SHEAR STRENGTH

8.1 INTRODUCTION

Soil is a *particulate* material composed of discrete particles which are relatively free to move with respect to one another. The *mineral skeleton* of soil is usually quite deformable due to interparticle sliding and rearrangement, even though the individual particles are very rigid.

Thus, when a compression load is gradually applied to a soil mass, it will eventually fail due to the movement of the individual soil particles relative to one another which occur along surfaces known as *shear surfaces. The maximum resisting stress offered by the soil particles to the deformation due to relative sliding of the particles immediately prior to failure of the soil mass is called as shearing strength of the soil mass.*

Thus, the structural strength of soil is primarily a function of its shear strength and as such the ability of a soil mass to support structural load is dependent on its shearing strength. Specially the knowledge of shear strength is essential for:

(i) Evaluation of bearing capacity (Chapter-11) used for the design of foundations.

(ii) Analysis of stability of slopes used for the design of embankments for dams, levees, roads, temporary/permanent excavations, etc. (Chapter-13), and

(iii) Estimation of lateral earth pressure required in the design of earth retaining structures such as retaining walls, bulk heads, sheetpile cofferdams, underground structures, etc. (Chapter-12).

The shear strength of a soil mass is essentially made up of:

(a) the structural resistance to movement of soil particles due to interlocking of the grains (i.e. density of the soil)

(b) the frictional resistance to sliding between the individual soil grains at their contact points (i.e. angle of internal friction, ϕ of the soil), and

(c) cohesion (adhesion) between surfaces of the soil grains. The cohesion (c) is the resistance due to the forces tending to hold the grains together in a soil mass.

Generally speaking, coarse-grained soils (sands, gravels and their mixtures) derive their shear strength almost entirely from internal friction (ϕ). On the other hand, fine-grained soils (clays, silts and their mixtures) have cohesion (c) as their major component of shear strength. Usually most of the natural soils are mixture of fine-grained and coarse-grained soils and as such their shear strength is dependent on both c and ϕ parameters.

It is therefore convenient to consider the following three conventional soil types for study of shear strength:

(i) Coarse-grained, frictional soil or cohesionless soils ($c = 0$ soils)

(ii) Fine-grained or cohesive soils ($\phi = 0$ soils)

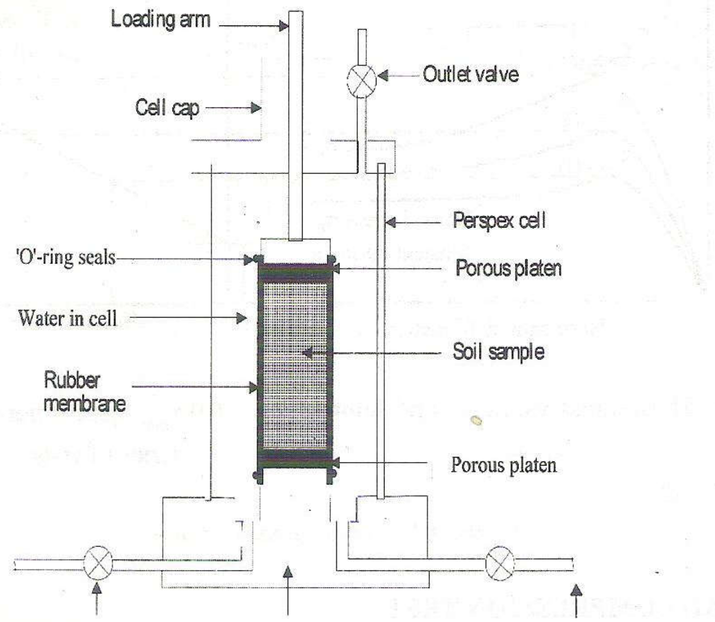
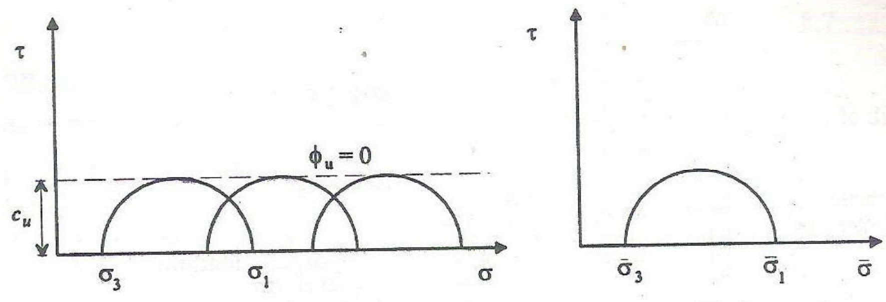


Figure 8.8 *Triaxial cell apparatus*

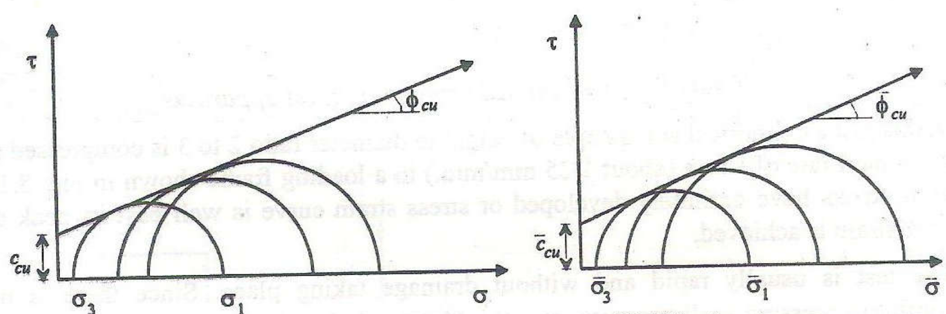
Comparison of direct shear and triaxial tests.

Direct shear	Triaxial
(i) Soil sample is made to fail along a pre-determined plane which may not be the weakest plane.	Sample is free to fail along the weakest plane. The failure plane is not pre-determined.
(ii) There is little control over drainage conditions. Arrangement for pore pressure measurement are not provided.	There is proper control over drainage conditions. Arrangement for measurement of pore pressure is provided.
(iii) Undrained test on sand cannot be performed properly and the results are not reliable.	Any type of test can be performed on any soil type.
(iv) There is unequal distribution of shear stress over the sliding plane	The stress distribution is relatively uniform.
(v) Effective stress cannot be computed.	Effective stress at various stages can be computed.



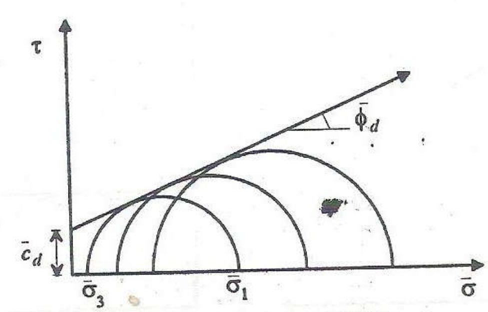
(i) Total stresses (ii) Effective stresses

(a) Unsonsolidated-undrained (UU) test on saturated soil



(i) Total stresses (ii) Effective stresses

(b) Consolidated-undrained (CU) test



(c) Consolidated-drained (CD) test

Figure 8.9 *Triaxial tests for various drainage conditions (see section* 8.10*)*

8.7 UNCONFINED COMPRESSION TEST (UCT)

This test actually is a special form of triaxial test where the confining cell pressure is kept zero during the test. Thus, the cylindrical soil sample is sheared to failure without applying any lateral pressure like a concrete cylinder crushing test. Although the test can be done in laboratory using triaxial apparatus, it is more usual to use a much simple portable piece of equipment known as unconfined compression test apparatus as shown in Fig 8.10. Due to its portable nature the equipment can be shifted to the site and UCT is generally done right in the field.

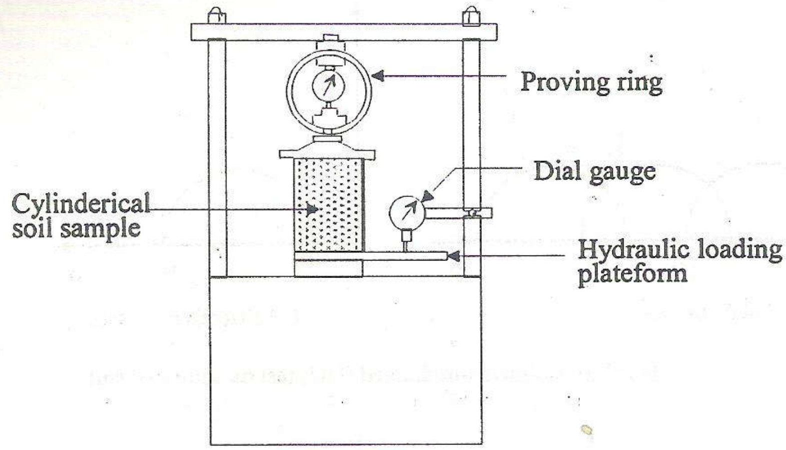
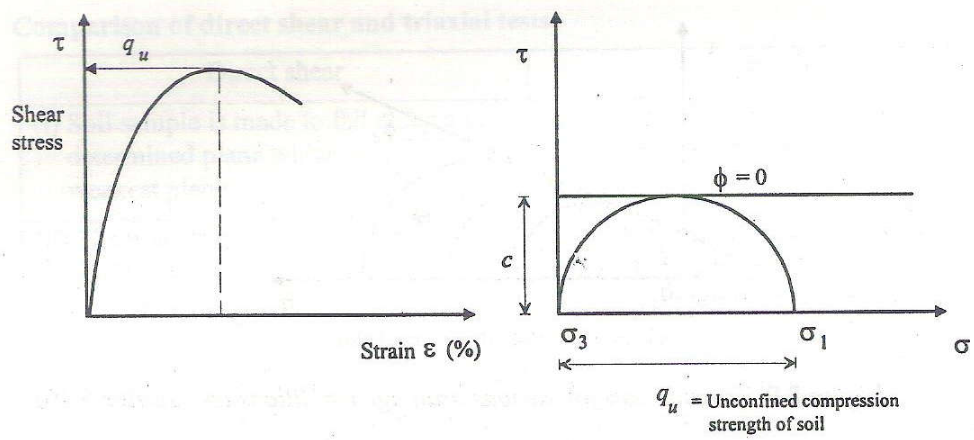


Figure 8.10 *Unconfined compression test apparatus*

In this test a cylindrical soil sample of height to diameter ratio 2 to 3 is compressed at a constant rate of strain (about 1.25 mm/min.) in a loading frame shown in Fig. 8.10 until cracks have definitely developed or stress strain curve is well past its peak or 15% strain is achieved.

The test is usually rapid and without drainage taking place. Since there is no confining pressure (cell pressure, σ_3), the Mohr's circle of stress passes through the origin as shown in Fig.8.11(ii).



(i) Stress-strain plot (ii) Mohr circle of stresses

Figure 8.11 *Unconfined compression test*

Thus $c = q_u/2$ 8.6

Since in this test it is assumed that the volume of sample remains constant, the cross-sectional area of the sample at any stage of test, A is given by:

$$A = \frac{A_o}{1-\varepsilon} \qquad 8.7$$

Where,

A_o = initial x-sectional area of the test sample = $\frac{\pi}{4}D^2$ (D being the sample dia.)

ε = axial strain at any stage of test, $\Delta L/L_o$

Where,

ΔL = change in sample length i.e. vertical deformation during the test at any stage of test.

L_o = initial sample length.

