# CHAPTER 7

# COMPRESSIBILITY AND CONSOLIDATION

## 7.1 INTRODUCTION

Structures are built on soils. They transfer loads to the subsoil through the foundations. The effect of the loads is felt by the soil normally up to a depth of about two to three times the width of the foundation. The soil within this depth gets compressed due to the imposed stresses. The compression of the soil mass leads to the decrease in the volume of the mass which results in the settlement of the structure.

The displacements that develop at any given boundary of the soil mass can be determined on a rational basis by summing up the displacements of small elements of the mass resulting from the strains produced by a change in the stress system. The compression of the soil mass due to the imposed stresses may be almost immediate or time dependent according to the permeability characteristics of the soil. Cohesionless soils which are highly permeable are compressed in a relatively short period of time as compared to cohesive soils which are less permeable. The compressibility characteristics of a soil mass might be due to any or a combination of the following factors:

1. Compression of the solid matter.
2. Compression of water and air within the voids.
3. Escape of water and air from the voids.

It is quite reasonable and rational to assume that the solid matter and the pore water are relatively incompressible under the loads usually encountered in soil masses. The change in volume of a mass under imposed stresses must be due to the escape of water if the soil is saturated. But if the soil is partially saturated, the change in volume of the mass is partly due to the compression and escape of air from the voids and partly due to the dissolution of air in the pore water.

The compressibility of a soil mass is mostly dependent on the rigidity of the soil skeleton. The rigidity, in turn, is dependent on the structural arrangement of particles and, in fine grained soils, on the degree to which adjacent particles are bonded together. Soils which possess a honeycombed structure possess high porosity and as such are more compressible. A soil composed predominantly of flat grains is more compressible than one containing mostly spherical grains. A soil in an undisturbed state is less compressible than the same soil in a remolded state.

Soils are neither truly elastic nor plastic. When a soil mass is under compression, the volume change is predominantly due to the slipping of grains one relative to another . The grains do not spring back to their original positions upon removal of the stress. However, a small elastic rebound under low pressures could be attributed to the elastic compression of the adsorbed water surrounding the grains.

Soil engineering problems are of two types. The first type includes all cases wherein there is no possibility of the stress being sufficiently large to exceed the shear strength of the soil, but wherein the strains lead to what may be a serious magnitude of displacement of individual grains leading to settlements within the soil mass. Chapter 7 deals with this type of problem. The second type includes cases in which there is danger of shearing stresses exceeding the shear strength of the soil. Problems of this type are called *Stability Problems* which are dealt with under the chapters of earth pressure, stability of slopes, and foundations.

Soil in nature may be found in any of the following states

1. Dry state.
2. Partially saturated state.
3. Saturated state.

Settlements of structures built on granular soils are generally considered only under two states, that is, either dry or saturated. The stress-strain characteristics of dry sand, depend primarily on the relative density of the sand, and to a much smaller degree on the shape and size of grains. Saturation does not alter the relationship significantly provided the water content of the sand can change freely. However, in very fine-grained or silty sands the water content may remain almost unchanged during a rapid change in stress. Under this condition, the compression is time-dependent. Suitable hypotheses relating displacement and stress changes in granular soils have not yet been formulated. However, the settlements may be determined by semi-empirical methods (Terzaghi, Peck and Mesri, 1996).

In the case of cohesive soils, the dry state of the soils is not considered as this state is only of a temporary nature. When the soil becomes saturated during the rainy season, the soil becomes more compressible under the same imposed load. Settlement characteristics of cohesive soils are, therefore, considered only under completely saturated conditions. It is quite possible that there are situations where the cohesive soils may remain partially saturated due to the confinement of air bubbles, gases etc. Current knowledge on the behavior of partially saturated cohesive soils under external loads is not sufficient to evolve a workable theory to estimate settlements of structures built on such soils.

## 7.2 CONSOLIDATION

When a saturated clay-water system is subjected to an external pressure, the pressure applied is initially taken by the water in the pores resulting thereby in an excess pore water pressure. If drainage is permitted, the resulting hydraulic gradients initiate a flow of water out of the clay mass and the mass begins to compress. A portion of the applied stress is transferred to the soil skeleton, which in turn causes a reduction in the excess pore pressure. This process, involving a gradual compression occurring simultaneously with a flow of water out of the mass and with a gradual transfer of the applied pressure from the pore water to the mineral skeleton is called *consolidation*. The process opposite to consolidation is called *swelling*, which involves an increase in the water content due to an increase in the volume of the voids.

Consolidation may be due to one or more of the following factors:

1. External static loads from structures.
2. Self-weight of the soil such as recently placed fills.
3. Lowering of the ground water table.
4. Desiccation.

The total compression of a saturated clay strata under excess effective pressure may be considered as the sum of

1. Immediate compression,
2. Primary consolidation, and
3. Secondary compression.

The portion of the settlement of a structure which occurs more or less simultaneously with the applied loads is referred to as the *initial* or *immediate settlement*. This settlement is due to the immediate compression of the soil layer under undrained condition and is calculated by assuming the soil mass to behave as an elastic soil.

If the rate of compression of the soil layer is controlled solely by the resistance of the flow of water under the induced hydraulic gradients, the process is referred to as *primary consolidation*. The portion of the settlement that is due to the primary consolidation is called *primary consolidation settlement* or *compression*. At the present time the only theory of practical value for estimating time-dependent settlement due to volume changes, that is under primary consolidation is *the one-dimensional theory*.

The third part of the settlement is due to secondary consolidation or compression of the clay layer. This compression is supposed to start after the primary consolidation ceases, that is after the excess pore water pressure approaches zero. It is often assumed that secondary compression proceeds linearly with the logarithm of time. However, a satisfactory treatment of this phenomenon has not been formulated for computing settlement under this category.

## The Process of Consolidation

The process of consolidation of a clay-soil-water system may be explained with the help of a mechanical model as described by Terzaghi and Frohlich (1936).

The model consists of a cylinder with a frictionless piston as shown in Fig. 7.1. The piston is supported on one or more helical metallic springs. The space underneath the piston is completely filled with water. The springs represent the mineral skeleton in the actual soil mass and the water below the piston is the pore water under saturated conditions in the soil mass. When a load of $p$ is placed on the piston, this stress is fully transferred to the water (as water is assumed to be incompressible) and the water pressure increases. The pressure in the water is

$$u = p$$

This is analogous to pore water pressure, $u$, that would be developed in a clay-water system under external pressures. If the whole model is leakproof without any holes in the piston, there is no chance for the water to escape. Such a condition represents a highly impermeable clay-water system in which there is a very high resistance for the flow of water. It has been found in the case of compact plastic clays that the minimum initial gradient required to cause flow may be as high as 20 to 30.

If a few holes are made in the piston, the water will immediately escape through the holes. With the escape of water through the holes a part of the load carried by the water is transferred to the springs. This process of transference of load from water to spring goes on until the flow stops

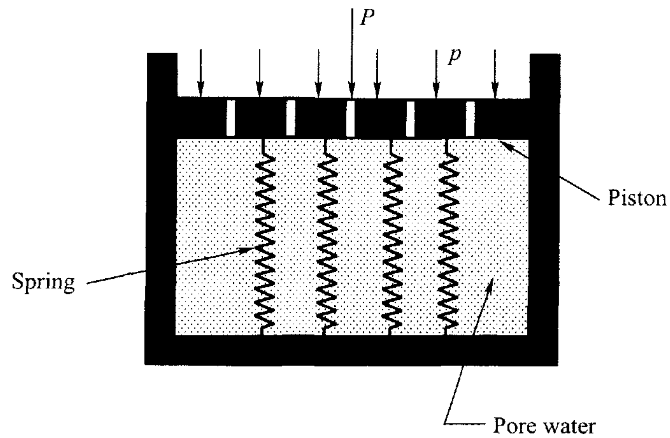



**Figure 7.1** Mechanical model to explain the process of consolidation

when all the load will be carried by the spring and none by the water. The time required to attain this condition depends upon the number and size of the holes made in the piston. A few small holes represents a clay soil with poor drainage characteristics.

When the spring-water system attains equilibrium condition under the imposed load, the settlement of the piston is analogous to the compression of the clay-water system under external pressures.

## One-Dimensional Consolidation

In many instances the settlement of a structure is due to the presence of one or more layers of soft clay located between layers of sand or stiffer clay as shown in Fig. 7.2A. The adhesion between the soft and stiff layers almost completely prevents the lateral movement of the soft layers. The theory that was developed by Terzaghi (1925) on the basis of this assumption is called the *one-dimensional consolidation theory.* In the laboratory this condition is simulated most closely by the *confined compression* or *consolidation test.*

The process of consolidation as explained with reference to a mechanical model may now be applied to a saturated clay layer in the field. If the clay strata shown in Fig 7.2 B(a) is subjected to an excess pressure $\Delta p$ due to a uniformly distributed load $p$ on the surface, the clay layer is compressed over

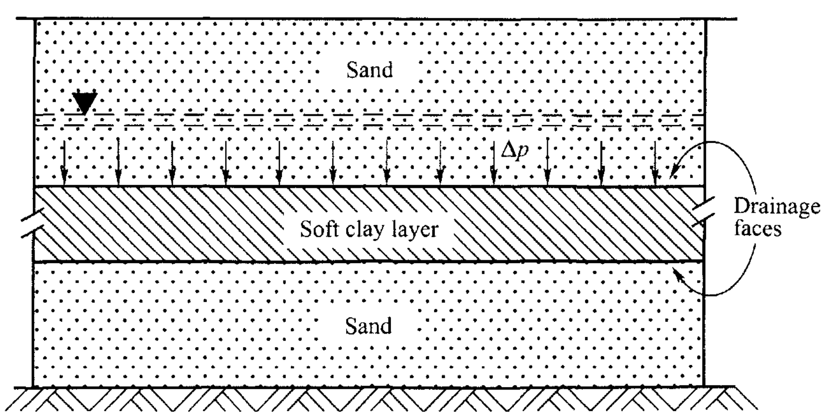



**Figure 7.2A** Clay layer sandwiched between sand layers

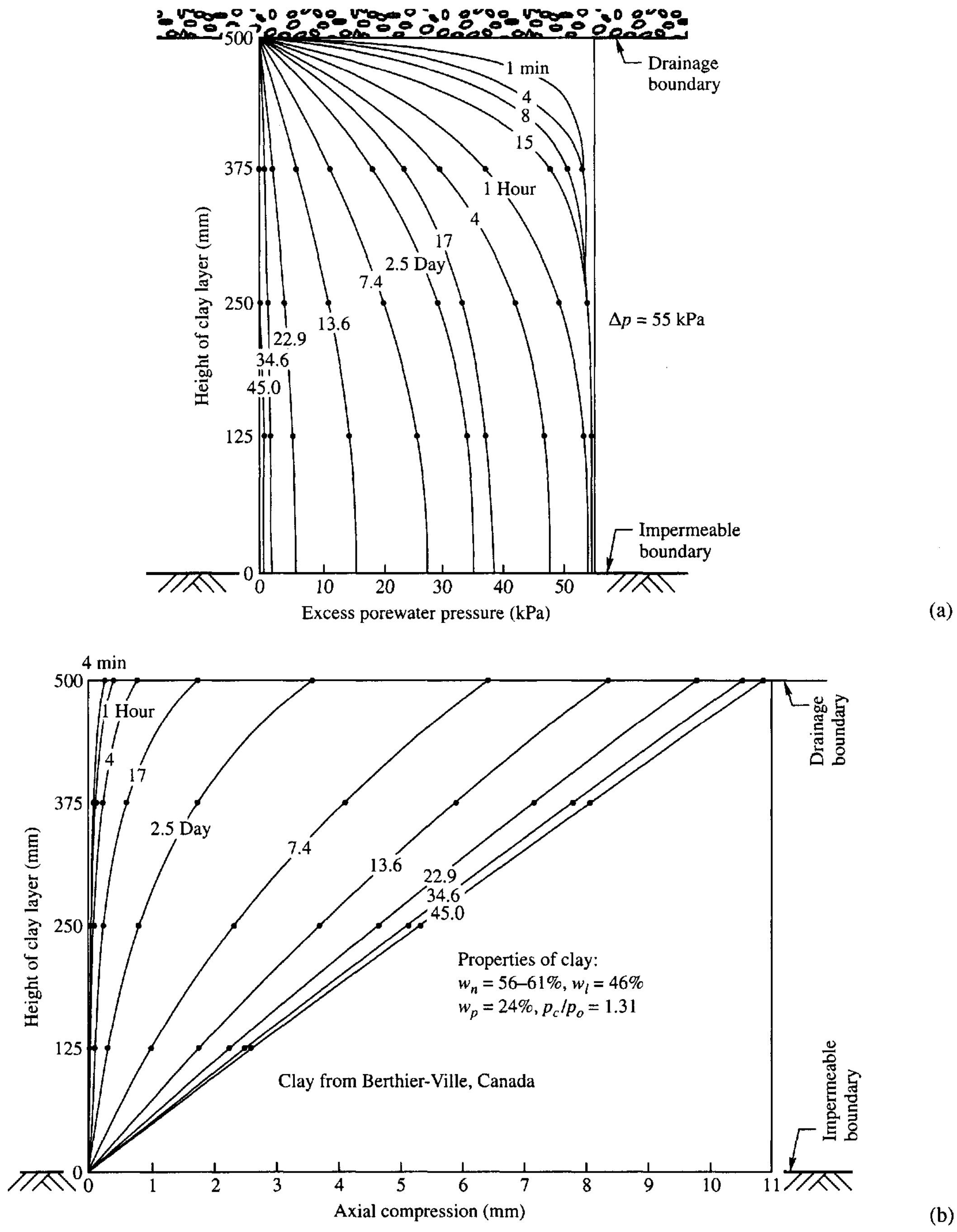



**Figure 7.2B** (a) Observed distribution of excess pore water pressure during consolidation of a soft clay layer; (b) observed distribution of vertical compression during consolidation of a soft clay layer (after Mesri and Choi, 1985, Mesri and Feng, 1986)

time and excess pore water drains out of it to the sandy layer. This constitutes the process of *consolidation*. At the instant of application of the excess load $\Delta p$, the load is carried entirely by water in the voids of the soil. As time goes on the excess pore water pressure decreases, and the effective vertical pressure in the layer correspondingly increases. At any point within the consolidating layer, the value $u$ of the excess pore water pressure at a given time may be determined from

$$u = u_i - \Delta p_z$$

where, $u$ = excess pore water pressure at depth $z$ at any time $t$

$u_i$ = initial total pore water pressure at time $t = 0$

$\Delta p_z$ = effective pressure transferred to the soil grains at depth $z$ and time $t$

At the end of primary consolidation, the excess pore water pressure $u$ becomes equal to zero. This happens when $u = 0$ at all depths.

The time taken for full consolidation depends upon the drainage conditions, the thickness of the clay strata, the excess load at the top of the clay strata etc. Fig. 7.2B (a) gives a typical example of an observed distribution of excess pore water pressure during the consolidation of a soft clay layer 50 cm thick resting on an impermeable stratum with drainage at the top. Figure 7.2B(b) shows the compression of the strata with the dissipation of pore water pressure. It is clear from the figure that the time taken for the dissipation of pore water pressure may be quite long, say a year or more.

## 7.3 CONSOLIDOMETER

The compressibility of a saturated, clay-water system is determined by means of the apparatus shown diagrammatically in Fig. 7.3(a). This apparatus is also known as an *oedometer*. Figure 7.3(b) shows a table top consolidation apparatus.

The consolidation test is usually performed at room temperature, in floating or fixed rings of diameter from 5 to 11 cm and from 2 to 4 cm in height. Fig. 7.3(a) is a fixed ring type. In a floating ring type, the ring is free to move in the vertical direction.

