds of rolled sections without cope holes. 7) Transverse splices in plates or flats tapered in width or in thickness with a slope ≤ ¼. Translation of welds to be machined notch free.	- The height of the weld convexity to be not greater than 10% of the weld width, with smooth transition to the plate surface. - Weld run-on and run-off pieces to be used and subsequently removed, plate edges to be ground flush in direction of stress. - Welded from both sides; checked by NDT. <u>Details 5 and 7:</u> Welds made in flat position.

Introduction to Fatigue : Detail Categories (EN 1993-1-9)

Table 8.4: Weld attachments and stiffeners

Detail category	Constructional detail		Description	Requirements
80	$L \leq 50mm$		Longitudinal attachments: 1) The detail category varies according to the length of the attachment L.	The thickness of the attachment must be less than its height. If not see Table 8.5, details 5 or 6.
71	$50 < L \leq 80mm$			
63	$80 < L \leq 100mm$			
56	$L > 100mm$			
71	$L > 100mm$ $\alpha < 45°$		2) Longitudinal attachments to plate or tube.	
80	$r > 150mm$	 reinforced	3) Longitudinal fillet welded gusset with radius transition to plate or tube; end of fillet weld reinforced (full penetration); length of reinforced weld > r.	Details 3) and 4): Smooth transition radius r formed by initially machining or gas cutting the gusset plate before welding, then subsequently grinding the weld area parallel to the direction of the arrow so that the transverse weld toe is fully removed.
90	$\frac{r}{L} \geq \frac{1}{3}$ or $r > 150mm$	 L: attachment length as above	4) Gusset plate, welded to the edge of a plate or beam flange.	
71	$\frac{1}{6} \leq \frac{r}{L} \leq \frac{1}{3}$			
50	$\frac{r}{L} < \frac{1}{6}$			