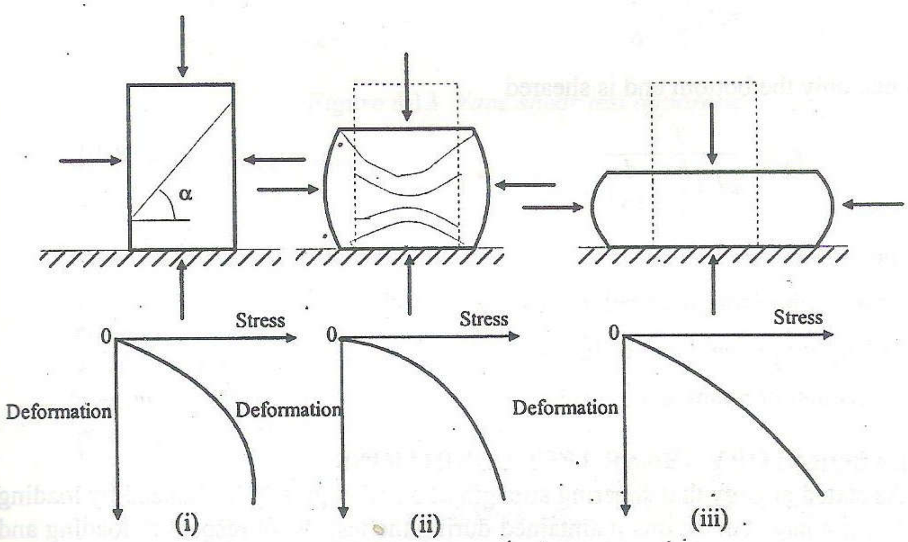
Let,

P = compression load at failure,

$q_u = P/A$ = unconfined compression strength 8.8

8.8 MODES OF FAILURE IN TRIAXIAL TEST AND UNCONFINED COMPRESSION TEST

Typical failure modes of triaxial/unconfined compression test sample are depicted in Fig. 8.12.



(a) Brittle failure with clearly defined failure plane. Common in OCC and dense sand.

(b) Intermediate failure with shear cones and bulging. Common for medium dense sand and lightly OCC.

(c) Plastic failure with considerable bulging. Common for loose sand and NCC.

Figure 8.12 *Typical failure modes of compression tests*

8.9 LABORATORY VANE SHEAR TEST (LVST)

This is also a rapid test, used either in field or laboratory to determine the undrained shear strength of soft cohesive sensitive clays. Sensitive clays are those soft clays which loose part of their shear strength when disturbed.

In this test a cruciform vane shown in Fig. 8.13 is used. A torque is applied to the shaft of the vane until failure occurs due to shearing on the cylinder of diameter *d*, and height, *h*. Vane blades are pushed into the soil and rotated at a constant rate of 1° per minute by a worm gear and wheel arrangement.

The undrained shear strength of clay (c_u) is computed using:

$$\text{Torque} = T = c_v(\pi dh)\frac{d}{2} + c_h(\pi\frac{d^2}{4})\frac{1}{3}d \times 2$$

(sides of cylinder) (ends of cylinder)

if $c_v = c_h$

$$c_u = \frac{T}{\pi d^2(\frac{h}{2}+\frac{d}{6})} \qquad 8.9$$

When only the bottom end is sheared

$$c_u = \frac{T}{\pi d^2(\frac{h}{2}+\frac{d}{12})} \qquad 8.10$$

Where,

T = maximum torque at failure in kg-cm or ft-lb

h = height of vane in cm or ft.

d = diameter of vane in cm or ft.

8.10 LABORATORY SHEAR TEST CONDITIONS

As stated already that shearing strength of a soil is greatly influenced by loading and drainage conditions maintained during the test. With respect to loading and drainage conditions, the laboratory tests can be divided into the following three main categories:

(a) Unconsolidated-Undrained (UU Test) or *Quick Shear Test.*

(b) Consolidated-Undrained (CU Test) or *Consolidated Quick Test.*

(c) Consolidated-Drained (CD Test) or *slow Test.*

Mohr's stress circles for these three test conditions have been presented in Fig. 8.9 and a brief description is given in the succeeding sections.

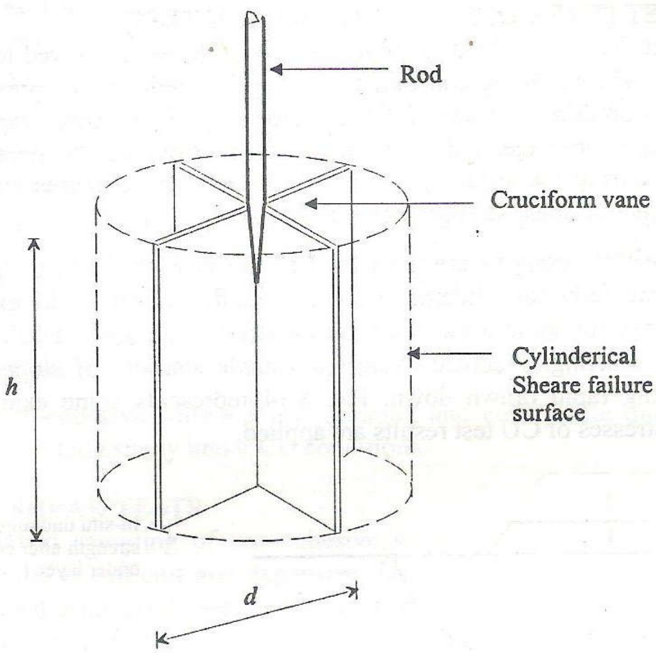


Figure 8.13 *Vane shear test apparatus*

• UU-Test (Quick Test)

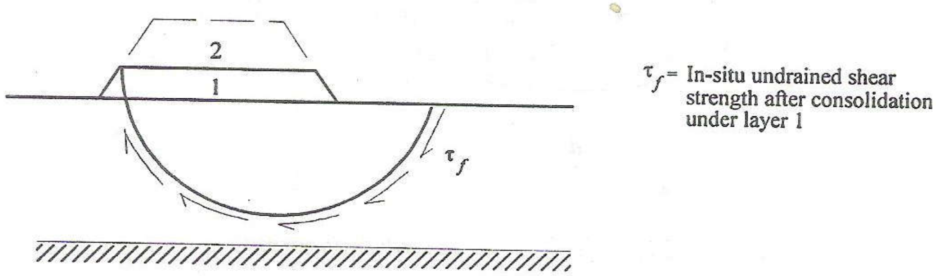
In this test no drainage is allowed as the testing proceeds to failure. Soil sample is sheared immediately after application of the normal load and no time for sample consolidation is allowed either before or during the shear test. To ensure that during testing the void ratio of the soil sample would change as little as possible, the shearing force is applied rapidly. The entire test is completed within a period of about 5 to 10 minutes and pore pressures are usually not measured. Hence the test yields total stress shear parameters (c_u, ϕ_u); but in principle, it is possible to measure pore pressure in UU tests.

In engineering practice, we mostly have to deal with a relatively quick shear loading where the excess pore pressure has no time to dissipate, or there is no time to adjust or equalize the pore pressure. Under such conditions the resistance of an earth mass to sliding under certain conditions result in smaller values than those obtained from tests. Therefore, the most unfavourable conditions are to be considered, the sudden loading of a soil mass till failure. Accordingly the shear test in the laboratory is to be performed quickly. Typical examples of the use of this test are the determination of shear strength in temporary excavations; calculation of bearing capacity of cohesive soils used in the design of foundations; and slope stability analysis of earth dams during construction.

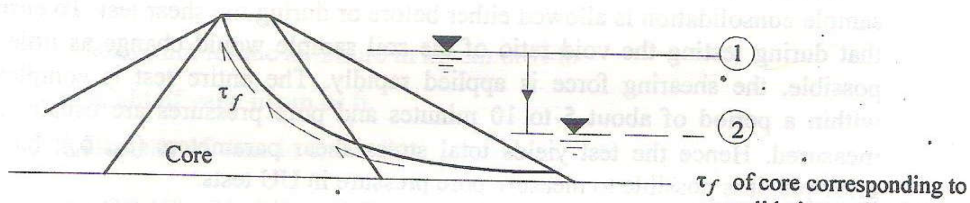
• CU-TEST (THE CONSOLIDATED-QUICK TEST)

In this test the normal load is applied and the sample is allowed to consolidate allowing drainage during consolidation process to reduce pore pressure to zero. Once this condition is reached, the deviator stress is increased rapidly without allowing any drainage, until the sample fails. Provided the pore pressure is measured during the shearing phase, the results can be expressed in terms of total or effective shear strength parameters (i.e. $\bar{c}_{cu}$ & $\bar{\phi}_{cu}$).

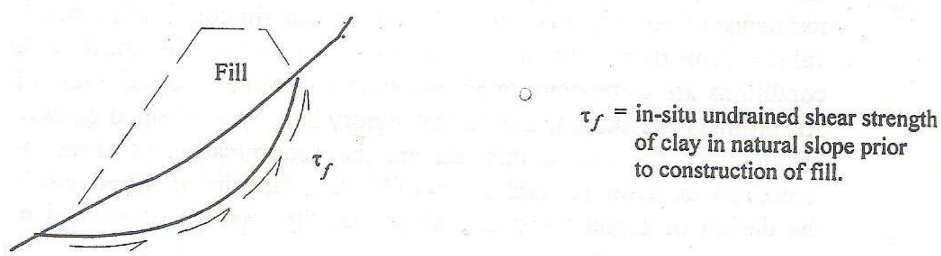
In practice CU strengths are used for stability problems where the soils have first become fully consolidated and are at equilibrium with the existing stress system. Then for some reasons *additional* stresses are applied quickly, with no drainage occurring. Practical examples include stability of slopes of earthen dams during rapid drawn down. Fig. 8.14 represents some examples where effective stresses of CU test results are applied.



(a) Embankment raised (2) subsequent to consolidation under its original height (1).



(b) Rapid drawdown behind an earth dam. No drainge of the core. Reservoir level falls from 1 to 2.



(c) Rapid construction of an embankment on a natural slope

Figure 8.14 *Some examples of CU analysis for clays (after Ladd, 1971)*

• **CD-Test (Slow Test)**

In CD test, soil consolidates under normal load and the drainage is allowed during consolidation. On completion of consolidation, drainage is allowed while the normal stress is increased at a rate such that no pore pressure can develop. The resulting shear strength parameters are in terms of effective stresses only (i.e. $\bar{c}_d$ & $\bar{\phi}_d$).

In practice, CD parameters are used in long term stability problems of clayey soil slopes and the long term lateral pressures on walls retaining cohesive soils. Examples of CD test use are given in Fig. 8.15.

Slow test on clays may take 4 to 6 weeks to complete and this test is usually used in research. This test is not very popular for clays:

Since non-cohesive soils are free draining and consolidate quickly, therefore these are tested usually under CD conditions.

8.11 FIELD SHEAR TESTS

Undisturbed sampling of non-cohesive soils and soft sensitive clays, if not impossible, is difficult and expensive. Usually shear strength of such soils is determined using field shear tests. Now-a-days numerous varieties of field shear tests are conducted for this purpose. Several routine tests are:-

(i) Standard Penetration Test, SPT (ASTM D1586).

This test provides fairly good estimate of shear strength of non-cohesive soils.

(ii) Cone Penetration Test, CPT (ASTM D3441)

Used for soft clays and loose to medium dense sands.