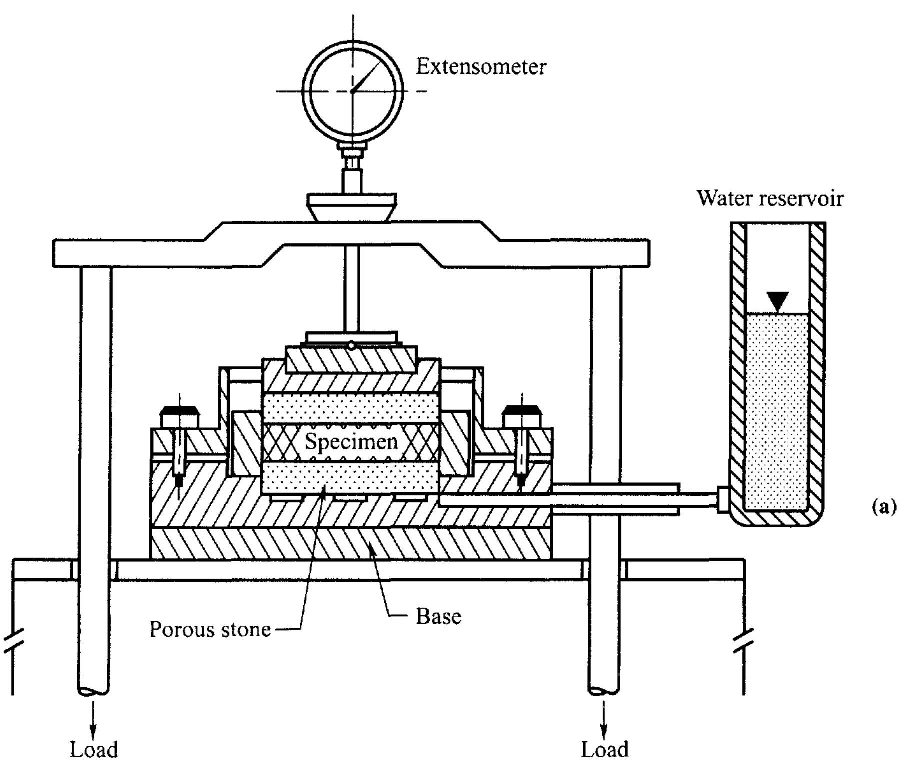



**Figure 7.3** (a) A schematic diagram of a consolidometer

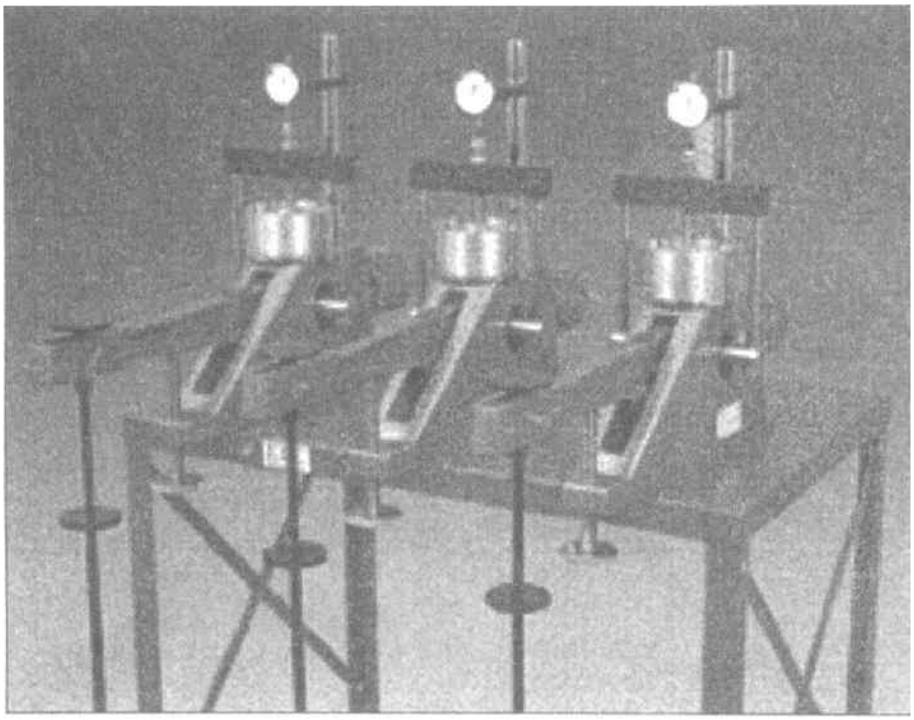


**Figure 7.3** (b) Table top consolidation apparatus (Courtesy: Soiltest, USA)

The soil sample is contained in the brass ring between two porous stones about 1.25 cm thick. By means of the porous stones water has free access to and from both surfaces of the specimen. The compressive load is applied to the specimen through a piston, either by means of a hanger and dead weights or by a system of levers. The compression is measured on a dial gauge.

At the bottom of the soil sample the water expelled from the soil flows through the filter stone into the water container. At the top, a well-jacket filled with water is placed around the stone in order to prevent excessive evaporation from the sample during the test. Water from the sample also flows into the jacket through the upper filter stone. The soil sample is kept submerged in a saturated condition during the test.

## 7.4 THE STANDARD ONE-DIMENSIONAL CONSOLIDATION TEST

The main purpose of the consolidation test on soil samples is to obtain the necessary information about the compressibility properties of a saturated soil for use in determining the magnitude and rate of settlement of structures. The following test procedure is applied to any type of soil in the standard consolidation test.

Loads are applied in steps in such a way that the successive load intensity, $p$, is twice the preceding one. The load intensities commonly used being 1/4, 1/2, 1, 2, 4, 8, and 16 tons/ft$^2$ (25, 50, 100, 200, 400, 800 and 1600 kN/m$^2$). Each load is allowed to stand until compression has practically ceased (no longer than 24 hours). The dial readings are taken at elapsed times of 1/4, 1/2, 1, 2, 4, 8, 15, 30, 60, 120, 240, 480 and 1440 minutes from the time the new increment of load is put on the sample (or at elpased times as per requirements). Sandy samples are compressed in a relatively short time as compared to clay samples and the use of one day duration is common for the latter.

After the greatest load required for the test has been applied to the soil sample, the load is removed in decrements to provide data for plotting the expansion curve of the soil in order to learn its elastic properties and magnitudes of plastic or permanent deformations. The following data should also be obtained:

1. Moisture content and weight of the soil sample before the commencement of the test.
2. Moisture content and weight of the sample after completion of the test.
3. The specific gravity of the solids.
4. The temperature of the room where the test is conducted.

## 7.5 PRESSURE-VOID RATIO CURVES

The pressure-void ratio curve can be obtained if the void ratio of the sample at the end of each increment of load is determined. Accurate determinations of void ratio are essential and may be computed from the following data:

1. The cross-sectional area of the sample $A$, which is the same as that of the brass ring.
2. The specific gravity, $G_s$, of the solids.
3. The dry weight, $W_s$, of the soil sample.
4. The sample thickness, $h$, at any stage of the test.

Let $V_s$ = volume of the solids in the sample
where

$$V_s = \frac{W}{G_s \gamma_w}$$

where $\gamma_w$ = unit weight of water
We can also write

$$V_s = h_s A \quad \text{or} \quad h_s = \frac{V_s}{A}$$

where, $h_s$ = thickness of solid matter.
If $e$ is the void ratio of the sample, then

$$e = \frac{Ah - Ah_s}{Ah_s} = \frac{h - h_s}{h_s} \tag{7.1}$$

In Eq. (7.1) $h_s$ is a constant and only $h$ is a variable which decreases with increment load. If the thickness $h$ of the sample is known at any stage of the test, the void ratio at all the stages of the test may be determined.

The equilibrium void ratio at the end of any load increment may be determined by the change of void ratio method as follows:

### Change of Void-Ratio Method

In one-dimensional compression the change in height $\Delta h$ per unit of original height $h$ equals the change in volume $\Delta V$ per unit of original volume $V$.

$$\frac{\Delta h}{h} = \frac{\Delta V}{V} \tag{7.2}$$

$V$ may now be expressed in terms of void ratio $e$.

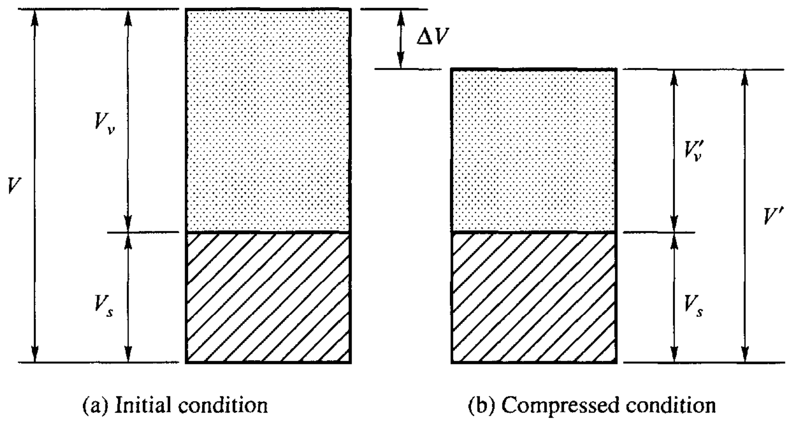



**Figure 7.4** Change of void ratio

We may write (Fig. 7.4),

$V_v = eV_s, \quad V = V_s(1 + e), \quad V_v' = e'V_s$

$V' = V_s(1+e')$

$$\frac{\Delta V}{V} = \frac{V - V'}{V} = \frac{V_s(1+e) - V_s(1+e')}{V} = \frac{e - e'}{1+e} = \frac{\Delta e}{1+e}$$

Therefore,

$$\frac{\Delta h}{h} = \frac{\Delta e}{1+e}$$

or

$$\Delta e = \frac{1+e}{h}\Delta h \qquad (7.3)$$

wherein, $\Delta e$ = change in void ratio under a load, $h$ = initial height of sample, $e$ = initial void ratio of sample, $e'$ = void ratio after compression under a load, $\Delta h$ = compression of sample under the load which may be obtained from dial gauge readings.

Typical pressure-void ratio curves for an undisturbed clay sample are shown in Fig. 7.5, plotted both on arithmetic and on semilog scales. The curve on the log scale indicates clearly two branches, a fairly horizontal initial portion and a nearly straight inclined portion. The coordinates of point $A$ in the figure represent the void ratio $e_0$ and effective overburden pressure $p_0$ corresponding to a state of the clay in the field as shown in the inset of the figure. When a sample is extracted by means of the best of techniques, the water content of the clay does not change significantly. Hence, the void ratio $e_0$ at the start of the test is practically identical with that of the clay in the ground. When the pressure on the sample in the consolidometer reaches $p_0$, the $e$-log $p$ curve should pass through the point $A$ unless the test conditions differ in some manner from those in the field. In reality the curve always passes below point $A$, because even the best sample is at least slightly disturbed.

The curve that passes through point $A$ is generally termed as a *field curve* or *virgin curve*. In settlement calculations, the field curve is to be used.

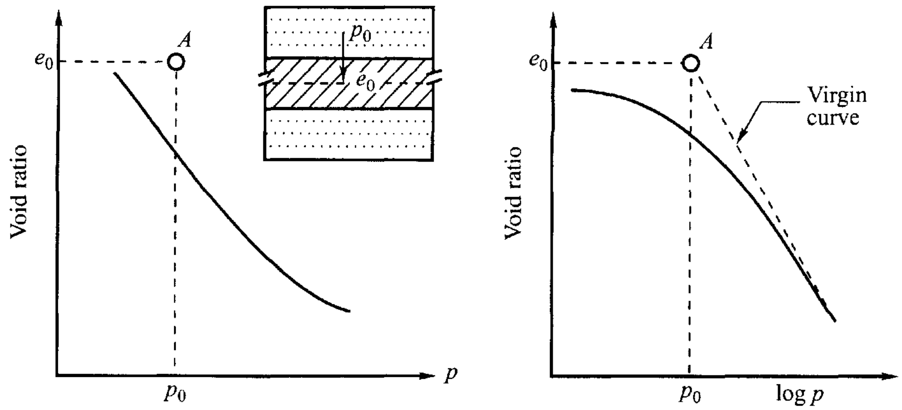



**Figure 7.5** Pressure-void ratio curves

## Pressure-Void Ratio Curves for Sand

Normally, no consolidation tests are conducted on samples of sand as the compression of sand under external load is almost instantaneous as can be seen in Fig. 7.6(a) which gives a typical curve showing the time versus the compression caused by an increment of load.

In this sample more than 90 per cent of the compression has taken place within a period of less than 2 minutes. The time lag is largely of a frictional nature. The compression is about the same whether the sand is dry or saturated. The shape of typical *e-p* curves for loose and dense sands are shown in Fig. 7.6(b). The amount of compression even under a high load intensity is not significant as can be seen from the curves.

## Pressure-Void Ratio Curves for Clays

The compressibility characteristics of clays depend on many factors.

The most important factors are

1. Whether the clay is normally consolidated or overconsolidated
2. Whether the clay is sensitive or insensitive.

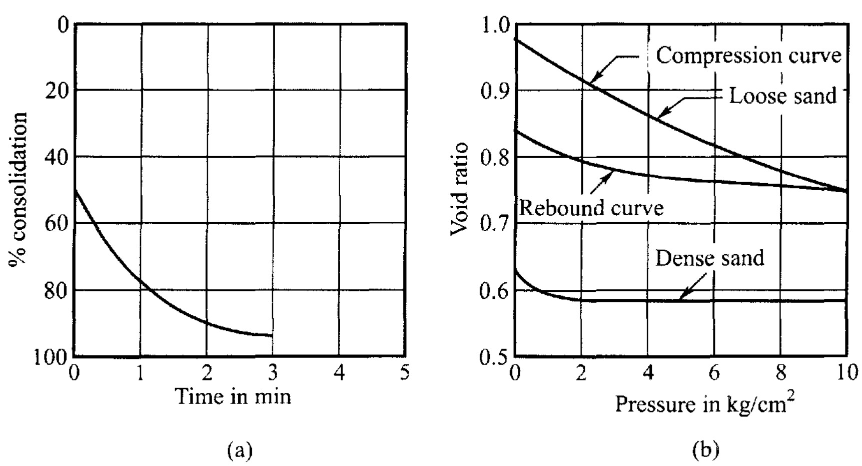



**Figure 7.6** Pressure-void ratio curves for sand

## Normally Consolidated and Overconsolidated Clays

A clay is said to be normally consolidated if the present effective overburden pressure $p_0$ is the maximum pressure to which the layer has ever been subjected at any time in its history, whereas a clay layer is said to be overconsolidated if the layer was subjected at one time in its history to a greater effective overburden pressure, $p_c$, than the present pressure, $p_0$. The ratio $p_c / p_0$ is called the *overconsolidation ratio* (OCR).

Overconsolidation of a clay stratum may have been caused due to some of the following factors

1. Weight of an overburden of soil which has eroded
2. Weight of a continental ice sheet that melted
3. Desiccation of layers close to the surface.

Experience indicates that the natural moisture content, $w_n$, is commonly close to the liquid limit, $w_l$, for normally consolidated clay soil whereas for the overconsolidated clay, $w_n$ is close to plastic limit $w_p$.

Fig. 7.7 illustrates schematically the difference between a normally consolidated clay strata such as *B* on the left side of Section *CC* and the overconsolidated portion of the same layer *B* on the right side of section *CC*. Layer *A* is overconsolidated due to desiccation.

All of the strata located above bed rock were deposited in a lake at a time when the water level was located above the level of the present high ground when parts of the strata were removed by erosion, the water content in the clay stratum *B* on the right hand side of section *CC* increased slightly, whereas that of the left side of section *CC* decreased considerably because of the lowering of the water table level from position $D_0D_0$ to *DD*. Nevertheless, with respect to the present overburden, the clay stratum *B* on the right hand side of section *CC* is overconsolidated clay, and that on the left hand side is normally consolidated clay.

While the water table descended from its original to its final position below the floor of the eroded valley, the sand strata above and below the clay layer *A* became drained. As a consequence, layer *A* gradually dried out due to exposure to outside heat. Layer *A* is therefore said to be overconsolidated by desiccation.

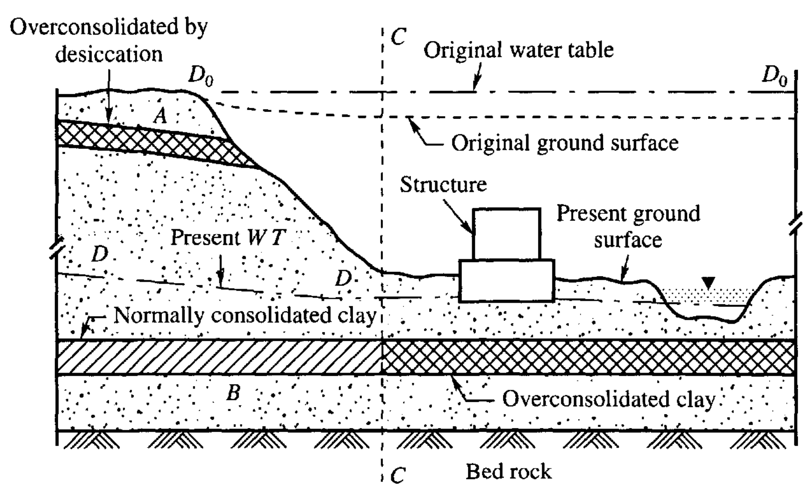



**Figure 7.7** Diagram illustrating the geological process leading to overconsolidation of clays (After Terzaghi and Peck, 1967)

## 7.6 DETERMINATION OF PRECONSOLIDATION PRESSURE

Several methods have been proposed for determining the value of the maximum consolidation pressure. They fall under the following categories. They are

1. Field method,
2. Graphical procedure based on consolidation test results.

### Field Method

The field method is based on geological evidence. The geology and physiography of the site may help to locate the original ground level. The overburden pressure in the clay structure with respect to the original ground level may be taken as the preconsolidation pressure $p_c$. Usually the geological estimate of the maximum consolidation pressure is very uncertain. In such instances, the only remaining procedure for obtaining an approximate value of $p_c$ is to make an estimate based on the results of laboratory tests or on some relationships established between $p_c$ and other soil parameters.

### Graphical Procedure

There are a few graphical methods for determining the preconsolidation pressure based on laboratory test data. No suitable criteria exists for appraising the relative merits of the various methods.

The earliest and the most widely used method was the one proposed by Casagrande (1936). The method involves locating the point of maximum curvature, $B$, on the laboratory $e$-log $p$ curve of an undisturbed sample as shown in Fig. 7.8. From $B$, a tangent is drawn to the curve and a horizontal line is also constructed. The angle between these two lines is then bisected. The abscissa of the point of intersection of this bisector with the upward extension of the inclined straight part corresponds to the preconsolidation pressure $p_c$.

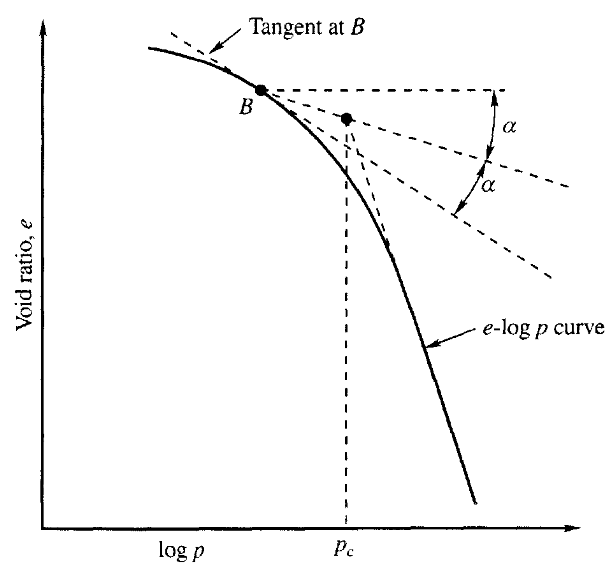



**Figure 7.8** Method of determining $p_c$ by Casagrande method

## 7.7 *e-log p* FIELD CURVES FOR NORMALLY CONSOLIDATED AND OVERCONSOLIDATED CLAYS OF LOW TO MEDIUM SENSITIVITY

It has been explained earlier with reference to Fig. 7.5, that the laboratory *e*-log *p* curve of an undisturbed sample does not pass through point *A* and always passes below the point. It has been found from investigation that the inclined straight portion of *e*-log *p* curves of undisturbed or remolded samples of clay soil intersect at one point at a low void ratio and corresponds to $0.4e_0$ shown as point *C* in Fig. 7.9 (Schmertmann, 1955). It is logical to assume the field curve labelled as $K_f$ should also pass through this point. The field curve can be drawn from point *A*, having coordinates $(e_0, p_0)$, which corresponds to the *in-situ* condition of the soil. The straight line *AC* in Fig. 7.9(a) gives the field curve $K_f$ for normally consolidated clay soil of low sensitivity.

The field curve for overconsolidated clay soil consists of two straight lines, represented by *AB* and *BC* in Fig. 7.9(b). Schmertmann (1955) has shown that the initial section *AB* of the field curve is parallel to the mean slope *MN* of the rebound laboratory curve. Point *B* is the intersection point of the vertical line passing through the preconsolidation pressure $p_c$ on the abscissa and the sloping line *AB*. Since point *C* is the intersection of the laboratory compression curve and the horizontal line at void ratio $0.4e_0$, line *BC* can be drawn. The slope of line *MN* which is the slope of the rebound curve is called the *swell index* $C_s$.

### Clay of High Sensitivity

If the sensitivity $S_t$ is greater than about 8 [sensitivity is defined as the ratio of unconfined compressive strengths of undisturbed and remolded soil samples refer to Eq. (3.50)], then the clay is said to be highly sensitive. The natural water contents of such clay are more than the liquid limits. The *e*-log *p* curve $K_u$ for an undisturbed sample of such a clay will have the initial branch almost flat as shown in Fig. 7.9(c), and after this it drops abruptly into a steep segment indicating there by a structural breakdown of the clay such that a slight increase of pressure leads to a large decrease in void ratio. The curve then passes through a point of inflection at *d* and its slope decreases. If a tangent is drawn at the point of inflection *d*, it intersects the line $e_0A$ at *b*. The pressure corresponding to *b* $(p_b)$ is approximately equal to that at which the structural breakdown takes place. In areas underlain by soft highly sensitive clays, the excess pressure $\Delta p$ over the layer should be limited to a fraction of the difference of pressure $(p_b - p_0)$. Soil of this type belongs mostly to volcanic regions.

## 7.8 COMPUTATION OF CONSOLIDATION SETTLEMENT

### Settlement Equations for Normally Consolidated Clays

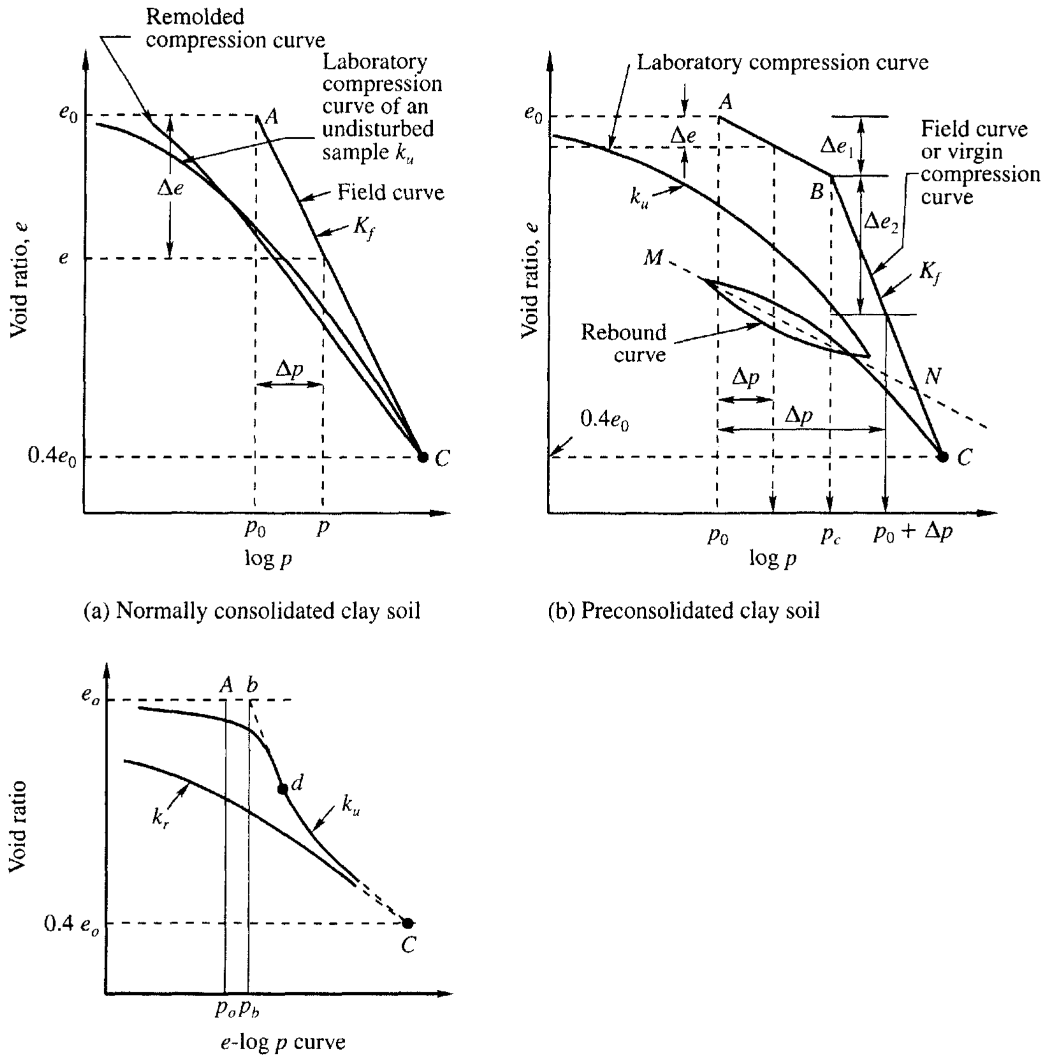
For computing the ultimate settlement of a structure founded on clay the following data are required

1. The thickness of the clay stratum, $H$
2. The initial void ratio, $e_0$
3. The consolidation pressure $p_0$ or $p_c$
4. The field consolidation curve $K_f$

The slope of the field curve $K_f$ on a semilogarithmic diagram is designated as the *compression index* $C_c$ (Fig. 7.9)

The equation for $C_c$ may be written as

$$C_c = \frac{e_0 - e}{\log p - \log p_0} = \frac{e_0 - e}{\log p/p_0} = \frac{\Delta e}{\log p/p_0} \tag{7.4}$$




(a) Normally consolidated clay soil

(b) Preconsolidated clay soil

(c) Typical e-log p curve for an undisturbed sample of clay of high sensitivity (Peck et al., 1974)

**Figure 7.9** Field $e$-log $p$ curves

In one-dimensional compression, as per Eq. (7.2), the change in height $\Delta H$ per unit of original $H$ may be written as equal to the change in volume $\Delta V$ per unit of original volume $V$ (Fig. 7.10).

$$\frac{\Delta H}{H} = \frac{\Delta V}{V} \tag{7.5}$$

Considering a unit sectional area of the clay stratum, we may write

$$V = H, \quad V_1 = H_1$$

$$\Delta V = (H - H_1) = H_s(1 + e_0) - H_s(1 + e_1) = H_s(e_0 - e_1)$$

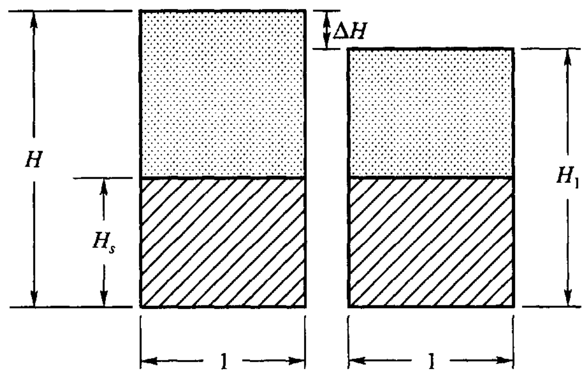



**Figure 7.10** Change of height due to one-dimensional compression

Therefore,

$$\frac{\Delta V}{V} = \frac{H_s(e_0 - e_1)}{H_s(1+e_0)} = \frac{e_0 - e_1}{(1+e_0)} = \frac{\Delta e}{1+e_0} \tag{7.6}$$

Substituting for $\Delta V/V$ in Eq. (7.5)

$$\Delta H = H\frac{\Delta e}{1+e_0} \tag{7.7}$$

If we designate the compression $\Delta H$ of the clay layer as the total settlement $S_t$ of the structure built on it, we have

$$\Delta H = S_t = H\frac{\Delta e}{1+e_0} \tag{7.8}$$

## Settlement Calculation from *e*-log *p* Curves

Substituting for $\Delta e$ in Eq. (7.8) we have

$$S_t = \frac{C_c}{1+e_0} H \log \frac{p}{p_0} \tag{7.9}$$

or $$S_t = \frac{C_c}{1+e_0} H \log \frac{p_0 + \Delta p}{p_0} \tag{7.10}$$

The net change in pressure $\Delta p$ produced by the structure at the middle of a clay stratum is calculated from the Boussinesq or Westergaard theories as explained in Chapter 6.

If the thickness of the clay stratum is too large, the stratum may be divided into layers of smaller thickness not exceeding 3 m. The net change in pressure $\Delta p$ at the middle of each layer will have to be calculated. Consolidation tests will have to be completed on samples taken from the middle of each of the strata and the corresponding compression indices will have to be determined. The equation for the total consolidation settlement may be written as

$$S_t = \quad H_i \frac{C_c}{1+e_0} \log \frac{p_0 + \Delta p}{p_0} \tag{7.11}$$

where the subscript $i$ refers to each layer in the subdivision. If there is a series of clay strata of thickness $H_1$, $H_2$, etc., separated by granular materials, the same Eq. (7.10) may be used for calculating the total settlement.

## Settlement Calculation from *e-p* Curves

We can plot the field *e-p* curves from the laboratory test data and the field *e*-log *p* curves. The weight of a structure or of a fill increases the pressure on the clay stratum from the overburden pressure $p_0$ to the value $p_0 + \Delta p$ (Fig. 7.11). The corresponding void ratio decreases from $e_0$ to $e$. Hence, for the range in pressure from $p_0$ to $(p_0 + \Delta p)$, we may write

$$e_0 - e = \Delta e = a_v \Delta p$$

or $$a_v(\text{cm}^2/\text{gm}) = \frac{\Delta e}{p(\text{cm}^2/\text{gm})} \tag{7.12}$$

where $a_v$ is called the *coefficient of compressibility.*

For a given difference in pressure, the value of the coefficient of compressibility decreases as the pressure increases. Now substituting for $\Delta e$ in Eq. (7.8) from Eq. (7.12), we have the equation for settlement

$$S_t = \frac{a_v H}{1+e_0}\Delta p = m_v H \ \Delta p \tag{7.13}$$

where $m_v = a_v/(1 + e_0)$ is known as the *coefficient of volume compressibility.*

It represents the compression of the clay per unit of original thickness due to a unit increase of the pressure.