(iii) Field Vane Shear Test, FVST (ASTM D2573)

(iv) Pressuremeter Test, PMT (Menard 1956)

Used for variety of soils and rocks.

(v) Dilatometer Test, DMT (ASTM D 6635-01, 2001)

Brief descriptions of these tests have been given in Chapter-15 of this book.

8.12 SHEAR STRENGTH OF SANDS AND CLAYS

• **Sands**

Due to relatively large particle size (less surface area), non-cohesive soils have little or no cohesion. Hence, their shear strength is mainly due to frictional resistance between the particles including sliding and rolling friction as wall as interlocking of the grains. Thus, the major contributing parameter towards the shear strength of granular soils such as sand is the internal friction angle, (ϕ) and the cohesion, $c = 0$

The most critical condition with regard to shear strength of non-cohesive soils occurs at construction stage or upon application of load. Any water contained in the voids during these stages will be drained out almost immediately due to high permeability of these soils. As a result, the shear strength parameters are governed by effective stress conditions. Thus, the shear strength of non-cohesive soils will remains more or less the same throughout the life of the structure.

The initial packing density of non-cohesive soils controls the strength and deformation behaviour during shearing. A dense sand tends to expand (dilates) during shear while a loose sand will decrease in volume. Dense sands have higher shear strength than that of loose sands.

Fig. 8.16 represents typical stress-strain curves for loose and dense sands of CD tests performed using triaxial apparatus.

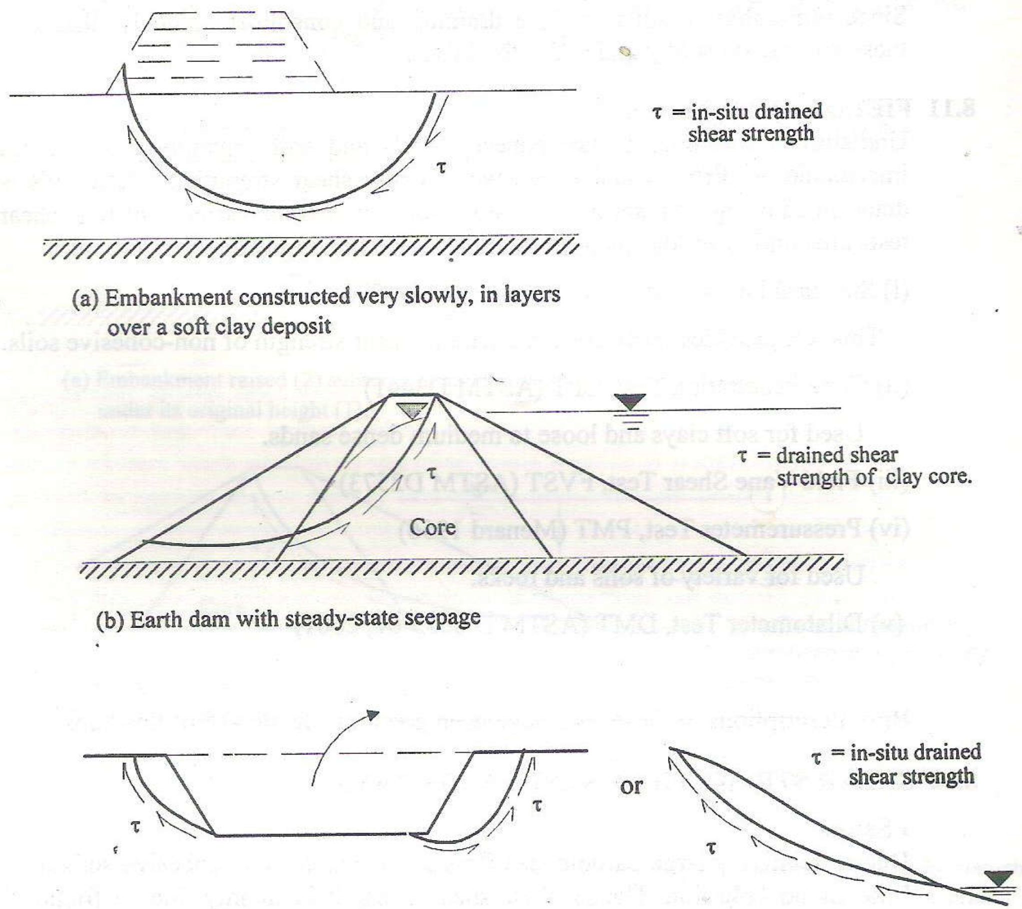


Figure 8.15 Some examples of CD analyses for clays (after Ladd, 1971)

Table 8.1 presents a list of some typical field problems with appropriate shear strength parameter to be used.

Table 8.1 Typical field problems and appropriate shear strength parameters

Type of construction	Load	Critical time	Analysis and comment
Foundation on saturated clay and passive earth pressures on retaining walls	Positive	End of construction	Total stress (c_u, $\phi_u = 0$), analysis gives reliable solutions.
Earth dam construction, Embankment fill, May involve construction with several loading periods.	Positive	End of construction	Effective stress ($\bar{c}$, $\bar{\phi}$). Stage construction will give some dissipation of excess pore-water pressure. Hence, measure pore-water pressure over field range to check the factor of safety for each stage.
Earth pressures on walls backfilled with partially saturated materials ($\phi_u \neq 0$)	Positive	End of construction or long-term	Total stress, (c_u, $\phi_u = 0$). Effective stress ($\bar{c}$, $\bar{\phi}$). Check any seepage pore-water pressures.
Active pressure on driven and the dredged sheet pile walls	Negative	Usually long-term but possibly during construction also.	Effective stress ($\bar{c}$, $\bar{\phi}$) Pore-water pressure may develop from water table behind the wall.
Permanent cuts and stability of natural slopes	Negative	Long-term	Effective stress ($\bar{c}$, $\bar{\phi}$) or preferably (c_d, ϕ_d). Pore-water pressures from steady seepage or static condition may develop. $\bar{c}$ is often not reliable and taken as zero. With some fissured over-consolidated clays the remoulded parameters. $c_r = 0$, ϕ_r should be used.
Temporary excavations, Slope stability, Base heave of intact clays.	Negative	During construction	Total stress (c_u, $\phi_u = 0$), c_u preferably measured using unloading triaxial test.
Temporary excavations, Slope stability, Base heave of non-intact clays.	Negative	During construction	Effective stress ($\bar{c}$, $\bar{\phi}$). Quick drainage of non-intact materials makes an undrained analysis unreliable. Often requires pore-water pressures to be estimated which may prove to be difficult.

Note: (i) Positive load is the structural load added due to construction.
(ii) Negative load is load removed during excavation.

Shear failure may be defined at:

(1) maximum principal stress difference, $(\sigma_1 - \sigma_3)_{max}$

(2) maximum principal effective stress ratio, $\left(\frac{\bar{\sigma}_1}{\bar{\sigma}_3}\right)_{max}$

(3) $\tau = \frac{(\sigma_1 - \sigma_3)}{2}$ at a prescribed strain.

Usually failure is defined as the *maximum principal stress difference*, which is the same as the compression strength of the sample.

(iii) Cohesive-frictional soils (c-ϕ soils)

Shear strength of a soil mass is greatly influenced by loading and drainage conditions during the loading. A number of different field and laboratory tests are used to evaluate the shear strength of soils. When used under comparable conditions these tests should give similar results.

This chapter is deputed for the study of shear strength.

8.2 COULOMB'S LAW OF SHEAR STRENGTH (1773)

As stated in the preceding section, the shear strength of a soil is made up of two major components — friction (ϕ) and cohesion (c).

The intergranular friction (ϕ) is directly proportional to the normal stress acting on shear surface. The cohesion (c) is dependent on the type, size, packing of grains and on the suction properties of the soil.

Coulomb (1773) proposed that the shearing strength of a soil, τ is governed by the straight line equation:

$$\tau = c + \sigma \tan\phi \qquad 8.1$$

Where,

c = apparent cohesion

σ = normal stress

ϕ = angle of internal friction or shearing resistance of the soil.

In 1773, when coulomb put forward equation 8.1, the concept of effective stress was not introduced. Following the introduction of this principle by Terzaghi, the equation 8.1 is now expressed in terms of effective stresses, thus:

$$\bar{\tau} = \bar{c} + \bar{\sigma} \tan\bar{\phi} \qquad 8.2$$

Where,

$\bar{c}$ = apparent cohesion w.r.t. effective stress

$\bar{\sigma}$ = $\sigma - u$, the effective normal stress (σ being the total stress and u the pore water pressure)

$\bar{\phi}$ = effective angle of internal friction.

The modified Coulomb's equation is diagrammatically shown in Fig. 8.1.

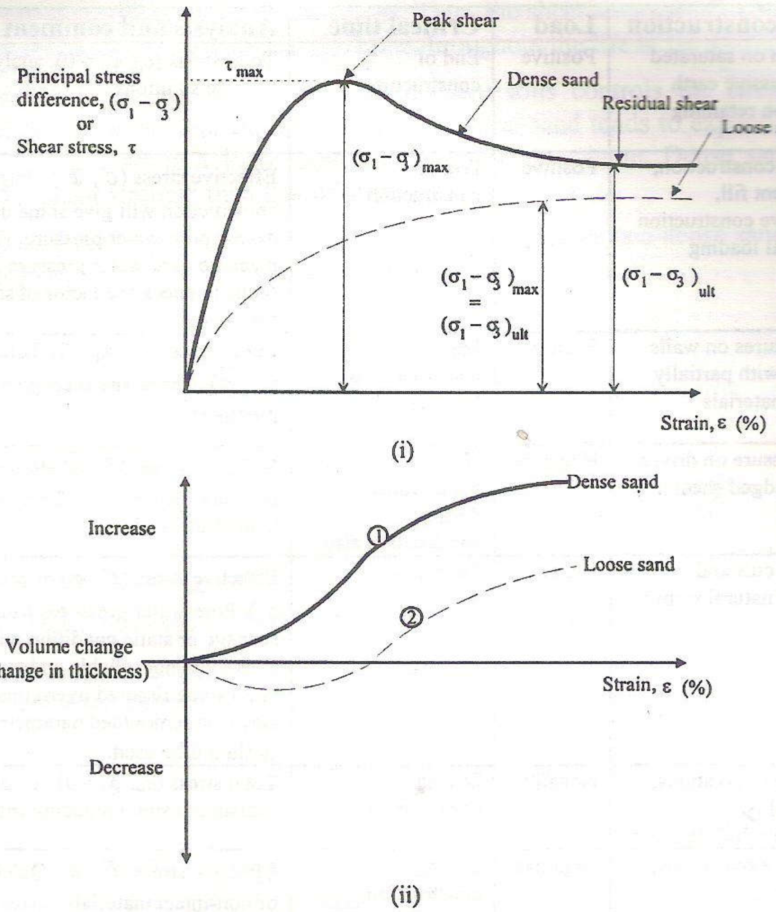


Figure 8.16 (i) *Shear strength-strain plots for sands* (ii) *Volume change-strain of sands*

When the *loose* sand is sheared, the principal stress difference $(\sigma_1-\sigma_3)$ gradually increases to a maximum or ultimate value, $(\sigma_1-\sigma_3)_{ult}$ as shown in Fig. 8.16 (i). For dense sand, on the contrary, $(\sigma_1-\sigma_3)$ increases to a peak or maximum value of $(\sigma_1-\sigma_3)_{max}$ after which it decreases to a value very close to $(\sigma_1-\sigma_3)_{ult}$ for the loose sand. Herochfeld (1963) pointed out that theoretically values of $(\sigma_1-\sigma_3)_{ult}$ for both the sands (loose and dense sands) should be the same. The difference is due to difficulties in precise measurement of ultimate stresses and due to non-uniform distribution of stresses in the test specimens.