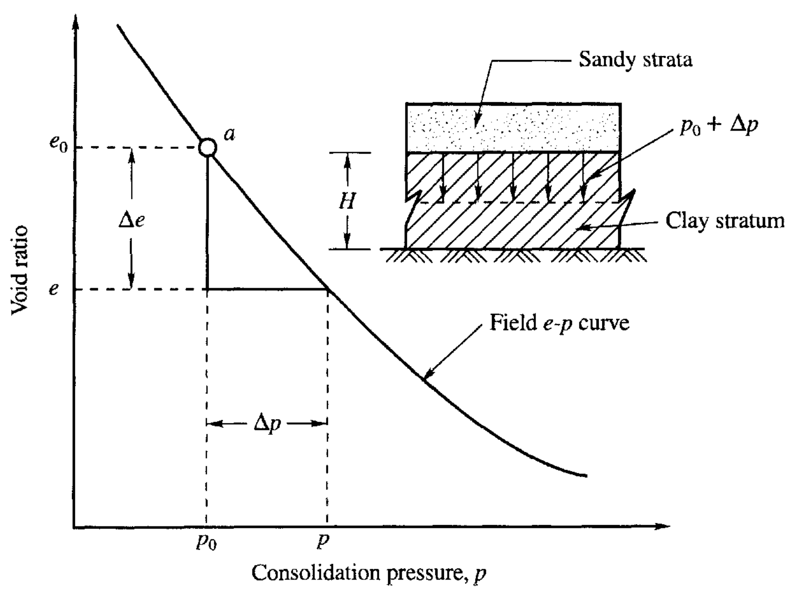



**Figure 7.11** Settlement calculation from *e-p* curve

### Settlement Calculation from *e*-log *p* Curve for Overconsolidated Clay Soil

Fig. 7.9(b) gives the field curve $K_f$ for preconsolidated clay soil. The settlement calculation depends upon the excess foundation pressure $\Delta p$ over and above the existing overburden pressure $p_0$.

### Settlement Computation, if $p_0 + \Delta p \le p_c$ (Fig. 7.9(b))

In such a case, use the sloping line *AB*. If $C_s$ = slope of this line (also called the swell index), we have

$$C_s = \frac{\Delta e}{\log \frac{(p_0 + \Delta p)}{p_0}} \tag{7.14a}$$

$$\text{or } \Delta e = C_s \log \frac{p_0 + \Delta p}{p_0} \tag{7.14b}$$

By substituting for $\Delta e$ in Eq. (7.8), we have

$$S_t = \frac{C_s H}{1 + e_0} \log \frac{p_0 + \Delta p}{p_0} \tag{7.15a}$$

### Settlement Computation, if $p_0 < p_c < p_0 + \Delta p$

We may write from Fig. 7.9(b)

$$\Delta e = \Delta e_1 + \Delta e_2 = C_s \log \frac{p_c}{p_0} + C_c \log \frac{p_0 + \Delta p}{p_c} \tag{7.15b}$$

In this case the slope of both the lines *AB* and *BC* in Fig. 7.9(b) are required to be considered. Now the equation for $S_t$ may be written as [from Eq. (7.8) and Eq. (7.15b)]

$$S_t = \frac{C_s H}{1 + e_0} \log \frac{p_c}{p_0} + \frac{C_c H}{1 + e_0} \log \frac{p_0 + \Delta p}{p_c} \tag{7.15c}$$

The swell index $C_s \approx$ 1/5 to 1/10 $C_c$ can be used as a check.

Nagaraj and Murthy (1985) have proposed the following equation for $C_s$ as

$$C_s = 0.0463 \ \frac{w_l}{100} \ G_s$$

where $w_l$ = liquid limit, $G_s$ = specific gravity of solids.

### Compression Index $C_c$—Empirical Relationships

Research workers in different parts of the world have established empirical relationships between the *compression index* $C_c$ and other soil parameters. A few of the important relationships are given below.

### Skempton's Formula

Skempton (1944) established a relationship between $C_c$ and liquid limits for remolded clays as

$$C_c = 0.007\ (w_l - 10) \tag{7.16}$$

where $w_l$ is in percent.

### Terzaghi and Peck Formula

Based on the work of Skempton and others, Terzaghi and Peck (1948) modified Eq. (7.16) applicable to normally consolidated clays of low to moderate sensitivity as

$$C_c = 0.009\,(w_l - 10) \tag{7.17}$$

### Azzouz et al., Formula

Azzouz et al., (1976) proposed a number of correlations based on the statistical analysis of a number of soils. The one of the many which is reported to have 86 percent reliability is

$$C_c = 0.37\,(e_0 + 0.003\,w_l + 0.0004\,w_n - 0.34) \tag{7.18}$$

where $e_0$ = *in-situ* void ratio, $w_l$ and $w_n$ are in per cent. For organic soil they proposed

$$C_c = 0.115\,w_n \tag{7.19}$$

### Hough's Formula

Hough (1957), on the basis of experiments on precompressed soils, has given the following equation

$$C_c = 0.3\,(e_0 - 0.27) \tag{7. 20}$$

### Nagaraj and Srinivasa Murthy Formula

Nagaraj and Srinivasa Murthy (1985) have developed equations based on their investigation as follows

$$C_c = 0.2343\,e_l \tag{7.21}$$

$$C_c = 0.39\,e_0 \tag{7.22}$$

where $e_l$ is the void ratio at the liquid limit, and $e_0$ is the *in-situ* void ratio.

In the absence of consolidation test data, one of the formulae given above may be used for computing $C_c$ according to the judgment of the engineer.

## 7.9 SETTLEMENT DUE TO SECONDARY COMPRESSION

In certain types of clays the secondary time effects are very pronounced to the extent that in some cases the entire time-compression curve has the shape of an almost straight sloping line when plotted on a semilogarithmic scale, instead of the typical inverted S-shape with pronounced primary consolidation effects. These so called secondary time effects are a phenomenon somewhat analogous to the creep of other overstressed material in a plastic state. A delayed progressive slippage of grain upon grain as the particles adjust themselves to a more dense condition, appears to be responsible for the secondary effects. When the rate of plastic deformations of the individual soil particles or of their slippage on each other is slower than the rate of decreasing volume of voids between the particles, then secondary effects predominate and this is reflected by the shape of the time compression curve. The factors which affect the rate of the secondary compression of soils are not yet fully understood, and no satisfactory method has yet been developed for a rigorous and reliable analysis and forecast of the magnitude of these effects. Highly organic soils are normally subjected to considerable secondary consolidation.

The rate of secondary consolidation may be expressed by the *coefficient of secondary compression,* $\overline{C}_\alpha$ as

$$\overline{C}_\alpha = \frac{\Delta e}{1+e_0}\frac{1}{\log(t_2/t_1)} = \frac{C_\alpha}{1+e_0} \text{ or } \Delta e = C_\alpha \log\frac{t_2}{t_1} \tag{7.23}$$

where $C_a$, the slope of the straight-line portion of the $e$-log $t$ curve, is known as the *secondary compression index*. Numerically $C_a$ is equal to the value of $\Delta e$ for a single cycle of time on the curve (Fig. 7.12(a)). Compression is expressed in terms of decrease in void ratio and time has been normalized with respect to the duration $t_p$ of the primary consolidation stage. A general expression for settlement due to secondary compression under the final stage of pressure $p_f$ may be expressed as

$$S_s = \frac{\Delta e}{1+e_0}H \tag{7.24}$$

The value of $\Delta e$ from $t/t_p = 1$ to any time $t$ may be determined from the $e$ versus $t/t_p$ curve corresponding to the final pressure $p_f$.

Eq. (7.23) may now be expressed as

$$\Delta e = C_\alpha \log\frac{t}{t_p} \tag{7.25}$$

For a constant value $C_\alpha$ between $t_p$ and $t$, Equation (7.24) may be expressed as

$$S_s = \frac{C_\alpha}{1+e_0}H\log\frac{t}{t_p} \tag{7.26}$$

where, $e_0$ = initial void ratio

$H$ = thickness of the clay stratum.

The value of $\overline{C}_\alpha$ for normally loaded compressible soils increases in a general way with the compressibility and hence, with the natural water content, in the manner shown in Fig. 7.12(b) (Mesri, 1973). Although the range in values for a given water content is extremely large, the relation gives a conception of the upper limit of the rate of secondary settlement that may be anticipated if the deposit is normally loaded or if the stress added by the proposed construction will appreciably exceed the preconsolidation stress. The rate is likely to be much less if the clay is strongly preloaded or if the stress after the addition of the load is small compared to the existing overburden pressure. The rate is also influenced by the length of time the preload may have acted,

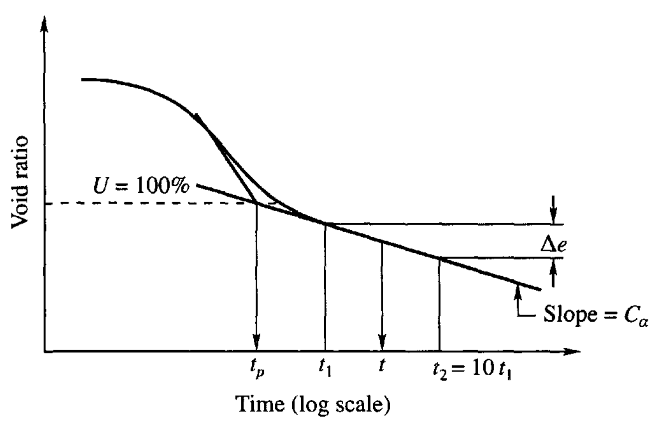



**Figure 7.12(a)** *e*-log *p* time curve representing secondary compression

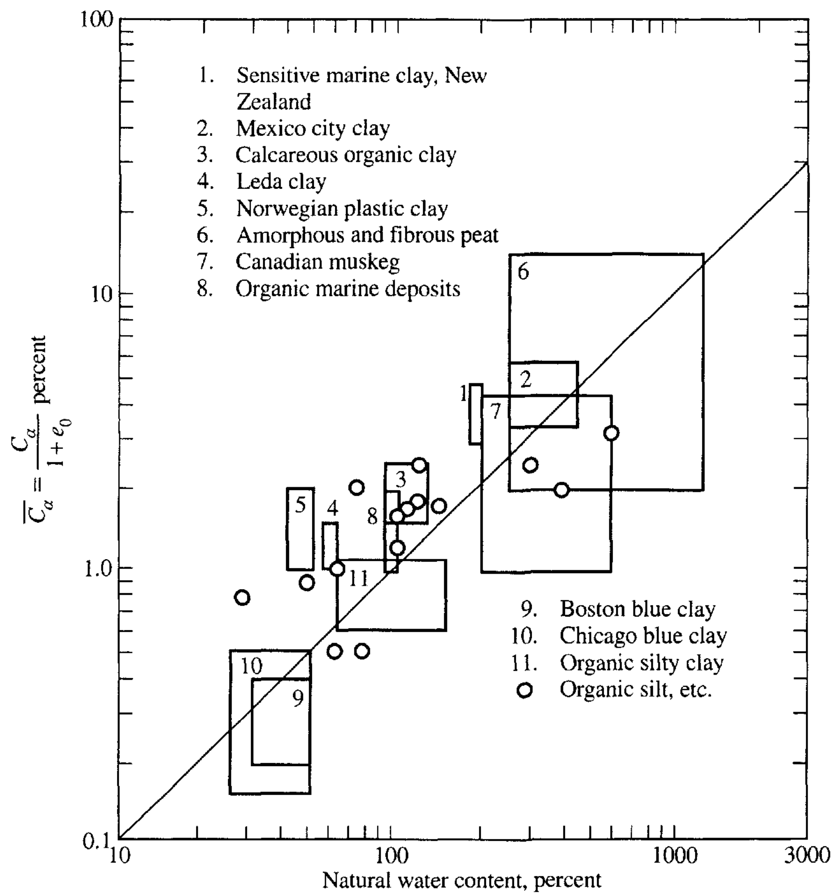



**Figure 7.12(b)** Relationship between coefficient of secondary consolidation and natural water content of normally loaded deposits of clays and various compressible organic soils (after Mesri, 1973)

by the existence of shearing stresses and by the degree of disturbance of the samples. The effects of these various factors have not yet been evaluated. Secondary compression is high in plastic clays and organic soils. Table 7.1 provides a classification of soil based on secondary compressibility. If 'young, normally loaded clay', having an effective overburden pressure of $p_0$ is left undisturbed for thousands of years, there will be creep or secondary consolidation. This will reduce the void ratio and consequently increase the preconsolidation pressure which will be much greater than the existing effective overburden pressure $p_0$. Such a clay may be called an *aged, normally consolidated clay*.

Mesri and Godlewski (1977) report that for any soil the ratio $C_\alpha/C_c$ is a constant (where $C_c$ is the compression index). This is illustrated in Fig. 7.13 for undisturbed specimens of brown Mexico City clay with natural water content $w_n$ = 313 to 340%, $w_l$ = 361%, $w_p$ = 91% and $p_c/p_o$ = 1.4

Table 7.2 gives values of $C_\alpha/C_c$ for some geotechnical materials (Terzaghi, et al., 1996).

It is reported (Terzaghi et al., 1996) that for all geotechnical materials $C_\alpha/C_c$ ranges from 0.01 to 0.07. The value 0.04 is the most common value for inorganic clays and silts.

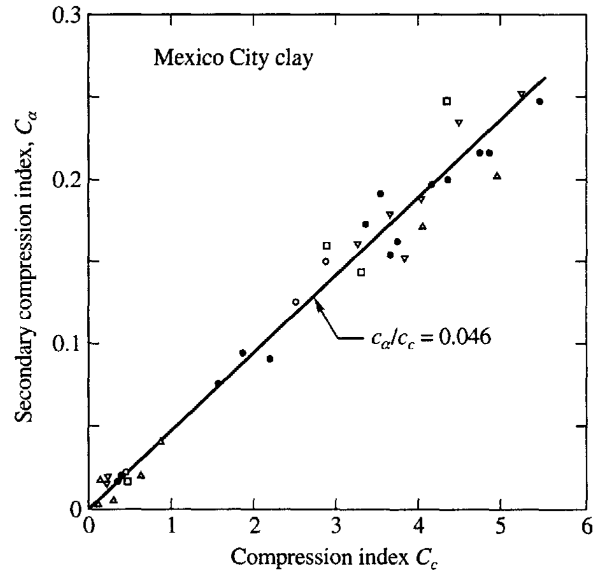



**Figure 7.13** An example of the relation between $C_\alpha$ and $C_c$ (after Mesri and Godlewski, 1977)

**Table 7.1** Classification of soil based on secondary compressibility (Terzaghi, et al., 1996)

| $C_\alpha$ | Secondary compressibility |
|---|---|
| $< 0.002$ | Very low |
| 0.004 | Low |
| 0.008 | Medium |
| 0.016 | High |
| 0.032 | Very high |
| 0.064 | Extremely high |

**Table 7.2** Values of $C_\alpha/C_c$ for geotechnical materials (Terzaghi, et al., 1996)

| Material | $C_\alpha/C_c$ |
|---|---|
| Granular soils including rockfill | $0.02 \pm 0.01$ |
| Shale and mudstone | $0.03 \pm 0.01$ |
| Inorganic clay and silts | $0.04 \pm 0.01$ |
| Organic clays and silts | $0.05 \pm 0.01$ |
| Peat and muskeg | $0.06 \pm 0.01$ |

## Example 7.1

During a consolidation test, a sample of fully saturated clay 3 cm thick ($= h_0$) is consolidated under a pressure increment of 200 kN/m$^2$. When equilibrium is reached, the sample thickness is reduced to 2.60 cm. The pressure is then removed and the sample is allowed to expand and absorb water. The final thickness is observed as 2.8 cm ($h_f$) and the final moisture content is determined as 24.9%.