Furthermore, during the process of shearing of dense sands, volume of the sample increases (i.e. the sand dilates) under the influence of shear strain (see Fig 8.16, ii). On the contrary, volume of loose sand decreases in the initial stages and then increases afterward.

At some intermediate state of density between loose and dense state, the shear strains do not bring about any change in volume, viz. density. *The density of sand at which no change in volume is brought about upon application of* *shearing strains is called the critical density. The porosity and void ratio corresponding to the critical density are called the critical porosity and the critical void ratio, respectively.*

Casagrande (1963) defined the critical void ratio as *the ultimate void ratio at which continuous deformation occurs with no change in principal stresses difference.*

The latest research indicates that volume change during shearing of sands is greatly dependent on void ratio and confining pressure (cell pressure, σ_3) during triaxial test. It is also evident that the shear strength of sands increases with increasing cell pressure σ_3.

FACTORS AFFECTING SHEAR STRENGTH OF SANDS

Following factors affect the shear strength of sands:

(i) Void ratio or relative density
(ii) Grain size and grain size distribution (*GSD*)
(iii) Grain surface roughness
(iv) Presence of water
(v) Intermediate principal stress, σ_2
(vi) Over consolidation or prestress.

Table 8.2 represents the summary of these factors.

Table 8.2 Summary of factors affecting ϕ

Factor	Effect
Void ratio, e	$e\uparrow$, $\phi\downarrow$
Angularity, A	$A\uparrow$, $\phi\uparrow$
Grain size distribution	$c_u\uparrow$, $\phi\uparrow$
Particle size, S	No effect (with constant e)
Surface roughness, R	$R\uparrow$, $\phi\uparrow$
Water, W	$W\uparrow$, $\phi\downarrow$ slightly
Intermediate principal stress	$\phi_{ps} \geq \phi_{tx}$
Over-consolidation or prestress	Little effect

The angle of internal friction values tabulated as in Table 8.3 or values from in-situ testing as in Table 15.6 are commonly used for both preliminary and final design studies (Bowles, 1996).

• Clays

Shear strength of clays is greatly influenced by drainage conditions during the testing and history of deposition of the clays i.e. over consolidated or normally consolidated clays.

For study of shear strength of clays, we shall divide clays into:

(a) Normally consolidated clays (*NCC*).
(b) Over consolidated clays (*OCC*)
(c) Sensitive clays (*SC*)

- Normally Consolidated Clays

A CD-test performed on *NCC* yields zero cohesion together with an angle of friction, ϕ and thus behaves as if it were a granular material.

A CU-test with pore pressure measurements gives a similar effective stress strength envelope.

A UU-test on the same unsaturated material with pore-pressure measurements will give strength which is considerably less than that obtained from a drained test since the effective stress is lower due to pore pressures. The resulting strength envelope is parallel to the normal stress axis indicating that the material possesses only cohesion (Fig. 8.9, i).

- Over consolidated clays

Over consolidated clays have higher shear strength than normally consolidated clays of similar composition. A *NCC* will undergo further consolidation whilst an *OCC* will expand during shear. Thus, when tested under undrained conditions they will develop negative pore pressure. This will lead to an increase in the effective stress with consequent increase in shear strength.

- Sensitive Clays

Marine or lake clays and organic silts with high water content may have no measurable remoulded strength. Disturbance during sampling may cause considerable decrease in shear strength of these soils and may convert the deposit into a viscous fluid. These soils are called as sensitive or quick clays.

The ratio of the undisturbed shear strength of a cohesive soil to the remoulded strength at the same water content is defined as the *sensitivity*, S_t.

$$S_t = \frac{\text{undisturbed strength}}{\text{remoulded strength}} \qquad 8.11$$

Clays may be classified as:

Soil type with respect to sensitivity	Degree of sensitivity S_t	Comments
Insensitive	$4 \le$	Majority of clays
Sensitive	$4 < S_t \le 8$	
Extra sensitive	> 8	Quick clays when $S_t > 16$

Thixotrophy is the regain of strength from remoulded state with time. It is the property of quick clays going from solution to gel to solution on agitation.

Table 8.3 Representative values for angle of internal friction ϕ (Bowles, 1996)

Soil	Type of test		
	Unconsolidated-undrained (UU)	Consolidated undrained (CU)	Consolidated drained (CD)
Gravel			
Medium size	40–55°		40–55°
Sandy	35–50°		35–50°
Sand			
Loose dry	28–34°		
Loose saturated	28–34°		
Dense dry	35–46°		43–50°
Dense saturated	1–2° less than dense dry		43–50°
Silt or silty sand			
Loose	20–22°		27–30°
Dense	25–30°		30–35°
Clay	0° if saturated	3–20°	20–42°

Notes:

1. use larger values as γ increases
2. Use larger values for more angular particles
3. Use larger values for well-graded sand and gravel mixtures
4. Average values for
   Gravel: 35–38°
   Sands: 32–34°

WORKED EXAMPLES

Ex. 8.1

Following are the data recorded during a series of direct shear tests performed on sandy clay:

Vertical load, P (kN)	Peak proving ring reading, R (Div.)*
0.361	17
0.721	26
1.081	35
1.441	44

* 1 Div. = 0.020 kN

Size of the direct shear box = 60 mm square

i) Determine the shear strength parameters (c & ϕ).
ii) Find the cohesion (c) which would be expected from an unconfined compression test on a sample of the same soil.
iii) If another specimen of this soil is subjected to an undrained triaxial test with confining pressure of 200 kPa, find total axial stress that would be needed for failure of the sample.

Solution

Vertical load P, (kN)	Normal stress, $\sigma = P/A$ (kPa)	Peak proving ring reading R, (div.)	Shear stress, $\tau = 0.020 \times R/A$ (kPa)	Remarks
0.361	100.3	17	94.4	A = shear box area
0.721	200.3	26	144.4	$= 0.06 \times 0.06$
1.081	300.3	35	194.4	$= 0.0036\ m^2$
1.441	400.3	44	244.4	

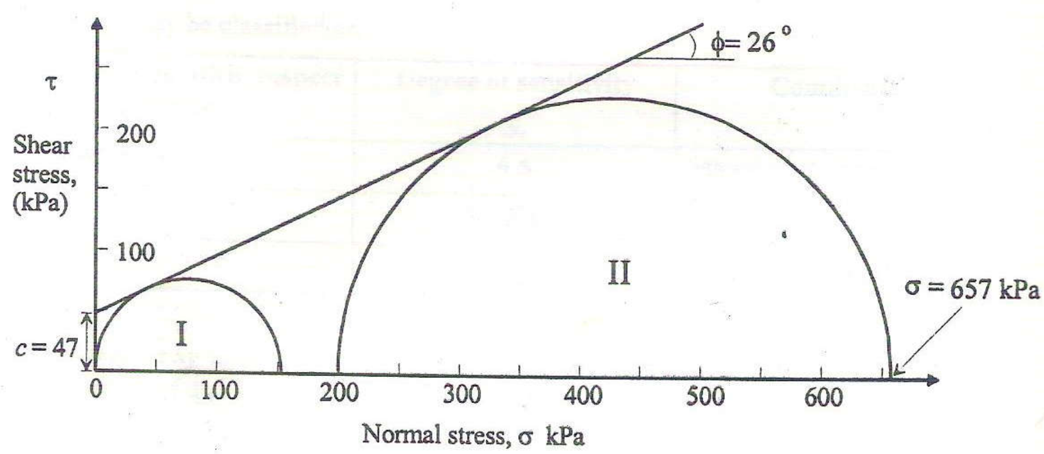


Figure Ex. 8.1 *Normal stress vs. shear stress plot*

i) Shear strength parameters: $c = 47$ kPa, $\phi = 26°$
ii) From circle-I (a circle drawn from origin which touches the failure envelope), read c value which is = 76 kPa

iii) From circle-II (a circle drawn from σ = 200 kPa which touches the failure envelope), read σ value which is = 657 kPa

Ex. 8.2

The following data were recorded during triaxial tests performed on undisturbed soil samples:

Test #	Cell pressure, σ_3 (kPa)	Peak proving ring dial reading, *R* (Div.)
I	50	66
II	150	106
III	250	147

Sample size: Diameter = 37.5 mm; Length = 75 mm
Load dial calibration factor, 1 division = 1.4×10^{-3} kN.
Compute c and ϕ.

Solution

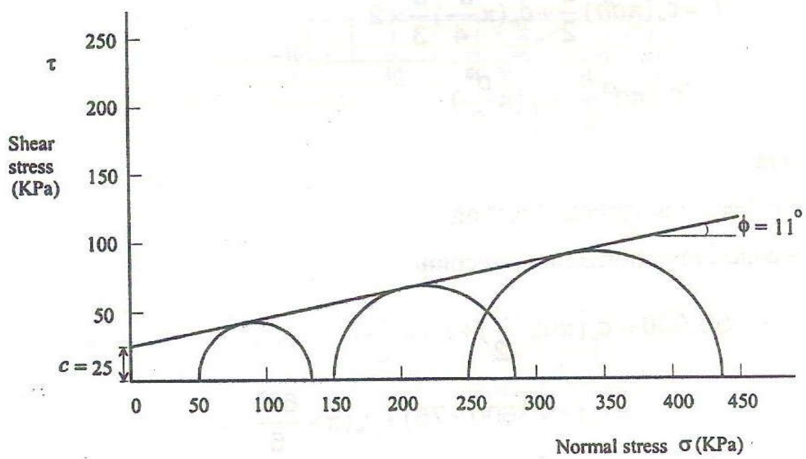


Figure Ex. 8.2 *Mohr circles diagram for triaxial tests*

Sample *x*-sectional area, $A = \dfrac{\pi}{4}D^2 = \dfrac{\pi\times37.5^2}{4\times10^6}\ \text{m}^2 = 1.104\times10^{-3}\ \text{m}^2$

Cell pressure, σ_3 (kPa)	Peak proving ring dial reading (Divisions)	Additional vertical stress (deviator stress), (kPa)	Total vertical stress, $\sigma_1 = \sigma_3$ + deviator pressure, (kPa)
50	66	83.7	133.4
150	106	134.4	284.4
250	147	186.4	436.4

Plot the Mohr's circles as shown in Fig. Ex. 8.2 and read the values of c and ϕ.

Ex. 8.3

In a vane shear test a torque of 50 Nm is needed to cause failure in a clay soil. The vane is 150 mm long and has a diameter of 60 mm. Compute the cohesion (c) of the soil.

When a vane of length 200 mm and diameter 90 mm is used in the same soil, the failure torque was recorded as 140 Nm. Calculate the ratio of shear strength of the clay in vertical direction to that in horizontal direction.

Solution

- **Original test**

$$c = \frac{T}{\pi d^2(\frac{h}{2}+\frac{d}{6})} = \frac{50\times10^3}{\pi\times60^2(\frac{150}{2}+\frac{60}{6})} = 52 \text{ KPa}$$

- **For both tests**

$$T = c_v(\pi dh)\frac{d}{2} + c_h(\pi\frac{d^2}{4})\frac{d}{3}\times2$$

$$= c_v(\pi d^2\frac{h}{2} + c_h(\pi\frac{d^3}{6})$$

Where,

c_v = cohesion in vertical direction

c_h = cohesion in horizontal direction

$$50{,}000 = c_v(\pi 60^2\frac{h}{2}) + c_h(\pi\frac{d^3}{6})$$

$$= c_v(\pi\times3600\times75) + c_h(\pi\times\frac{60^3}{6}$$

$$= (848230)c_v + c_h(113097)$$

$$5 = 84.823\ c_v + 11.3097\ c_h \qquad \text{(i)}$$

Similarly

$$140\times10^3 = c_v(\pi\times90^2\times\frac{200}{2}) + c_h(\pi\times\frac{90^3}{6})$$

$$14 = c_v\,(254.5) + c_h\,(38.2) \qquad \text{(ii)}$$

From equations (i) and (ii):

$c_h = 0.233$ N/mm^2

$c_v = 0.028$ N/mm^2

$\therefore\ c_h/c_v = 8.32$

Ex. 8.4

Given. Figure E8.4-1.

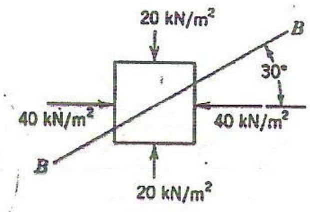


Fig. E8.4-1

Find. Stresses on plane B–B.
Solution. Use Fig. E8.4-2.

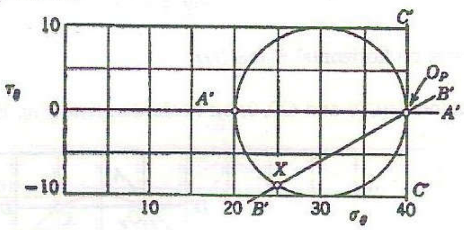


Fig. E8.4-2

1. Locate points with co-ordinates (40, 0) and (20, 0).
2. Draw circle, using these points to define diameter.
3. Draw line $A'A'$ through point (20, 0) and parallel to plane on which stress (20, 0) acts.
4. Intersection of $A'A'$ with Mohr circle at point (40, 0) is the origin of planes.
5. Draw line $B'B'$ through O_P parallel to BB.
6. Read coordinates of point X where $B'B'$ intersects Mohr circle.

Answer. See Fig. E8.4-3.

$$\text{on } BB \quad \begin{cases} \sigma = 25 \text{ kN/m}^2 \\ \tau = -8.7 \text{ kN/m}^2 \end{cases}$$

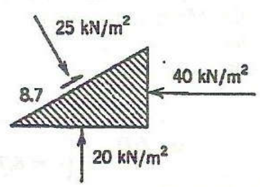


Fig. 8.4-3

Alternate Solution. Steps 1 and 2 same as above.

3. Draw line $C'C'$ through (40, 0) parallel to plane on which stress (40, 0) acts. $C'C'$ is vertical.

4. $C'C'$ intersects Mohr circle only at (40, 0), so this point is O_P. Steps 5 and 6 same as above.

Using equations. 8.3 and 8.4

$$\sigma_1 = 40 \text{ kN/m}^2 \quad \sigma_3 = 20 \text{ kN/m}^2 \quad \theta = 120°$$

$$\sigma_\theta = \frac{40+20}{2} + \frac{40-20}{2}\cos 240° = 30 - 10\cos 60° = 25 \text{ kN/m}^2$$

$$\tau_\theta = \frac{40-20}{2}\sin 240° = -10\sin 60° = -8.66 \text{ kN/m}^2$$

(*Questions for student.* Why is $\theta = 120°$? Would result be different if $\theta = 300°$?)

Ex. 8.5

Given. Figure E8.5-1.

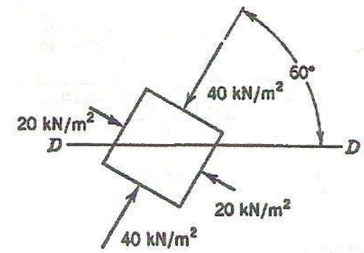


Fig. E8.5-1

Find. Stresses on horizontal plane *DD*.
Solution.
1. Locate points (40, 0) and (20, 0) on Mohr diagram (Fig. E8.5-2).

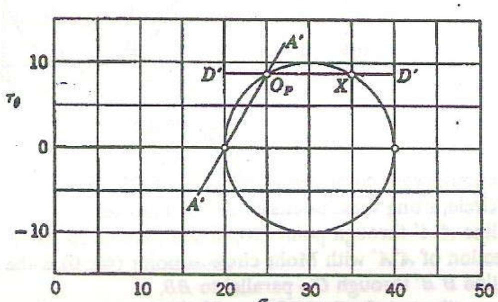


Fig. E8.5-2

2. Draw Mohr circle.
3. Draw line $A'A'$ through (20, 0) parallel to plane upon which stress (20, 0) acts.
4. Intersection of $A'A'$ with Mohr circle gives O_P.
5. Draw line $D'D'$ parallel to plane *DD*.
6. Intersection *X* gives desired stresses

Answer. See Fig. E8.5-3.

$$\text{on } DD \begin{cases} \sigma = 35 \text{ kN/m}^2 \\ \tau = 8.7 \text{ kN/m}^2 \end{cases}$$

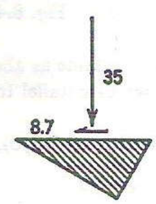


Fig. E8.5-3

Ex. 8.6

Given. Figure E8.6-1.

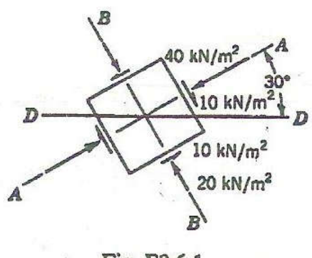


Fig. E8.6-1

Find. Magnitude and direction of the principal stresses.
Solution. Use Fig. E8.6-2.

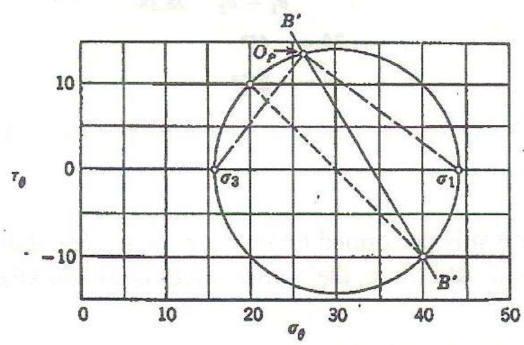


Fig. E8.6-2

1. Locate points (40, −10) and (20, 10).
2. Erect diameter and draw Mohr circle.
3. Draw $B'B'$ through (40,−10) parallel to BB.
4. Intersection of $B'B'$ with circle gives O_P.
5. Read σ_1 and σ_3 from graph.
6. Line through O_P and σ_1 gives plane on which σ_1 acts, etc. (see Fig. E8.6-3).

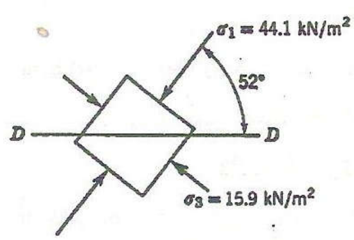


Fig. E8.6-3

Solution by Equations.
1. First make use of fact that sum of normal stresses is a constant:

$$\frac{\sigma_1 + \sigma_3}{2} = \frac{\Sigma\sigma_\theta}{2} = \frac{40 + 20}{2} = 30 \text{ kN/m}^2$$

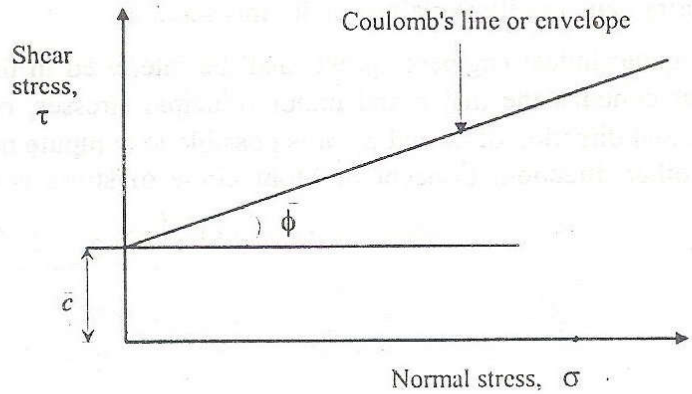


Figure 8.1 *Diagrammatic representation of Coulomb's equation* 8.2.

From equation 8.2, it is evident that the pore water pressure (u) has a major influence on the shearing strength of soils. For coarse-grained soils where the drainage is very good, the total stress (σ) is usually equal to the effective stress ($\bar{\sigma}$). With the fine-grained soils, however, the drainage is very poor and usually considerable time is required before the effective stress increase is equal to the total stress increase. Hence, the rate and length of time of testing is important in the determination of the shear strength of the fine-grained soils. Fig. 8.2 represents the relation between testing time and the effective stress:

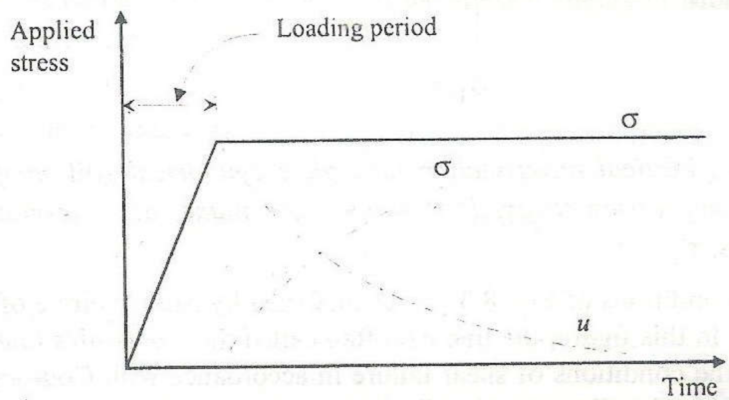


Figure 8.2 *Pore pressure/effective stress/time relationship*

Thus, in order to obtain realistic results within a reasonable time for fine-grained soils, pore pressures must continuously be recorded during the progress of the test. For shear strength of fine-grained soils, it is vital that the conditions applicable to the field problem must be considered before deciding the type of test to be used. This is to ensure that the testing conditions in laboratory simulate the site conditions.

8.3 MOHR CIRCLE OF STRESS

In 1887, O Mohr presented the concept of Mohr circle of stress according to which the stress at any point within a material at equilibrium can be represented by a circle provided the shear stress and the normal stress are plotted using same scale. Mohr's circle of stress represents the complete two-dimensional state of stress *at equilibrium*

2. Use relation

$$\left(\frac{\sigma_1 - \sigma_3}{2}\right) = \sqrt{\left[\sigma_\theta - \left(\frac{\sigma_1 + \sigma_3}{2}\right)\right]^2 + [\tau_\theta]^2}$$

with either pair of given stresses

$$\left(\frac{\sigma_1 - \sigma_3}{2}\right) = \sqrt{[20 - 30]^2 + [10]^2} = \sqrt{200} = 14.14 \text{ kN/m}^2$$

3.
$$\sigma_1 = \left(\frac{\sigma_1 + \sigma_3}{2}\right) + \left(\frac{\sigma_1 - \sigma_3}{2}\right) = 44.14 \text{ kN/m}^2$$

$$\sigma_3 = \left(\frac{\sigma_1 + \sigma_3}{2}\right) - \left(\frac{\sigma_1 - \sigma_3}{2}\right) = 15.86 \text{ kN/m}^2$$

4. Use stress pair in which σ_θ is largest; i.e. (40, −10)

$$\sin 2\theta = \frac{2\tau_\theta}{\sigma_1 - \sigma_3} = \frac{-20}{28.28} = -0.707$$

$$2\theta = -45°$$

$$\theta = -22\tfrac{1}{2}°$$

5. Angle from horizontal to major principal stress direction $= 30° - \theta = 52\tfrac{1}{2}°$. ◄

Ex. 8.7

A direct shear test is performed on medium dense sand with normal stress, σ_n = 60 kPa; $K_o = 0.5$. At failure, the normal stress is still 60 kPa and the shear stress is 40 kPa.