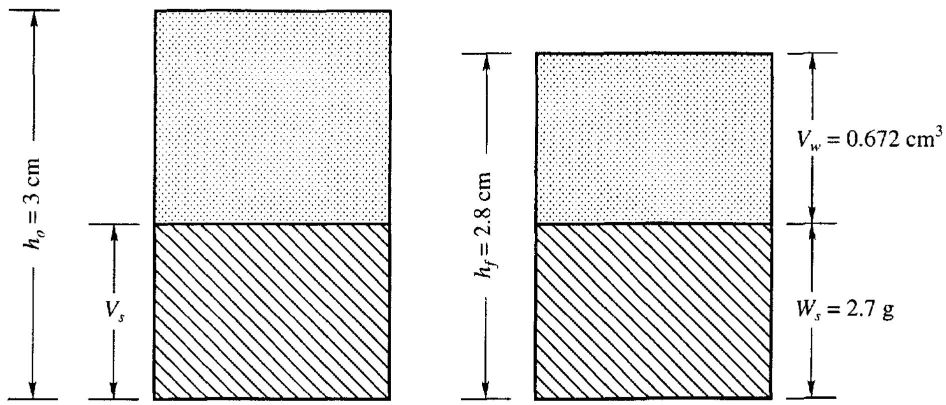



**Figure Ex. 7.1**

If the specific gravity of the soil solids is 2.70, find the void ratio of the sample before and after consolidation.

### Solution

Use equation (7.3)

$$\Delta e = \frac{1+e}{h}\Delta h$$

1. *Determination of* $e_f$

   Weight of solids $= W_s = V_s G_s \gamma_\omega = 1 \times 2.70 \times 1 = 2.70$ g.

   $$\frac{W_w}{W_s} = 0.249 \text{ or } W_w = 0.249 \times 2.70 = 0.672 \text{ gm, } e_f = V_w = 0.672.$$

2. *Changes in thickness from final stage to equilibrium stage with load on*

   $$\Delta h = 2.80 - 2.60 = 0.20 \text{ cm,} \qquad \Delta e = \frac{(1+0.672)\ 0.20}{2.80} = 0.119.$$

   Void ratio after consolidation $= e_f - \Delta e = 0.672 - 0.119 = 0.553.$

3. *Change in void ratio from the commencement to the end of consolidation*

   $$\Delta e = \frac{1+0.553}{2.6}(3.00-2.60) = \frac{1.553}{2.6} \times 0.40 = 0.239.$$

   Void ratio at the start of consolidation $= 0.553 + 0.239 = 0.792$

## Example 7.2

A recently completed fill was 32.8 ft thick and its initial average void ratio was 1.0. The fill was loaded on the surface by constructing an embankment covering a large area of the fill. Some months after the embankment was constructed, measurements of the fill indicated an average void ratio of 0.8. Estimate the compression of the fill.

### Solution

Per Eq. (7.7), the compression of the fill may be calculated as

$$\Delta H = \frac{\Delta e}{1+e_0} H_0$$

where $\Delta H$ = the compression, $\Delta e$ = change in void ratio, $e_0$ = initial void ratio, $H_0$ = thickness of fill.

Substituting, $\Delta H = \dfrac{1.0-0.8}{1+1.0} \times 32.8 = 3.28$ ft.

## Example 7.3

A stratum of normally consolidated clay 7 m thick is located at a depth 12 m below ground level. The natural moisture content of the clay is 40.5 per cent and its liquid limit is 48 per cent. The specific gravity of the solid particles is 2.76. The water table is located at a depth 5 m below ground surface. The soil is sand above the clay stratum. The submerged unit weight of the sand is 11 kN/m$^3$ and the same weighs 18 kN/m$^3$ above the water table. The average increase in pressure at the center of the clay stratum is 120 kN/m$^2$ due to the weight of a building that will be constructed on the sand above the clay stratum. Estimate the expected settlement of the structure.

### Solution

1. Determination of $e$ and $\gamma_b$ for the clay [Fig. Ex. 7.3]

$$\frac{W_w}{W_s} = w_n\,, \quad W_s = V_s G_s \gamma_w = 1 \times 2.76 \times 1 = 2.76 \text{ g}$$

$$W_w = \frac{40.5}{100} \times 2.76 = 1.118 \text{ g}$$

$$V_w = \frac{W_w}{\gamma_w} = \frac{1.118}{1} = 1.118 \text{ cm}^3$$

$$e_0 = \frac{V_w}{V_s} = \frac{1.118}{1} = 1.118$$

$$W = W_w + W_s = 1.118 + 2.76 = 3.878 \text{ g}$$

$$\gamma_t = \frac{W}{1+e_0} = \frac{3.88}{2.118} = 1.83 \text{ g/cm}^3$$

$$\gamma_b = (1.83 - 1) = 0.83 \text{ g/cm}^3 .$$

2. Determination of overburden pressure $p_0$

$$p_0 = \gamma_1 h_1 + \gamma_2 h_2 + \gamma_3 h_3 \text{ or}$$

$$p_0 = 0.83 \times 9.81 \times 3.5 + 11 \times 7 + 18 \times 5 = 195.5 \text{ kN/m}^2$$

3. Compression index [Eq. 11.17]

$$C_c = 0.009(w_l - 10) = 0.009 \times (48 - 10) = 0.34$$

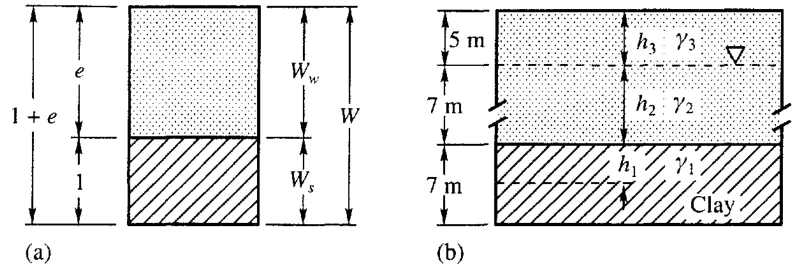



**Fig. Ex. 7.3**

4. Excess pressure

$$\Delta p = 120 \text{ kN/m}^2$$

5. Total Settlement

$$S_t = \frac{C_c}{1+e_0} H \log \frac{p_0 + \Delta p}{p_0}$$

$$= \frac{0.34}{2.118} \times 700 \log \frac{195.5 + 120}{195.5} = 23.3 \text{ cm}$$

Estimated settlement = 23.3 cm.

## Example 7.4

A column of a building carries a load of 1000 kips. The load is transferred to sub soil through a square footing of size 16 × 16 ft founded at a depth of 6.5 ft below ground level. The soil below the footing is fine sand up to a depth of 16.5 ft and below this is a soft compressible clay of thickness 16 ft. The water table is found at a depth of 6.5 ft below the base of the footing. The specific gravities of the solid particles of sand and clay are 2.64 and 2.72 and their natural moisture contents are 25 and 40 percent respectively. The sand above the water table may be assumed to remain saturated. If the plastic limit and the plasticity index of the clay are 30 and 40 percent respectively, estimate the probable settlement of the footing (see Fig. Ex. 7.4)

### Solution

1. Required $\Delta p$ at the middle of the clay layer using the Boussinesq equation

$$\frac{z}{B} = \frac{24.5}{16} = 1.53 < 3.0$$

Divide the footing into 4 equal parts so that $Z/B > 3$

The concentrated load at the center of each part = 250 kips

Radial distance, $r$ = 5.66 ft

By the Boussinesq equation the excess pressure $\Delta p$ at depth 24.5 ft is ($I_B$ = 0.41)

$$\Delta p = 4 \times \frac{Q}{z^2} I_B = \frac{4 \times 250}{24.5^2} \times 0.41 = 0.683 \text{ k/ft}^2$$

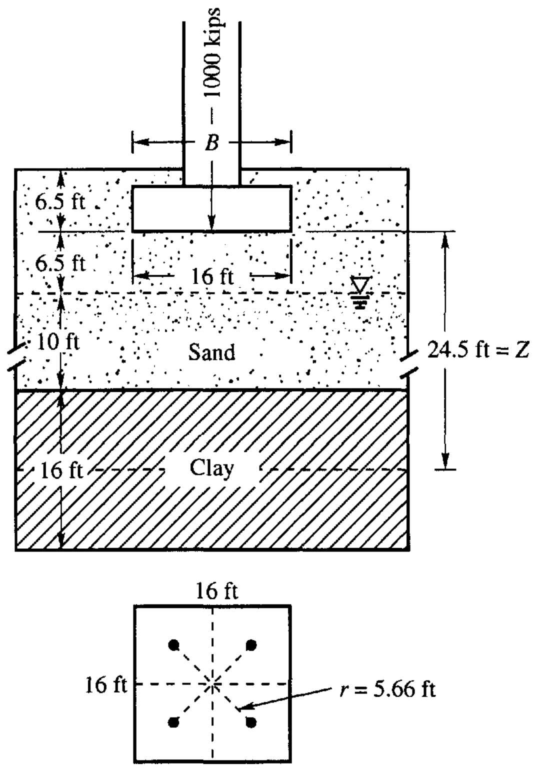



**Figure Ex. 7.4**

2. Void ratio and unit weights

   Per the procedure explained in Ex. 7.3

   For sand $\gamma_t = 124 \text{ lb/ft}^3$ $\gamma_b = 61.6 \text{ lb/ft}^3$

   For clay $\gamma_b = 51.4 \text{ lb/ft}^3$ $e_0 = 1.09$

3. Overburden pressure $p_0$

   $p_0 = 8 \times 51.4 + 10 \times 62 + 13 \times 124 = 2639 \text{ lb/ft}^2$

4. Compression index

   $w_l = I_p + w_p = 40 + 30 = 70\%,$ $C_c = 0.009\,(70 - 10) = 0.54$

   Settlement $S_t = \dfrac{0.54}{1+1.09} \times 16 \times \log \dfrac{2639+683}{2639} = 0.413 \text{ ft} = 4.96 \text{ in}.$

## Example 7.5

Soil investigation at a site gave the following information. Fine sand exists to a depth of 10.6 m and below this lies a soft clay layer 7.60 m thick. The water table is at 4.60 m below the ground surface. The submerged unit weight of sand $\gamma_b$ is 10.4 kN/m$^3$, and the wet unit weight above the water table is 17.6 kN/m$^3$. The water content of the normally consolidated clay $w_n$ = 40%, its liquid limit $w_l$ = 45%, and the specific gravity of the solid particles is 2.78. The proposed construction will transmit a net stress of 120 kN/m$^2$ at the center of the clay layer. Find the average settlement of the clay layer.

### Solution

For calculating settlement [Eq. (7.15a)]

$$S_t = \frac{C_c}{1+e_0} H \log \frac{p_0 + \Delta p}{p_0} \qquad \text{where } \Delta p = 120 \text{ kN/m}^2$$

From Eq. (7.17), $C_c = 0.009 (w_l - 10) = 0.009(45 - 10) = 0.32$

From Eq. (3.14a), $e_0 = \frac{wG}{S} = wG = 0.40 \times 2.78 = 1.11 \quad$ since $S = 1$

$\gamma_b$, the submerged unit weight of clay, is found as follows

$$\gamma_{sat} = \frac{\gamma_w (G_s + e_0)}{1+e_0} = \frac{9.81(2.78+1.11)}{1+1.11} = 18.1 \text{ kN/m}^3$$

$$\gamma_b = \gamma_{sat} - \gamma_w = 18.1 - 9.81 = 8.28 \text{ kN/m}^3$$

The effective vertical stress $p_0$ at the mid height of the clay layer is

$$p_0 = 4.60 \times 17.6 + 6 \times 10.4 + \frac{7.60}{2} \times 8.28 = 174.8 \text{ kN/m}^2$$

Now $\quad S_t = \frac{0.32 \times 7.60}{1+1.11} \log \frac{174.8+120}{174.8} = 0.26 \text{ m} = 26 \text{ cm}$

Average settlement = 26 cm.

## Example 7.6

A soil sample has a compression index of 0.3. If the void ratio $e$ at a stress of 2940 lb/ft$^2$ is 0.5, compute (i) the void ratio if the stress is increased to 4200 lb/ft$^2$, and (ii) the settlement of a soil stratum 13 ft thick.

### Solution

Given: $C_c = 0.3$, $e_1 = 0.50$, $p_1 = 2940$ lb/ft$^2$, $p_2 = 4200$ lb/ft$^2$.

(i) Now from Eq. (7.4),

$$C_c = \frac{e_1 - e_2}{\log p_2 - \log p_1} = \frac{e_1 - e_2}{\log p_2/p_1}$$

or $e_2 = e_1 - C_c \log p_2/p_1$

substituting the known values, we have,

$$e_2 = 0.5 - 0.3 \log \frac{4200}{2940} = 0.454$$

(ii) The settlement per Eq. (7.10) is

$$S = \frac{C_c}{1+e_1} H \log \frac{p_2}{p_1} = \frac{0.3 \times 13 \times 12}{1.5} \log \frac{4200}{2940} = 4.83 \text{ in.}$$

**Example 7.7**

Two points on a curve for a normally consolidated clay have the following coordinates.

Point 1: $e_1 = 0.7$, $p_1 = 2089$ lb/ft$^2$
Point 2: $e_2 = 0.6$, $p_2 = 6266$ lb/ft$^2$

If the average overburden pressure on a 20 ft thick clay layer is 3133 lb/ft$^2$, how much settlement will the clay layer experience due to an induced stress of 3340 lb/ft$^2$ at its middepth.

**Solution**

From Eq. (7.4) we have

$$C_c = \frac{e_1 - e_2}{\log p_2/p_1} = \frac{0.7-0.6}{\log(6266/2089)} = 0.21$$

We need the initial void ratio $e_0$ at an overburden pressure of 3133 lb/ft$^2$.

$$C_c = \frac{e_0 - e_2}{\log p_2/p_0} = 0.21$$

or $(e_0 - 0.6) = 0.21 \log (6266/3133) = 0.063$
or $e_0 = 0.6 + 0.063 = 0.663$.

Settlement, $S = \dfrac{C_c}{1+e_0} H \log \dfrac{p_0 + \Delta p}{p_0}$

Substituting the known values, with $\Delta p = 3340$ lb/ft$^2$

$$S = \frac{0.21\times 20\times 12}{1.663}\log\frac{3133+3340}{3133} = 9.55 \text{ in}$$

## 7.10 RATE OF ONE-DIMENSIONAL CONSOLIDATION THEORY OF TERZAGHI

One dimensional consolidation theory as proposed by Terzaghi is generally applicable in all cases that arise in practice where

1. Secondary compression is not very significant,
2. The clay stratum is drained on one or both the surfaces,
3. The clay stratum is deeply buried, and
4. The clay stratum is thin compared with the size of the loaded areas.

The following assumptions are made in the development of the theory:

1. The voids of the soil are completely filled with water,
2. Both water and solid constituents are incompressible,
3. Darcy's law is strictly valid,
4. The coefficient of permeability is a constant,
5. The time lag of consolidation is due entirely to the low permeability of the soil, and
6. The clay is laterally confined.

### Differential Equation for One-Dimensional Flow

Consider a stratum of soil infinite in extent in the horizontal direction (Fig. 7.14) but of such thickness $H$, that the pressures created by the weight of the soil itself may be neglected in comparison to the applied pressure.

Assume that drainage takes place only at the top and further assume that the stratum has been subjected to a uniform pressure of $p_0$ for such a long time that it is completely consolidated under that pressure and that there is a hydraulic equilibrium prevailing, i.e., the water level in the piezometric tube at any section $XY$ in the clay stratum stands at the level of the water table (piezometer tube in Fig. 7.14).

Let an increment of pressure $\Delta p$ be applied. The total pressure to which the stratum is subjected is

$$p_1 = p_0 + \Delta p \tag{7.27}$$

Immediately after the increment of load is applied the water in the pore space throughout the entire height, $H$, will carry the additional load and there will be set up an excess hydrostatic pressure $u_i$ throughout the pore water equal to $\Delta p$ as indicated in Fig. 7.14.

After an elapsed time $t = t_1$, some of the pore water will have escaped at the top surface and as a consequence, the excess hydrostatic pressure will have been decreased and a part of the load transferred to the soil structure. The distribution of the pressure between the soil and the pore water, $p$ and $u$ respectively at any time $t$, may be represented by the curve as shown in the figure. It is evident that

$$p_1 = p + u \tag{7.28}$$

at any elapsed time t and at any depth $z$, and $u$ is equal to zero at the top. The pore pressure $u$, at any depth, is therefore a function of $z$ and $t$ and may be written as

$$u = f(z, t) \tag{7.29}$$

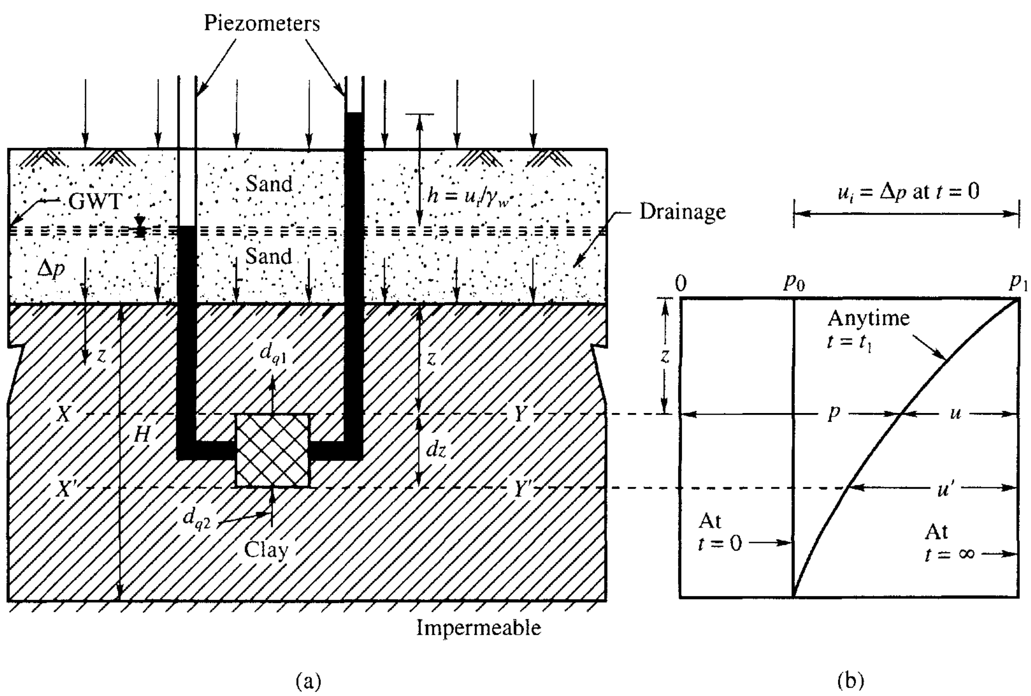



**Figure 7.14** One-dimensional consolidation

Consider an element of volume of the stratum at a depth $z$, and thickness $dz$ (Fig. 7.14). Let the bottom and top surfaces of this element have unit area.

The consolidation phenomenon is essentially a problem of non-steady flow of water through a porous mass. The difference between the quantity of water that enters the lower surface at level $X'Y'$ and the quantity of water which escapes the upper surface at level $XY$ in time element $dt$ must equal the volume change of the material which has taken place in this element of time. The quantity of water is dependent on the hydraulic gradient which is proportional to the slope of the curve $t$.

The hydraulic gradients at levels $XY$ and $X'Y'$ of the element are

$$i = \frac{1}{\gamma_w}\frac{\partial u}{\partial z}$$

$$i' = \frac{1}{\gamma_w}\frac{\partial}{\partial z}\ u + \frac{\partial u}{\partial z}dz \ = \frac{1}{\gamma_w}\frac{\partial u}{\partial z} + \frac{1}{\gamma_w}\frac{\partial^2 u}{\partial z^2}dz \tag{7.30}$$

If $k$ is the hydraulic conductivity the outflow from the element at level $XY$ in time $dt$ is

$$dq_1 = ikdt = \frac{k}{\gamma_w}\frac{\partial u}{\partial z}dt \tag{7.31}$$

The inflow at level $X'Y'$ is

$$dq_2 = ikdt = \frac{k}{\gamma_w}\frac{\partial u}{\partial z}dt + \frac{\partial^2 u}{\partial z^2}dz\ dt \tag{7.32}$$

The difference in flow is therefore

$$dq = dq_1 - dq_2 = -\frac{k}{\gamma_w}\frac{\partial^2 u}{\partial z^2}dz\ dt \tag{7.33}$$

From the consolidation test performed in the laboratory, it is possible to obtain the relationship between the void ratios corresponding to various pressures to which a soil is subjected. This relationship is expressed in the form of a pressure-void ratio curve which gives the relationship as expressed in Eq. (7.12)

$$de = a_v dp \tag{7.34}$$

The change in volume $\Delta dv$ of the element given in Fig. 7.14 may be written as per Eq. (7.7).

$$\Delta dv = \Delta dz = \frac{de}{1+e}dz \tag{7.35}$$

Substituting for $de$, we have

$$\Delta dv = \frac{a_v}{1+e}dp\ dz \tag{7.36}$$

Here $dp$ is the change in effective pressure at depth $z$ during the time element $dt$. The increase in effective pressure $dp$ is equal to the decrease in the pore pressure, $du$.

Therefore, $$dp = -du = \frac{\partial u}{\partial t}dt \tag{7.37}$$

Hence, $\Delta dv = -\frac{a_v}{1+e}\frac{\partial u}{\partial t}dt\,dz = -m_v\frac{\partial u}{\partial t}dt\,dz$ (7.38)

Since the soil is completely saturated, the volume change $\Delta dv$ of the element of thickness $dz$ in time $dt$ is equal to the change in volume of water $dq$ in the same element in time $dt$.

Therefore, $dq = \Delta dv$ (7.39)

or $$-\frac{k}{\gamma_w}\frac{\partial^2 u}{\partial z^2}dz\,dt = -\frac{a_v}{1+e}\frac{\partial u}{\partial t}dz\,dt$$

or $$\frac{k(1+e)}{\gamma_w a_v}\frac{\partial^2 u}{\partial z^2} = \frac{\partial u}{\partial t} = c_v\frac{\partial^2 u}{\partial z^2} \quad (7.40)$$

where $$c_v = \frac{k(1+e)}{\gamma_w a_v} = \frac{k}{\gamma_w m_v} \quad (7.41)$$

is defined as the *coefficient of consolidation.*

Eq. (7.40) is the differential equation for one-dimensional flow. The differential equation for three-dimensional flow may be developed in the same way. The equation may be written as

$$\frac{\partial u}{\partial t} = \frac{1+e}{\gamma_w a_v}\left(k_x\frac{\partial^2 u}{\partial x^2} + k_y\frac{\partial^2 u}{\partial y^2} + k_z\frac{\partial^2 u}{\partial z^2}\right) \quad (7.42)$$

where $k_x$, $k_y$ and $k_z$ are the coefficients of permeability (hydraulic conductivity) in the coordinate directions of $x$, $y$ and $z$ respectively.

As consolidation proceeds, the values of $k$, $e$ and $a_v$ all decrease with time but the ratio expressed by Eq. (7.41) may remain approximately constant.

## Mathematical Solution for the One-Dimensional Consolidation Equation

To solve the consolidation Eq. (7.40) it is necessary to set up the proper boundary conditions. For this purpose, consider a layer of soil having a total thickness $2H$ and having drainage facilities at both the top and bottom faces as shown in Fig. 7.15. Under this condition no flow will take place across the center line at depth $H$. The center line can therefore be considered as an impervious barrier. The boundary conditions for solving Eq. (7.40) may be written as

1. $u = 0$ when $z = 0$
2. $u = 0$ when $z = 2H$
3. $u = \Delta p$ for all depths at time $t = 0$

On the basis of the above conditions, the solution of the differential Eq. (7.40) can be accomplished by means of Fourier Series.

The solution is

$$u = \sum_{N=0}^{N=\infty}\frac{2\Delta p}{m}\sin\frac{mz}{H}\,\varepsilon^{-m^2T} \quad (7.43)$$

where $m = \frac{(2N+1)\pi}{2}$, $T = \frac{c_v t}{H^2}$ = a non-dimensional time factor.

Eq. (7.43) can be expressed in a general form as

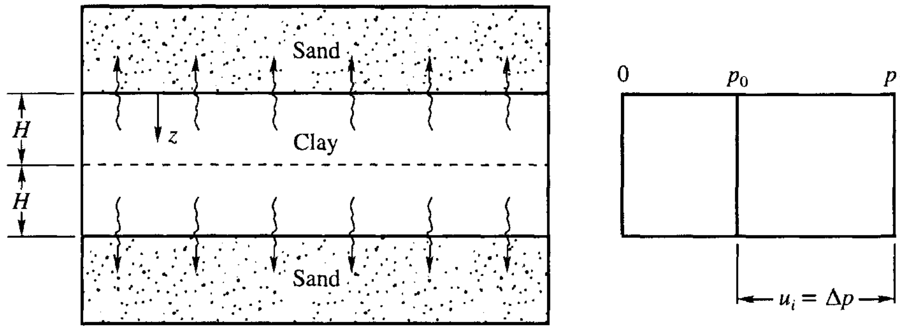



**Figure 7.15** Boundary conditions

$$\frac{u}{\Delta p} = f\left(\frac{z}{H}, T\right) \tag{7.44}$$

Equation (7.44) can be solved by assuming $T$ constant for various values of $z/H$. Curves corresponding to different values of the *time factor* $T$ may be obtained as given in Fig. 7.16. It is of interest to determine how far the consolidation process under the increment of load $\Delta p$ has progressed at a time $t$ corresponding to the *time factor* $T$ at a given depth $z$. The term $U_z$ is used to express this relationship. It is defined as the ratio of the amount of consolidation which has already taken place to the total amount which is to take place under the load increment.

The curves in Fig. 7.16 shows the distribution of the pressure $\Delta p$ between solid and liquid phases at various depths. At a particular depth, say $z/H = 0.5$, the stress in the soil skeleton is represented by $AC$ and the stress in water by $CB$. $AB$ represents the original excess hydrostatic pressure $u_i = \Delta p$. The degree of consolidation $U_z$ percent at this particular depth is then

$$U_z\% = 100 \times \frac{AC}{AB} = \frac{\Delta p - u}{\Delta p} = 100\left(1 - \frac{u}{\Delta p}\right) \tag{7.45}$$

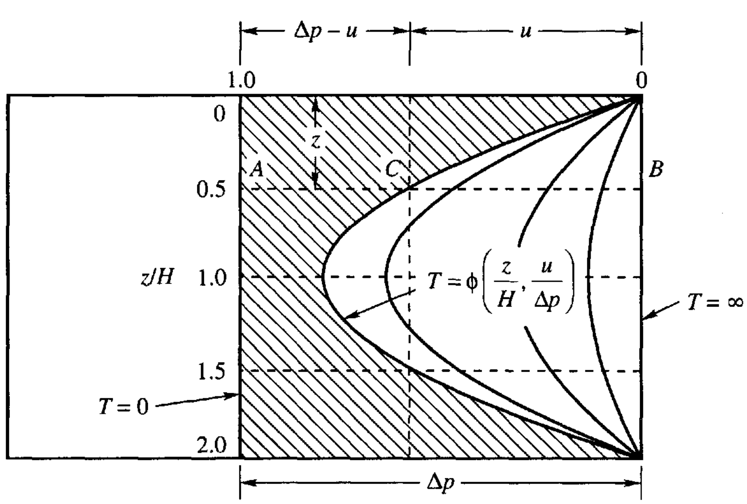



**Figure 7.16** Consolidation of clay layer as a function *T*

Following a similar reasoning, the average degree of consolidation $U\%$ for the entire layer at a time factor $T$ is equal to the ratio of the shaded portion (Fig. 7.16) of the diagram to the entire area which is equal to $2H\,\Delta p$.

Therefore

$$U\% = \frac{\int_0^{2H} (\Delta p - u)dz}{2H\Delta p} \times 100$$

$$\text{or} \quad U\% = \frac{100}{2H}\left(2H - \frac{1}{\Delta p}\int_0^{2H} udz\right) \tag{7.46}$$

Hence, Eq. (7.46) after integration reduces to

$$U\% = 100\left(1 - \sum_{N=0}^{N=\infty} \frac{2}{m^2}\varepsilon^{-m^2T}\right) \tag{7.47}$$

It can be seen from Eq. (7.47) that the degree of consolidation is a function of the time factor $T$ only which is a dimensionless ratio. The relationship between $T$ and $U\%$ may therefore be established once and for all by solving Eq. (7.47) for various values of $T$. Values thus obtained are given in Table 7.3 and also plotted on a semilog plot as shown in Fig. 7.17.

For values of $U\%$ between 0 and 60%, the curve in Fig. 7.17 can be represented almost exactly by the equation

$$T = \frac{\pi}{4}\left(\frac{U\%}{100}\right)^2 \tag{7.48}$$

which is the equation of a parabola. Substituting for $T$, Eq. (7.48) may be written as

$$\frac{U\%}{100} = \sqrt{\frac{4c_v}{\pi H^2}}\sqrt{t} \tag{7.49}$$

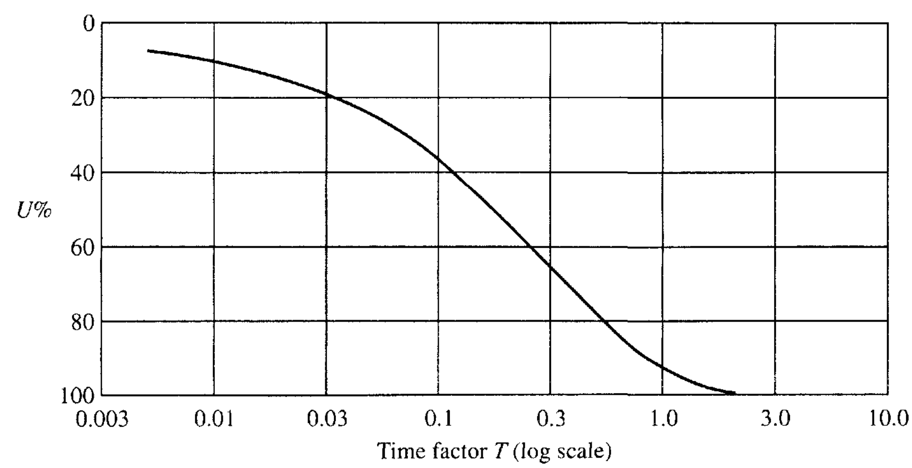


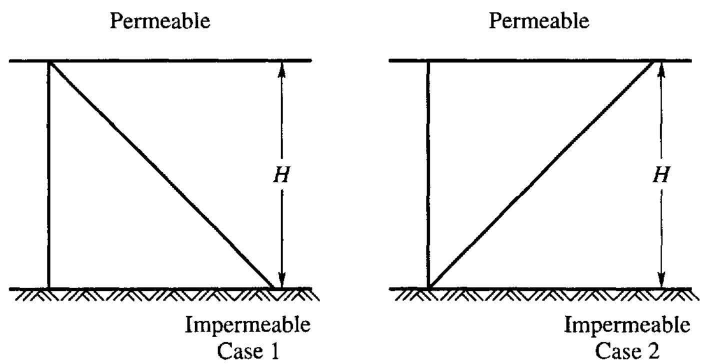

**Figure 7.17** *U* versus *T*

**Table 7.3** Relationship between $U$ and $T$

| $U$% | $T$ | $U$% | $T$ | $U$% | $T$ |
|---|---|---|---|---|---|
| 0 | 0 | 40 | 0.126 | 75 | 0.477 |
| 10 | 0.008 | 45 | 0.159 | 80 | 0.565 |
| 15 | 0.018 | 50 | 0.197 | 85 | 0.684 |
| 20 | 0.031 | 55 | 0.238 | 90 | 0.848 |
| 25 | 0.049 | 60 | 0.287 | 95 | 1.127 |
| 30 | 0.071 | 65 | 0.342 | 100 | ∞ |
| 35 | 0.096 | 70 | 0.405 | | |

In Eq. (7.49), the values of $c_v$ and $H$ are constants. One can determine the time required to attain a given degree of consolidation by using this equation. It should be noted that $H$ represents half the thickness of the clay stratum when the layer is drained on both sides, and it is the full thickness when drained on one side only.