Draw Mohr circles for initial conditions and at failure and determine:

(i) The principal stresses at failure.

(ii) The failure plane orientation

(iii) The orientation of the major principal plane at failure.

(iv) The orientation of the plane of maximum shear stress at failure.

Solution

Since $K_o = \sigma_h / \sigma_v = 0.5$

∴ Initial horizontal stress, $\sigma_{3i} = (0.5)(60) = 30$ kPa

Thus to draw initial stress circle (circle *i*) use

$\sigma_{3i} = 30$ kPa and $\sigma_{1i} = 60$ kPa

(i) The initial circle is drawn (Fig. Ex. 8.7) as circle *i*. As the normal stress is held at 60 kPa at failure and the shear stress is recorded as 40 kPa at failure, the failure point *C* is plotted with coordinates (60, 40) as shown in Fig. Ex. 8.7. When point *C* is joined with origin, ϕ can be determined as shown.

To draw the failure circle *f*, find the center of the failure circle by drawing a perpendicular at point *C* cutting the x-axis at point *D*; the center of the *f* circle. From circle *f* scale off the principal stresses as:

σ_{3f} = 39 kPa and σ_{1f} = 139 kPa

(ii) The state of stress at failure point *C* is (60, 40) kPa, and the failure plane is assumed to be horizontal for direct shear test.

(iii) To find the orientation of the major principal plane at failure, draw a horizontal line through point *C* intersecting the *f* circle at point *E* which is the pole of the Mohr circle. Join *E* with *F* and record the angle of line *EF* with the horizontal = 63°.

(iv) Line *EM* is the orientation of the plane of maximum shear stress.= 17°.

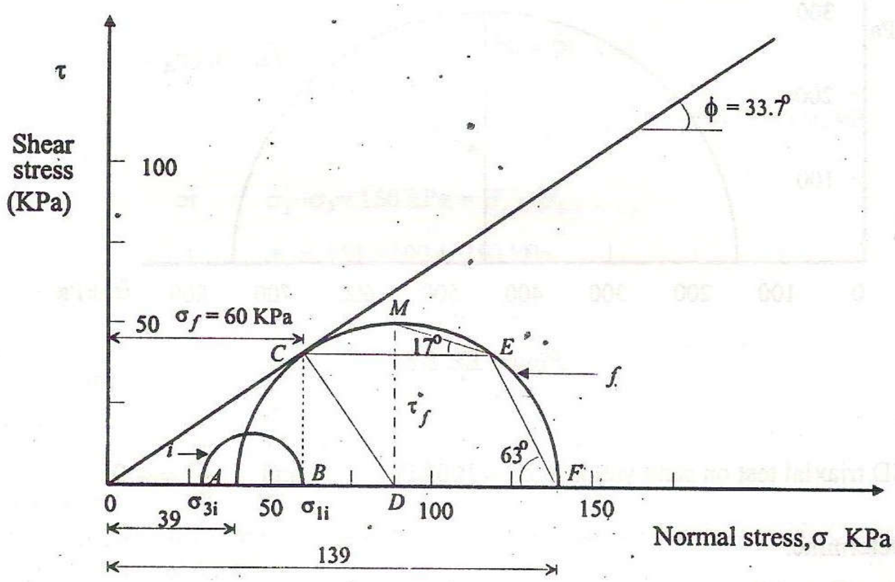


Figure Ex. 8.7

Ex. 8.8

A CD triaxial test is performed on a granular soil and following data recorded:

$\frac{\overline{\sigma}_1}{\overline{\sigma}_3} = 5, \qquad \overline{\sigma}_3 = 150\,\text{kPa}$

(i) Compute $\overline{\phi}$

(ii) What is the deviator stress $(\overline{\sigma}_1 - \overline{\sigma}_3)$ at failure?

(iii) Plot the Mohr circle.

Solution

(i) $$\frac{\overline{\sigma}_1}{\overline{\sigma}_3} = \frac{1+\sin\overline{\phi}}{1-\sin\overline{\phi}} = \tan^2(45^\circ + \overline{\phi}/2) = 5$$

$$= \tan(45^\circ + \bar{\phi}/2) = \sqrt{5} \quad \text{and}$$

$$45^\circ + \bar{\phi}/2 = 65.91 \quad \text{and} \quad \bar{\phi} = 41.8^\circ \cong 42^\circ$$

(ii) $\bar{\sigma}_1 - \bar{\sigma}_3 = (\frac{\bar{\sigma}_1}{\bar{\sigma}_3} - 1)\bar{\sigma}_3 = (5\text{-}1)\ 150 = 600$ kPa

(iii) For $(\bar{\sigma}_1 - \bar{\sigma}_3) = 600$ kPa $\qquad \bar{\sigma}_3 = 150$ kPa

$$\bar{\sigma}_1 = (\bar{\sigma}_1 - \bar{\sigma}_3) + \bar{\sigma}_3 = 750 \text{ kPa}$$

See the Mohr circle in Fig. Ex. 8.8.

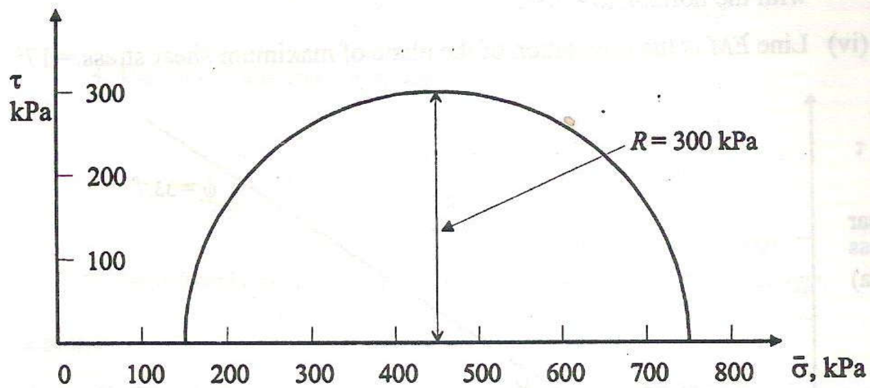


Figure Ex. 8.8

Ex. 8.9

CD triaxial test on sand yields $= \bar{\sigma}_3 = 100$ kPa and $\frac{\bar{\sigma}_1}{\bar{\sigma}_3} = 4.0$

Determine:

(i) $\bar{\sigma}_1$ at failure

(ii) $(\bar{\sigma}_1 - \bar{\sigma}_3)$ at failure, and

(iii) $\bar{\phi}$

Solution

(i) $\frac{\bar{\sigma}_1}{\bar{\sigma}_3} = 4.0 \qquad \therefore \bar{\sigma}_1$ at failure $= 4 \times 100 = 400$ kPa

(ii) $(\bar{\sigma}_1 - \bar{\sigma}_3) = \bar{\sigma}_3(\frac{\bar{\sigma}_1}{\bar{\sigma}_3} - 1) = 100(4-1) = 300$ kPa

(iii) $\frac{\bar{\sigma}_1}{\bar{\sigma}_3} = \tan^2(45 + \bar{\phi}/2)$

$\therefore \quad \tan(45 + \bar{\phi}/2) = 2$ and

$\bar{\phi} = 36.87^\circ \quad$ say $= 37^\circ$

Ex. 8.10

Assume the test specimen of Ex.8.9 was sheared undrained at the same total cell pressure (100 kPa). The pore pressure induced, $u = 50$ kPa at failure.

Calculate:

(i) $\bar{\sigma}_1$ at failure

(ii) $(\bar{\sigma}_1 - \bar{\sigma}_3)$ at failure

(iii) ϕ in term of total stress.

Solution

(i) and (ii) $\quad (\frac{\bar{\sigma}_1}{\bar{\sigma}_3} - 1)\bar{\sigma}_3 = \sigma_1 - \sigma_3 \quad$ and

$$\bar{\sigma}_3 = \sigma_3 - u = 100 - 50 = 50 \text{ kPa}$$

$$\therefore \quad \sigma_1 - \sigma_3 = \bar{\sigma}_1 - \bar{\sigma}_3 = \bar{\sigma}_3\left(\frac{\bar{\sigma}_1}{\bar{\sigma}_3} - 1\right) = 50\ (4-1) = 150 \text{ kPa}$$

$$\text{or} \quad \sigma_1 - \sigma_3 = 150 \text{ kPa} = \bar{\sigma}_1 - \bar{\sigma}_3$$

$$\therefore \quad \sigma_1 = 150 + 100 = 250 \text{ kPa}$$

$$\bar{\sigma}_1 = 250 - 50 = 200 \text{ kPa}$$

(iii) $$\frac{\sigma_1}{\sigma_3} = \tan^2(45 + \phi/2) = \frac{250}{100} = 2.5$$

$$\therefore \quad \phi = 25.4°$$

Ex. 8.11

A CU triaxial test performed on a normally consolidated clay (NCC) yielded the following data:

Consolidating cell pressure, $\sigma_3 = 150$ kPa

Deviator stress at failure, $\sigma_1 - \sigma_3 = 100$ kPa

Pore pressured induced at failure, $u = 88$ kPa

Determine the shear strength parameters in terms of total stress and effective stress:

(i) Analytically
(ii) Graphically
(iii) Draw total and effective Mohr circles and failure envelopes.
(iv) Compute $\bar{\sigma}_1/\bar{\sigma}_3$ and σ_1/σ_3 at failure.
(v) Determine the theoretical angle of the failure plane in the specimen.

Assume $\bar{c} = c =$ negligible $= 0$ as soil is NCC.