**TABLE 7.4** Relation between U% and T (Special Cases)




| | Time Factors, $T$ | |
|---|---|---|
| $U$% | Consolidation pressure increase with depth | Consolidation pressure decreases with depth |
| 00 | 0 | 0 |
| 10 | 0.047 | 0.003 |
| 20 | 0.100 | 0.009 |
| 30 | 0.158 | 0.024 |
| 40 | 0.221 | 0.048 |
| 50 | 0.294 | 0.092 |
| 60 | 0.383 | 0.160 |
| 70 | 0.500 | 0.271 |
| 80 | 0.665 | 0.44 |
| 90 | 0.94 | 0.72 |
| 95 | 1 | 0.8 |
| 100 | ∞ | ∞ |

For values of $U\%$ greater than 60%, the curve in Fig. 7.17 may be represented by the equation

$$T = 1.781 - 0.933 \log (100 - U\%) \tag{7.50}$$

### Effect of Boundary Conditions on Consolidation

A layer of clay which permits drainage through both surfaces is called an *open layer*. The thickness of such a layer is always represented by the symbol $2H$, in contrast to the symbol $H$ used for the thickness of half-closed layers which can discharge their excess water only through one surface.

The relationship expressed between $T$ and $U$ given in Table 7.3 applies to the following cases:

1. Where the clay stratum is drained on both sides and the initial consolidation pressure distribution is uniform or linearly increasing or decreasing with depth.
2. Where the clay stratum is drained on one side but the consolidation pressure is uniform with depth.

Separate relationships between $T$ and $U$ are required for half closed layers with thickness $H$ where the consolidation pressures increase or decrease with depth. Such cases are exceptional and as such not dealt with in detail here. However, the relations between $U\%$ and $T$ for these two cases are given in Table 7.4.

## 7.11 DETERMINATION OF THE COEFFICIENT OF CONSOLIDATION

The coefficient of consolidation $c_v$ can be evaluated by means of laboratory tests by fitting the experimental curve with the theoretical.

There are two laboratory methods that are in common use for the determination of $c_v$. They are

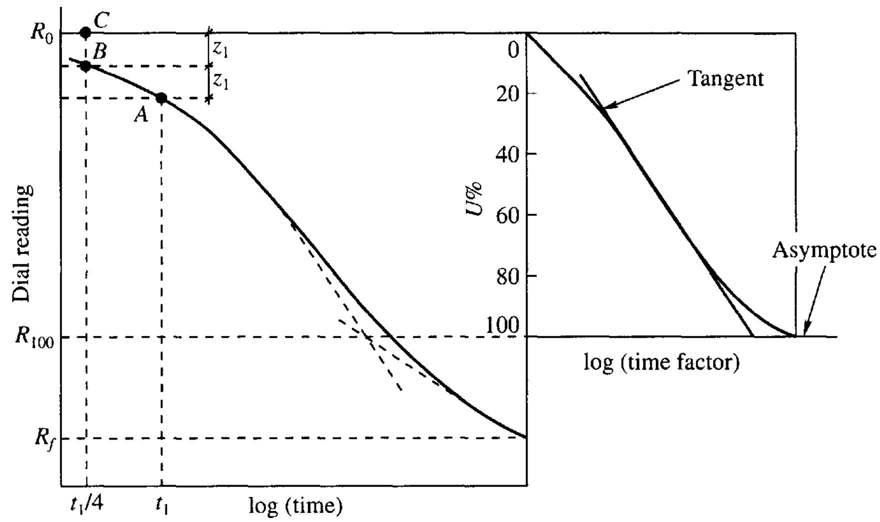
1. Casagrande Logarithm of Time Fitting Method.
2. Taylor Square Root of Time Fitting Method.

### Logarithm of Time Fitting Method

This method was proposed by Casagrande and Fadum (1940).

Figure 7.18 is a plot showing the relationship between compression dial reading and the logarithm of time of a consolidation test. The theoretical consolidation curve using the log scale for the time factor is also shown. There is a similarity of shape between the two curves. On the laboratory curve, the intersection formed by the final straight line produced backward and the tangent to the curve at the point of inflection is accepted as the 100 per cent primary consolidation point and the dial reading is designated as $R_{100}$. The time-compression relationship in the early stages is also parabolic just as the theoretical curve. The dial reading at zero primary consolidation $R_0$ can be obtained by selecting any two points on the parabolic portion of the curve where times are in the ratio of 1 : 4. The difference in dial readings between these two points is then equal to the difference between the first point and the dial reading corresponding to zero primary consolidation. For example, two points $A$ and $B$ whose times 10 and 2.5 minutes respectively, are marked on the curve. Let $z_1$ be the ordinate difference between the two points. A point C is marked vertically over $B$ such that $BC = z_1$. Then the point $C$ corresponds to zero primary consolidation. The procedure is repeated with several points. An average horizontal line is drawn through these points to represent the theoretical zero percent consolidation line.

The interval between 0 and 100% consolidation is divided into equal intervals of percent consolidation. Since it has been found that the laboratory and the theoretical curves have better




(a) Experimental curve (b) Theoretical curve

**Figure 7.18** Log of time fitting method

correspondence at the central portion, the value of $c_v$ is computed by taking the time $t$ and time factor $T$ at 50 percent consolidation. The equation to be used is

$$T_{50} = \frac{c_v t_{50}}{H_{dr}^2} \quad \text{or} \quad c_v = \frac{T_{50}}{t_{50}} H_{dr}^2 \tag{7.51}$$

where $H_{dr}$ = drainage path

From Table 7.3, we have at $U = 50\%$, $T = 0.197$. From the initial height $H_i$ of specimen and compression dial reading at 50% consolidation, $H_{dr}$ for double drainage is

$$H_{dr} = \frac{H_i - \Delta H}{2} \tag{7.52}$$

where $\Delta H$= Compression of sample up to 50% consolidation.

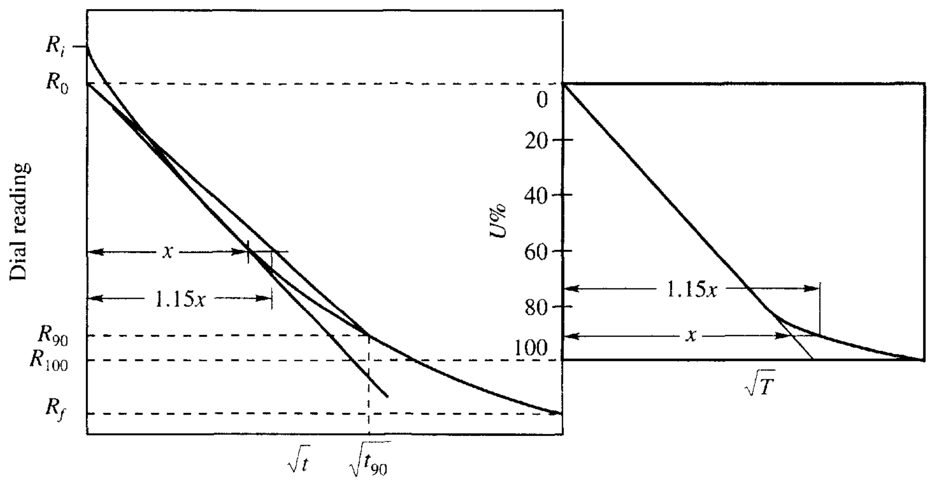
Now the equation for $c_v$ may be written as

$$c_v = 0.197 \frac{H_{dr}^2}{t_{50}} \tag{7.53}$$

## Square Root of Time Fitting Method

This method was devised by Taylor (1948). In this method, the dial readings are plotted against the square root of time as given in Fig. 7.19(a). The theoretical curve $U$ versus $\sqrt{T}$ is also plotted and shown in Fig. 7.19(b). On the theoretical curve a straight line exists up to 60 percent consolidation while at 90 percent consolidation the abscissa of the curve is 1.15 times the abscissa of the straight line produced.

The fitting method consists of first drawing the straight line which best fits the early portion of the laboratory curve. Next a straight line is drawn which at all points has abscissa 1.15 times as great as those of the first line. The intersection of this line and the laboratory curve is taken as the 90 percent ($R_{90}$) consolidation point. Its value may be read and is designated as $t_{90}$.




(a) Experimental curve (b) Theoretical curve

**Figure 7.19** Square root of time fitting method

Usually the straight line through the early portion of the laboratory curve intersects the zero time line at a point ($R_o$) differing somewhat from the initial point ($R_i$). This intersection point is called the *corrected zero point*. If one-ninth of the vertical distance between the corrected zero point and the 90 per cent point is set off below the 90 percent point, the point obtained is called the "*100 percent primary compression point*" ($R_{100}$). The compression between zero and 100 per cent point is called "*primary compression*".

At the point of 90 percent consolidation, the value of $T = 0.848$. The equation of $c_v$ may now be written as

$$c_v = 0.848\frac{H_{dr}^2}{t_{90}} \tag{7.54}$$

where $H_{dr}$ = drainage path (average)

## 7.12 RATE OF SETTLEMENT DUE TO CONSOLIDATION

It has been explained that the ultimate settlement $S_t$ of a clay layer due to consolidation may be computed by using either Eq. (7.10) or Eq. (7.13). If $S$ is the settlement at any time $t$ after the imposition of load on the clay layer, the degree of consolidation of the layer in time $t$ may be expressed as

$$U\% = \frac{S}{S_t} \times 100 \text{ percent} \tag{7.55}$$

Since $U$ is a function of the time factor $T$, we may write

$$U\% = 100f(T) = \frac{S}{S_t} \times 100 \tag{7.56}$$

The rate of settlement curve of a structure built on a clay layer may be obtained by the following procedure:

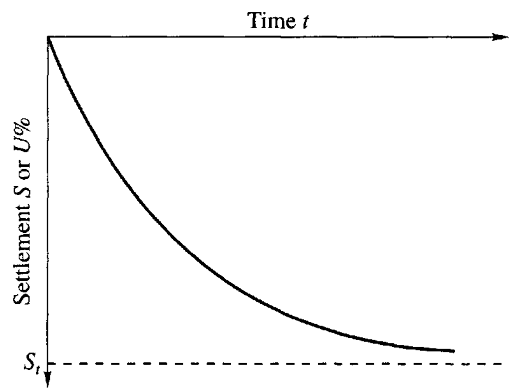



**Figure 7.20** Time-settlement curve

1. From consolidation test data, compute $m_v$ and $c_v$.
2. Compute the total settlement $S_t$ that the clay stratum would experience with the increment of load $\Delta p$.
3. From the theoretical curve giving the relation between $U$ and $T$, find $T$ for different degrees of consolidation, say 5, 10, 20, 30 percent etc.
4. Compute from equation $t = \frac{TH_{dr}^2}{c_v}$ the values of $t$ for different values of T. It may be noted here that for drainage on both sides $H_{dr}$ is equal to half the thickness of the clay layer.
5. Now a curve can be plotted giving the relation between $t$ and $U\%$ or $t$ and $S$ as shown in Fig. 7.20.

## 7.13 TWO- AND THREE-DIMENSIONAL CONSOLIDATION PROBLEMS

When the thickness of a clay stratum is great compared with the width of the loaded area, the consolidation of the stratum is three-dimensional. In a three-dimensional process of consolidation the flow occurs either in radial planes or else the water particles travel along flow lines which do not lie in planes. The problem of this type is complicated though a general theory of three-dimensional consolidation exists (Biot, et al., 1941). A simple example of three-dimensional consolidation is the consolidation of a stratum of soft clay or silt by providing sand drains and surcharge for accelerating consolidation.

The most important example of two dimensional consolidation in engineering practice is the consolidation of the case of a hydraulic fill dam. In two-dimensional flow, the excess water drains out of the clay in parallel planes. Gilboy (1934) has analyzed the two dimensional consolidation of a hydraulic fill dam.

### Example 7.8

A 2.5 cm thick sample of clay was taken from the field for predicting the time of settlement for a proposed building which exerts a uniform pressure of 100 kN/m$^2$ over the clay stratum. The sample was loaded to 100 kN/m$^2$ and proper drainage was allowed from top and bottom. It was seen that 50 percent of the total settlement occurred in 3 minutes. Find the time required for 50 percent of the total settlement of the building, if it is to be constructed on a 6 m thick layer of clay which extends from the ground surface and is underlain by sand.

### Solution

$T$ for 50% consolidation = 0.197.

The lab sample is drained on both sides. The coefficient of consolidation $c_v$ is found from

$$c_v = \frac{TH_{dr}^2}{t} = 0.197 \times \frac{(2.5)^2}{4} \times \frac{1}{3} = 10.25 \times 10^{-2}\ \text{cm}^2/\text{min}.$$

The time $t$ for 50% consolidation in the field will be found as follows.

$$t = \frac{0.197 \times 300 \times 300 \times 100}{10.25 \times 60 \times 24} = 120\ \text{days}.$$

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## Example 7.9

The void ratio of a clay sample A decreased from 0.572 to 0.505 under a change in pressure from 122 to 180 kN/m$^2$. The void ratio of another sample $B$ decreased from 0.61 to 0.557 under the same increment of pressure. The thickness of sample $A$ was 1.5 times that of $B$. Nevertheless the time taken for 50% consolidation was 3 times larger for sample $B$ than for $A$. What is the ratio of coefficient of permeability of sample $A$ to that of $B$?

### Solution

Let $H_a$ = thickness of sample $A$, $H_b$ = thickness of sample $B$, $m_{va}$ = coefficient of volume compressibility of sample $A$, $m_{vb}$ = coefficient of volume compressibility of sample $B$, $c_{va}$ = coefficient of consolidation for sample $A$, $c_{vb}$ = coefficient of consolidation for sample $B$, $\Delta p_a$ = increment of load for sample $A$, $\Delta p_b$ = increment of load for sample $B$, $k_a$ = coefficient of permeability for sample $A$, and $k_b$ = coefficient of permeability of sample $B$.