Solution

(i) As $\quad \sigma_1 - \sigma_3 = 100$ kPa, $\quad$ and $\quad \sigma_3 = 150$ kPa

$$\therefore \quad \sigma_1 = (\sigma_1 - \sigma_3) + \sigma_3 = 100 + 150 = 250 \text{ kPa}$$

Also $\overline{\sigma}_3 = \sigma_3 - u = 150 - 88 = 62$ kPa

$\overline{\sigma}_1 = \sigma_1 - u = 250 - 88 = 162$ kPa

Now $\dfrac{\overline{\sigma}_1}{\overline{\sigma}_3} = \tan^2(45 + \dfrac{\overline{\phi}}{2}) = \dfrac{162}{62} = 2.61$

$\therefore$ $\overline{\phi} = 26.5°$

Similarly $\phi = 14.5°$

(ii) & (iii) the graphical solution including the failure envelopes is shown in Fig. Ex. 8.11

(iv) $\dfrac{\overline{\sigma}_1}{\overline{\sigma}_3} = \dfrac{162}{62} = 2.61$

$\dfrac{\sigma_1}{\sigma_3} = \dfrac{250}{150} = 1.67$

(iv) $\alpha_f = 45 + \dfrac{\overline{\phi}}{2} = 45 + 13.25 = 58.25°$

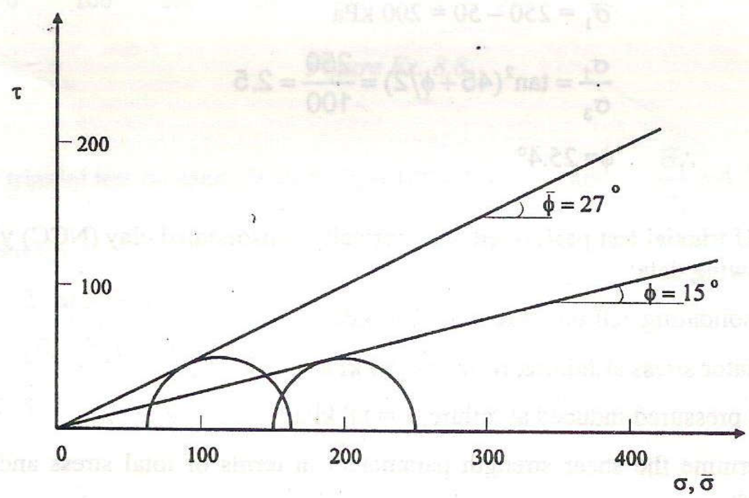


Figure Ex. 8.11

Ex. 8.12

An unconfined compression test was performed on an undisturbed sample of clay and following data recorded:

Sample size: Diameter = 35 mm Height = 80 mm

Compression load at failure, $P = 15$ N

Axial deformation, $\Delta H = 10$ mm

Calculate the unconfined compressive strength q_u and undrained shear strength, c_u.

Solution

$$A = \frac{A_o}{1-\varepsilon} \quad \text{but} \quad \varepsilon = \frac{\Delta H}{H_o} = \frac{10}{80} \times 100 = 12.5\%$$

$$A_o = \frac{\pi}{4}(35)^2 = 962.1\ \text{mm}^2$$

$$\therefore \quad A = \frac{962.1}{1-0.125} = 1099.54\ \text{mm}^2$$

$$\therefore \quad q_u = \frac{P}{A} = \frac{15}{1099.54} = 13.64\ \text{kPa}$$

$$c_u = \frac{q_u}{2} = 6.82\ \text{kPa}$$

PROBLEMS

8-1 What would be the shear strength parameters of a sand specimen which when subjected to a normal stress of 1 kg/cm^2 failed at a shear stress of 0.8 kg/cm^2. (*Ans:* $\phi = 38.66°$, $c = 0$)

8-2 A clay soil sample is subjected to an unconfined compression test. The sample fails at a pressure of 2540 psf {i.e. unconfined compressive strength (q_u) = 2540 psf}. Determine the cohesion of the clay soil. (*Ans:* 1270 psf).

8-3 Three soils were tested in a constant rate of strain shear box. The cross-sectional area of the box is 2600 mm^2. The results obtained at failure were:

Soil	Test #	Horizontal shearing force (kN)	Vertical loading (kN)
K	1	0.081	0.027
	2	0.085	0.040
	3	0.090	0.067
L	4	0.056	0.089
	5	0.083	0.133
	6	0.125	0.200
M	7	0.051	0.111
	8	0.052	0.178
	9	0.053	0.222

Plot the above results and obtain the apparent cohesion and angle of shearing resistance. What is the probable soil type of each sample?
(*Ans: K is c-ϕ*, $c = 28$ kN/m^2, $\phi = 13$ to $14°$)
L is ϕ soil, $c = 0$, $\phi = 32\text{-}33°$), *M* is *c* soil, $c = 20$ kN/m^2, $\phi = 0$)

8-4 A series of direct shear tests was performed on a soil. Each test was carried out until the soil sample sheared. Laboratory data for the tests are listed below:

Specimen #	Normal stress (ksf)	Shearing stress (ksf)
1	0.25	0.35
2	0.50	0.56
3	1.00	0.94

Determine the cohesion and the angle of internal friction of the soil.
(*Ans:* $c = 0.16$ ksf, $\phi = 38°$)

8-5 A series of direct shear tests was performed on a soil. Each test was carried out until the soil sample sheared. Laboratory data for the tests are tabulated below. Determine the cohesion and the angle of internal friction of the soil.

Specimen #	Normal stress (psf)	Shearing stress (psf)
1	200	450
2	400	520
3	600	590
4	1000	740

(*Ans:* $c = 380$ psf, $\phi = 19°$)

8-6 Triaxial compression tests on three specimens of the same soil were performed in a soil laboratory. Each test was carried out until the sample failed. The data obtained in the tests are tabulated below:

Specimen #	Minor principal stress, σ_3, (confining pressure) (ksf)	Major principal stress $\sigma_1 = \Delta P + \sigma_3$ (ksf)
1	2	11.0
2	4	15.2
3	6	18.8

Determine the cohesion and the angle of internal friction of the soil.
(*Ans:* $c = 2.5$ ksf, $\phi = 19.5°$)

8-7 A sample of dry cohesionless soil (i.e. $c = 0$) is subjected to a triaxial test. The angle of internal friction is estimated to be 37°. If the minor principal stress (σ_3) is 14 psi, at what values of the maximum deviator stress (ΔP) and major principal stress (σ_1) is the sample likely to fail? (*Ans:* Major principal stress = 56.3 psi, Deviator stress = 42.3 psi).

8-8 The data shown below were obtained in triaxial compression tests on identical soil samples. Find the cohesion and the angle of internal friction of the soil.

Specimen #	Minor principal stress, σ_3 (psi)	Major principal stress, σ_1 (psi)
1	5	23.0
2	10	38.5
3	15	53.6

(*Ans:* $\phi = 38.5°$, $c = 2.5$ psi)

8-9 A cohesionless soil sample was subjected to a triaxial test. The sample failed when the minor principal stress (confining pressure) was 1200 psf and the maximum deviator stress (ΔP) was 3000 psf. Find the angle of internal friction of the soil. (Ans: $\phi = 33.7°$)

8-10 A sample of dry cohesionless soil is known to have the angle of internal friction of 35°. If the minor principal stress (σ_3) is 15 psi, at what values of the maximum deviator stress (ΔP), major principal stress (σ_1) and normal stress (σ_n) is the sample likely to fail?
(*Ans:* $\Delta P = 40$ psi, $\sigma_1 = 55$ psi, $\sigma_n = 23.5$ psi).

8-11 A soil has an effective angle of shearing resistance ϕ' of 20° and an effective cohesion c' of 20 kPa. What value would you expect of the vertical stress at failure if the soil is subjected to drained triaxial compression with a cell pressure of 250 kPa? (*Ans.* 576 kPa) in an element or at a point. The concept of Mohr's circle is very useful in geotechnical engineering; and briefly explained under this section.

More specifically in geotechnical engineering, we shall be interested in the state of stress in a plane that contains the major and minor principal stresses, σ_1 and σ_3. Given the magnitude and direction of σ_1 and σ_3, it is possible to compute normal and shear stress in any other direction. Concept of Mohr circle of stress is explained below.

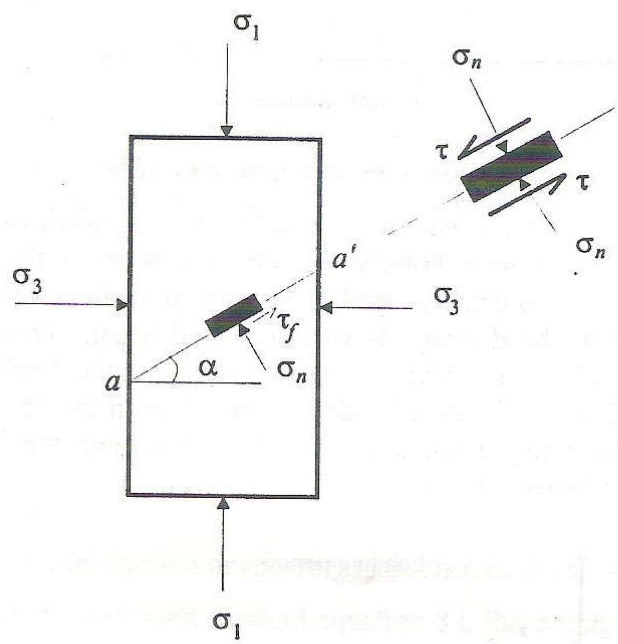


Figure 8.3 *Vertical cross-section through a cylindrical soil sample subjected to confined compression (triaxial) showing shear plane, aa' , normal stress,* σ_n*, and shear stress,* τ.

The stress conditions of Fig. 8.3 can be analyzed by Mohr's circle of stress as shown in Fig. 8.4. In this figure, the line *FE* often called the *Coulomb's line* or *rupture line*, represents the conditions of shear failure in accordance with Coulomb's law. Mohr's circle touching this line at point *E*, represents a condition of incipient failure. Any circle falling below line *FE* would denote stable soil condition.

The normal stress, σ_n and the shear stress, τ on an inclined shear plane (*aa'*) as shown in Fig. 8.3 can be geometrically demonstrated on Mohr's graph as follows:

$$\sigma_n = OB = OC - CB$$

But $$OC = \frac{\sigma_1 + \sigma_3}{2}$$

$$CB = (CE)\cos 2\alpha,$$

And $$CE = CA = CD = \frac{\sigma_1 - \sigma_3}{2}$$

Therefore,

$$\sigma_n = OB = \frac{\sigma_1 + \sigma_3}{2} + \frac{\sigma_1 - \sigma_3}{2}\cos 2\alpha \qquad 8.3$$

Similarly,

$$\tau = BE = (CE)\sin 2\alpha = \frac{\sigma_1 - \sigma_3}{2}\sin 2\alpha \qquad 8.4$$

Thus each point on the circle gives the pair of stresses acting on a rupture plane of specific inclination, α.

A tangent *t-t*, drawn to the circle at point *E*, has the equation $\tau = c + \sigma\tan\phi$

Coulomb's equation for shear strength of soils

The slope of this line (tanϕ) physically means the co-efficient of internal friction of the soil; ϕ is the angle of internal friction, and *c* is the cohesion.

c and ϕ are actually test co-efficients obtained by special apparatus and by special methods of testing.

Fig. 8.5 represents the Mohr's circle of stress for non-cohesive and cohesive soils. In this method, the normal stresses (σ_1, σ_3, σ_n) acting on a cylindrical soil sample (triaxial test) are plotted as abscissa and shear stress (τ) as ordinates using same scale along the axes. The difference in the principal stresses, ($\sigma_1-\sigma_3$) is called as deviator stress. A circle of radius $(\frac{\sigma_1 - \sigma_3}{2})$ is drawn. Since the shear stresses are zero on planes where principal stresses are acting, the ends of the stress diameter or Mohr's circle of stress have the co-ordinates (σ_1, 0) and (σ_3, 0).

In coarse-grained soils (dry sands and gravels), cohesion is insignificant or negligible and the Mohr's circle of stress takes the shape of Fig. 8.5 (i). In this case the ratio of the principal stresses, σ_1/σ_3 is the same for all the circles tangent to the line *OE*.

$$\sin\phi = \frac{CE}{OC} = \frac{1/2\,(\text{difference of principal stresses})}{1/2(\text{sum of principal stresses})} = \frac{\sigma_1 - \sigma_3}{\sigma_1 + \sigma_3}$$

From this it is deduced that:

$$\frac{\sigma_1}{\sigma_3} = \frac{1+\sin\phi}{1-\sin\phi} \qquad 8.5$$

Equation 8.5 is used in the calculation of earth pressures.

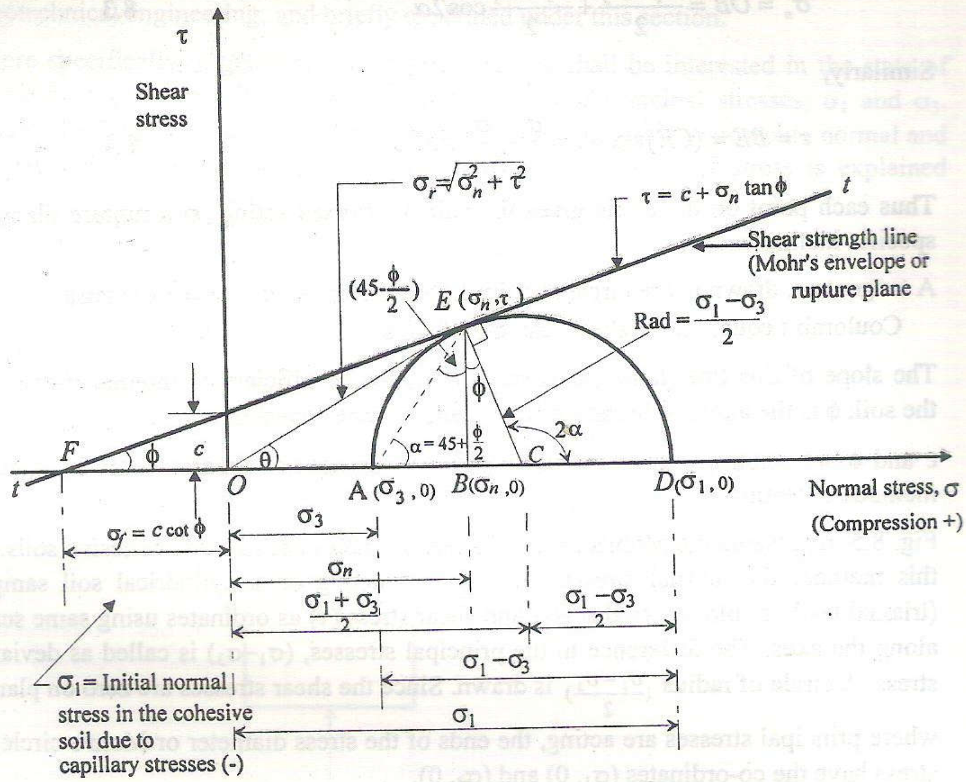


Figure 8.4 *Mohr's circle of stresses*

Given σ_1 and σ_3 alongwith their directions, one can find stresses in any other direction by Mohr circle or the magnitude and direction of the principal stresses can be found if σ_n and τ are given. The notion of the origin of planes is especially useful in such constructions. The origin of planes is a point on the Mohr circle, denoted by O_P, with the following property: a line through O_P and any point A on the Mohr's circle will be parallel to the plane on which the stresses given by point A act. Examples 8.4 to 8.6 illustrate the use of the Mohr's circle and origin of planes.

8.4 METHODS OF DETERMINING SHEAR STRENGTH

Shear strength of soils can be determined using:

(i) Laboratory tests; and

(ii) Field tests.

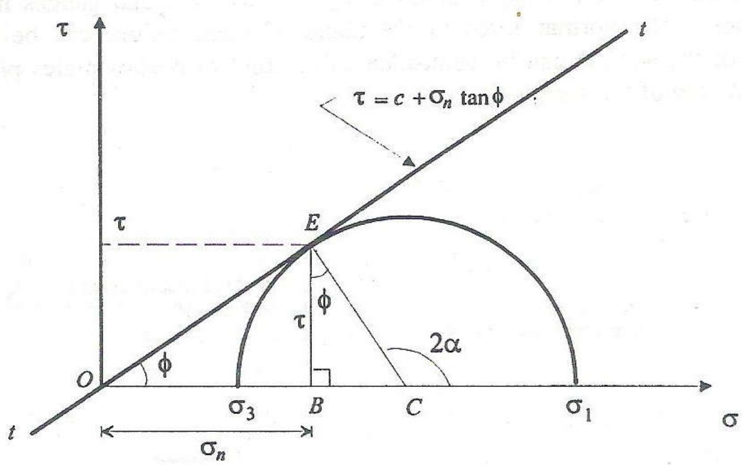
- **Laboratory tests**

(a) Direct shear test (ASTM D3080)

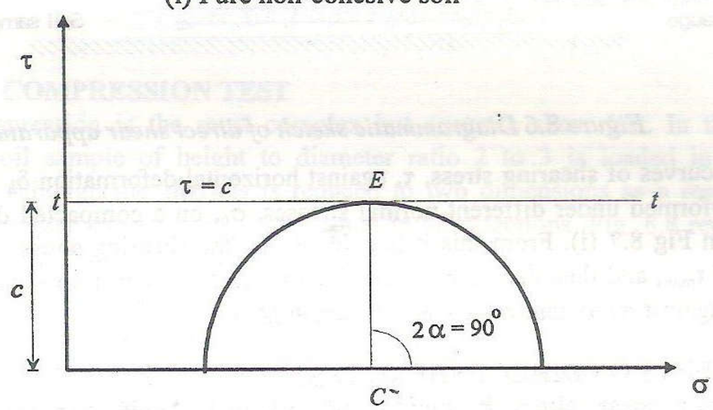
(b) Unconfined compression test (ASTM D2166)

(c) Triaxial compression test (ASTM D2850)

(d) Vane shear test (ASTM D2573)



(i) Pure non-cohesive soil



(ii) Pure cohesive soil

Figure 8.5 Mohr's circle of stresses for different soils

Unconfined compression test can only be used for determining shear strength of cohesive soils and the vane shear is suitable only for soft clays particularly sensitive clays. Direct shear and triaxial test, however, can be used to investigate cohesive and cohesionless soils.

For details of shear tests, the readers may refer to any soil mechanics laboratory testing manual or respective test standards shown against each test. Brief description of each test is presented in the succeeding sections.

8.5 DIRECT SHEAR TEST

This is relatively a simple shear test in which the shearing force is applied at a constant rate of strain until shearing failure occurs. In this test, soil samples are placed in a metal shear box of square or round shape spliced at mid height, as shown in Fig. 8.6. Thus in this test, the sample is made to fail on a *pre-determined horizontal shear plane*. The shearing force is measured by means of a horizontal proving ring from which the peak shearing stress is determined. Horizontal and vertical deformations can be recorded using displacement dial gauges installed for this purpose. The normal force to the plane of shear failure can be varied and drainage of the sample can be controlled using solid or porous plates placed at the bottom and top of the sample.

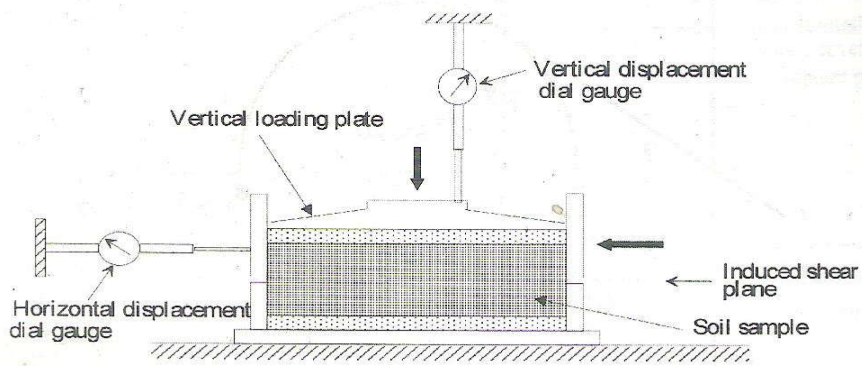


Figure 8.6 *Diagrammatic sketch of direct shear apparatus*

Typical curves of shearing stress, τ, against horizontal deformation δ_h for a series of tests performed under different normal stresses, σ_n, on a compacted dense sand are shown in Fig 8.7 (i). From this it is evident that the shearing stress reaches a peak value of τ_{max}, and then decreases while shearing still continues. In Fig 8.7 (ii), τ_{max} is plotted against σ_n to determine c & ϕ parameters.

COMMENTS ON DIRECT SHEAR TEST

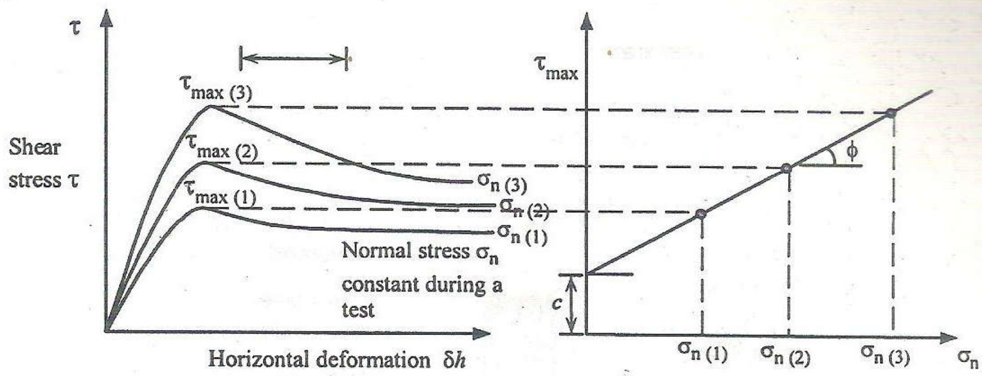
The direct shear although simple and relatively rapid has following major disadvantages:

(i) There is a little control over the drainage conditions. The measurement of vertical displacement and hence of volume change is not accurate.

(ii) The failure plane is pre-determined which may not be the weakest plane.

(iii) The actual distribution of shearing stress over the failure plane is not known.

(iv) The area of failure plane decreases during the test. The corrected area should be used to compute normal and shear stresses.



(i) Shear stress vs. horizontal deformation (ii) τ_{max} against normal stress for a series of tests

Figure 8.7 *Direct shear test plots*

8.6 TRIAXIAL COMPRESSION TEST

Triaxial compression is the most complex but accurate shear test. In this test a cylindrical soil sample of height to diameter ratio 2 to 3 is loaded in all three dimensions, although the analysis is reduced to two dimensions as a result of the lateral stresses (cell pressure, σ_3) being equal in all directions. Fig. 8.8 represents a typical test cell layout.

The soil sample enclosed in a rubber membrane and generally has porous platens on each end, is placed in the water tight perspex cell. Water is pumped into the cell and its pressure raised to σ_3 (cell pressure) which acts in all directions. A vertical load is then applied and recorded using proving ring, until shear failure occurs. Since the cell pressure σ_3 was acting all around the sample, and additional vertical stress of $(\sigma_1-\sigma_3)$ will cause the failure of the sample and as such this additional stress is known as *deviator stress*. The major and minor principal stresses would be σ_1 and σ_3 respectively. Vertical displacement of the sample can be recorded using strain gauges or dial gauge. If desired, the pore water pressure and volume changes can also be monitored during the test.

Tests are carried out under different cell pressures and results plotted as Mohr's circles. The tangent drawn to the circles gives the values of c & ϕ as shown in Fig. 8.9.