We may write the following relationship

$$m_{va} = \frac{\Delta e_a}{1+e_a}\frac{1}{\Delta p_a},\quad m_{vb} = \frac{\Delta e_b}{1+e_b}\frac{1}{\Delta p_b}$$

where $e_a$ is the void ratio of sample $A$ at the commencement of the test and $\Delta e_a$ is the change in void ratio. Similarly $e_b$ and $\Delta e_b$ apply to sample $B$.

$$\frac{m_{va}}{m_{vb}} = \frac{\Delta p_b}{\Delta p_a}\frac{\Delta e_a}{\Delta e_b}\frac{1+e_b}{1+e_a},\ \text{and}\ T_a = \frac{c_{va}t_a}{H_a^2},\ T_b = \frac{c_{vb}t_b}{H_b^2}$$

wherein $T_a$, $t_a$, $T_b$ and $t_b$ correspond to samples $A$ and $B$ respectively. We may write

$$\frac{c_{va}}{c_{vb}} = \frac{T_a}{T_b}\frac{H_a^2}{H_b^2}\frac{t_b}{t_a},\qquad k_a = c_{va}m_{va}\gamma_w,\qquad k_b = c_{vb}m_{vb}\gamma_w$$

Therefore, $\dfrac{k_a}{k_b} = \dfrac{c_{va}}{c_{vb}}\dfrac{m_{va}}{m_{vb}}$

Given $e_a = 0.572$, and $e_b = 0.61$

$$\Delta e_a = 0.572 - 0.505 = 0.067,\ \Delta e_b = 0.610 - 0.557 = 0.053$$

$$\Delta p_a = \Delta p_b = 180 - 122 = 58 \text{ kN/m}^2, \qquad H_a = 1.5\, H_b$$

But $t_b = 3t_a$

We have, $$\frac{m_{va}}{m_{vb}} = \frac{0.067}{0.053} \times \frac{1+0.61}{1+0.572} = 1.29$$

$$\frac{c_{va}}{c_{vb}} = 1.5^2 \times 3 = 6.75$$

Therefore, $$\frac{k_a}{k_b} = 6.75 \times 1.29 = 8.7$$

The ratio is 8.7 : 1.

## Example 7.10

A strata of normally consolidated clay of thickness 10 ft is drained on one side only. It has a hydraulic conductivity of $k = 1.863 \times 10^{-8}$ in/sec and a coefficient of volume compressibility $m_v = 8.6 \times 10^{-4}$ in$^2$/lb. Determine the ultimate value of the compression of the stratum by assuming a uniformly distributed load of 5250 lb/ft$^2$ and also determine the time required for 20 percent and 80 percent consolidation.

### Solution

Total compression,

$$S_t = m_v H \Delta p = 8.6 \times 10^{-4} \times 10 \times 12 \times 5250 \times \frac{1}{144} = 3.763 \text{ in.}$$

For determining the relationship between $U\%$ and $T$ for 20% consolidation use the equation

$$T = \frac{\pi}{4}\left(\frac{U\%}{100}\right)^2 \text{ or } T = \frac{3.14}{4} \times \left(\frac{20}{100}\right)^2 = 0.0314$$

For 80% consolidation use the equation

$$T = 1.781 - 0.933 \log (100 - U\%)$$

Therefore $T = 1.781 - 0.933 \log_{10} (100 - 80) = 0.567$.

The coefficient of consolidation is

$$c_v = \frac{k}{\gamma_w m_v} = \frac{1.863 \times 10^{-8}}{3.61 \times 10^{-2} \times 8.6 \times 10^{-4}} = 6 \times 10^{-4} \text{ in}^2/\text{sec}$$

The times required for 20% and 80% consolidation are

$$t_{20} = \frac{H_{dr}^2 T}{c_v} = \frac{(10 \times 12)^2 \times 0.0314}{6 \times 10^{-4} \times 60 \times 60 \times 24} = 8.72 \text{ days}$$

$$t_{80} = \frac{H_{dr}^2 T}{c_v} = \frac{(10 \times 12)^2 \times 0.567}{6 \times 10^{-4} \times 60 \times 60 \times 24} = 157.5 \text{ days}$$

## Example 7.11

The loading period for a new building extended from May 1995 to May 1997. In May 2000, the average measured settlement was found to be 11.43 cm. It is known that the ultimate settlement will be about 35.56 cm. Estimate the settlement in May 2005. Assume double drainage to occur.

### Solution

For the majority of practical cases in which loading is applied over a period, acceptable accuracy is obtained when calculating time-settlement relationships by assuming the time datum to be midway through the loading or construction period.

$S_t = 11.43$ cm when $t = 4$ years and $S = 35.56$ cm.

The settlement is required for $t = 9$ years, that is, up to May2005. Assuming as a starting point that at $t = 9$ years, the degree of consolidation will be = 0.60. Under these conditions per Eq. (7.48), $U = 1.13\sqrt{T}$.

If $S_{t_1}$ = settlement at time $t_1$, $S_{t_2}$ = settlement at time $t_2$

$$\frac{S_{t_1}}{S_{t_2}} = \frac{U_1}{U_2} = \sqrt{\frac{T_1}{T_2}} = \sqrt{\frac{t_1}{t_2}} \quad \text{since} \quad T = \frac{c_v t}{H_{dr}^2}$$

where $\frac{c_v}{H_{dr}^2}$ is a constant. Therefore $\frac{11.43}{S_{t_2}} = \sqrt{\frac{4}{9}}$ or $S_{t_2} = 17.15$ cm

Therefore at $t = 9$ years, $U = \frac{17.5}{35.56} = 0.48$

Since the value of $U$ is less than 0.60 the assumption is valid. Therefore the estimated settlement is 17.15 cm. In the event of the degree of consolidation exceeding 0.60, equation (7.50) has to be used to obtain the relationship between $T$ and $U$.

## Example 7.12

An oedometer test is performed on a 2 cm thick clay sample. After 5 minutes, 50% consolidation is reached. After how long a time would the same degree of consolidation be achieved in the field where the clay layer is 3.70 m thick? Assume the sample and the clay layer have the same drainage boundary conditions (double drainage).

### Solution

The time factor $T$ is defined as $T = \frac{c_v t}{H_{dr}^2}$

where $H_{dr}$ = half the thickness of the clay for double drainage.

Here, the time factor $T$ and coefficient of consolidation are the same for both the sample and the field clay layer. The parameter that changes is the time $t$. Let $t_1$ and $t_2$ be the times required to reach 50% consolidation both in the oedometer and field respectively. $t_1 = 5$ min

$$\text{Therefore} \quad \frac{c_v t_1}{H_{dr(1)}^2} = \frac{c_v t_2}{H_{dr(2)}^2}$$

$$\text{Now } t_2 = \left(\frac{H_{dr(2)}}{H_{dr(1)}}\right)^2 t_1 = \left(\frac{370}{2}\right)^2 \times 5 \times \frac{1}{60} \times \frac{1}{24} \text{ days} \approx 119 \text{ days}.$$

## Example 7.13

A laboratory sample of clay 2 cm thick took 15 min to attain 60 percent consolidation under a double drainage condition. What time will be required to attain the same degree of consolidation for a clay layer 3 m thick under the foundation of a building for a similar loading and drainage condition?

### Solution

Use Eq. (7.50) for $U > 60\%$ for determining $T$

$$T = 1.781 - 0.933 \log (100 - U\%)$$

$$= 1.781 - 0.933 \log (100 - 60) = 0.286.$$

From Eq. (7.51) the coefficient of consolidation, $c_v$ is

$$c_v = \frac{TH_{dr}^2}{t} = \frac{0.286 \times (1)^2}{15} = 1.91 \times 10^{-2} \text{ cm}^2/\text{min}.$$

The value of $c_v$ remains constant for both the laboratory and field conditions. As such, we may write,

$$\left(\frac{TH_{dr}^2}{t}\right)_{lab} = \left(\frac{TH_{dr}^2}{t}\right)_{field}$$

where $H_{dr}$ = half the thickness = 1 cm for the lab sample and 150 cm for field stratum, and $t_{lab}$ = 15 min.

Therefore,

$$\left(\frac{T(1)^2}{0.25}\right)_{lab} = \left(\frac{T(150)^2}{t}\right)_{field}$$

or $t_f = (150)^2 \times 0.25 = 5625$ hr or 234 days (approx).
for the field stratum to attain the same degree of consolidation.

## 7.14 PROBLEMS

7.1 A bed of sand 10 m thick is underlain by a compressible of clay 3 m thick under which lies sand. The water table is at a depth of 4 m below the ground surface. The total unit weights of sand below and above the water table are 20.5 and 17.7 kN/m$^3$ respectively. The clay has a natural water content of 42%, liquid limit 46% and specific gravity 2.76. Assuming the clay to be normally consolidated, estimate the probable final settlement under an average excess pressure of 100 kN/m$^2$.

7.2 The effective overburden pressure at the middle of a saturated clay layer 12 ft thick is 2100 lb/ft$^2$ and is drained on both sides. The overburden pressure at the middle of the clay stratum is expected to be increased by 3150 lb/ft$^2$ due to the load from a structure at the ground surface. An undisturbed sample of clay 20 mm thick is tested in a consolidometer. The total change in thickness of the specimen is 0.80 mm when the applied pressure is 2100 lb/ft$^2$. The final water content of the sample is 24 percent and the specific gravity of the solids is 2.72. Estimate the probable final settlement of the proposed structure.

7.3 The following observations refer to a standard laboratory consolidation test on an undisturbed sample of clay.

| Pressure $kN/m^2$ | Final Dial Gauge Reading $\times 10^{-2}$ mm | Pressure $kN/m^2$ | Final Dial Gauge Reading $\times 10^{-2}$ mm |
|---|---|---|---|
| 0 | 0 | 400 | 520 |
| 50 | 180 | 100 | 470 |
| 100 | 250 | 0 | 355 |
| 200 | 360 | | |

The sample was 75 mm in diameter and had an initial thickness of 18 mm. The moisture content at the end of the test was 45.5%; the specific gravity of solids was 2.53.

Compute the void ratio at the end of each loading increment and also determine whether the soil was overconsolidated or not. If it was overconsolidated, what was the overconsolidation ratio if the effective overburden pressure at the time of sampling was 60 $kN/m^2$?

7.4 The following points are coordinates on a pressure-void ratio curve for an undisturbed clay.

| $p$ | 0.5 | 1 | 2 | 4 | 8 | 16 | $kips/ft^2$ |
|---|---|---|---|---|---|---|---|
| $e$ | 1.202 | 1.16 | 1.06 | 0.94 | 0.78 | 0.58 | |

Determine (i) $C_c$, and (ii) the magnitude of compression in a 10 ft thick layer of this clay for a load increment of 4 $kips/ft^2$. Assume $e_0 = 1.320$, and $p_0 = 1.5$ $kips/ft^2$

7.5 The thickness of a compressible layer, prior to placing of a fill covering a large area, is 30 ft. Its original void ratio was 1.0. Sometime after the fill was constructed measurements indicated that the average void ratio was 0.8. Determine the compression of the soil layer.

7.6 The water content of a soft clay is 54.2% and the liquid limit is 57.3%. Estimate the compression index, by equations (7.17) and (7.18). Given $e_0 = 0.85$

7.7 A layer of normally consolidated clay is 20 ft thick and lies under a recently constructed building. The pressure of sand overlying the clay layer is 6300 $lb/ft^2$, and the new construction increases the overburden pressure at the middle of the clay layer by 2100 $lb/ft^2$. If the compression index is 0.5, compute the final settlement assuming $w_n = 45\%$, $G_s = 2.70$, and the clay is submerged with the water table at the top of the clay stratum.

7.8 A consolidation test was made on a sample of saturated marine clay. The diameter and thickness of the sample were 5.5 cm and 3.75 cm respectively. The sample weighed 650 g at the start of the test and 480 g in the dry state after the test. The specific gravity of solids was 2.72. The dial readings corresponding to the final equilibrium condition under each load are given below.

| Pressure, $kN/m^2$ | *DR* cm $\times 10^{-4}$ | Pressure, $kN/m^2$ | *DR* cm $\times 10^{-4}$ |
|---|---|---|---|
| 0 | 0 | 106 | 1880 |
| 6.7 | 175 | 213 | 3340 |
| 11.3 | 275 | 426 | 5000 |
| 26.6 | 540 | 852 | 6600 |
| 53.3 | 965 | | |

(a) Compute the void ratios and plot the $e$-log $p$ curve.
(b) Estimate the maximum preconsolidation pressure by the Casagrande method.
(c) Draw the field curve and determine the compression index.

7.9 The results of a consolidation test on a soil sample for a load increased from 200 to 400 kN/m$^2$ are given below:

| Time in Min. | Dial reading division | Time in Min. | Dial reading division |
|---|---|---|---|
| 0 | 1255 | 16 | 1603 |
| 0.10 | 1337 | 25 | 1632 |
| 0.25 | 1345 | 36 | 1651 |
| 0.50 | 1355 | 49 | 1661 |
| 1.00 | 1384 | 64 | 1670 |
| 2.25 | 1423 | 81 | 1677 |
| 4.00 | 1480 | 100 | 1682 |
| 9.00 | 1557 | 121 | 1687 |

The thickness of the sample corresponding to the dial reading 1255 is 1.561 cm. Determine the value of the coefficient of consolidation using the square root of time fitting method in cm$^2$/min. One division of dial gauge corresponds to $2.5 \times 10^{-4}$ cm. The sample is drained on both faces.

7.10 A 2.5 cm thick sample was tested in a consolidometer under saturated conditions with drainage on both sides. 30 percent consolidation was reached under a load in 15 minutes. For the same conditions of stress but with only one way drainage, estimate the time in days it would take for a 2 m thick layer of the same soil to consolidate in the field to attain the same degree of consolidation.

7.11 The dial readings recorded during a consolidation test at a certain load increment are given below.

| Time min | Dial Reading cm × $10^{-4}$ | Time min | Dial Reading cm × $10^{-4}$ |
|---|---|---|---|
| 0 | 240 | 15 | 622 |
| 0.10 | 318 | 30 | 738 |
| 0.25 | 340 | 60 | 842 |
| 0.50 | 360 | 120 | 930 |
| 1.00 | 385 | 240 | 975 |
| 2.00 | 415 | 1200 | 1070 |
| 4.00 | 464 | | |
| 8.00 | 530 | - | - |

Determine $c_v$ by both the square root of time and log of time fitting methods. The thickness of the sample at DR 240 = 2 cm and the sample is drained both sides.

7.12 In a laboratory consolidation test a sample of clay with a thickness of 1 in. reached 50% consolidation in 8 minutes. The sample was drained top and bottom. The clay layer from which the sample was taken is 25 ft thick. It is covered by a layer of sand through which water can escape and is underlain by a practically impervious bed of intact shale. How long will the clay layer require to reach 50 per cent consolidation?

7.13 The following data were obtained from a consolidation test performed on an undisturbed clay sample 3 cm in thickness:

(i) $p_1 = 3.5$ kips/ft$^2$, $e_1 = 0.895$

(ii) $p_2 = 6.5$ kips/ft$^2$, $e_2 = 0.782$

By utilizing the known theoretical relationship between percent consolidation and time factor, compute and plot the decrease in thickness with time for a 10 ft thick layer of this clay, which is drained on the upper surface only. Given : $e_0$ = 0.92 $p_0$ = 4.5 kips/ft$^2$, $\Delta p$ = 1.5 kips/ft$^2$, $c_v = 4.2 \times 10^{-5}$ ft$^2$/min.

7.14 A structure built on a layer of clay settled 5 cm in 60 days after it was built. If this settlement corresponds to 20 percent average consolidation of the clay layer, plot the time settlement curve of the structure for a period of 3 years from the time it was built. Given : Thickness of clay layer = 3m and drained on one side

7.15 A 30 ft thick clay layer with single drainage settles 3.5 in. in 3.5 yr. The coefficient consolidation for this clay was found to be $8.43 \times 10^{-4}$ in.$^2$/sec. Compute the ultimate consolidation settlement and determine how long it will take to settle to 90% of this amount.

7.16 The time factor T for a clay layer undergoing consolidation is 0.2. What is the average degree of consolidation (consolidation ratio) for the layer?

7.17 If the final consolidation settlement for the clay layer in Prob. 7.16 is expected to be 1.0 m, how much settlement has occurred when the time factor is (a) 0.2 and (b) 0.7?

7.18 A certain compressible layer has a thickness of 12 ft. After 1 yr when the clay is 50% consolidated, 3 in. of settlement has occurred. For similar clay and loading conditions, how much settlement would occur at the end of 1 yr and 4 yr, if the thickness of this new layer were 20 ft?

7.19 A layer of normally consolidated clay 14 ft thick has an average void ratio of 1.3. Its compression index is 0.6. When the induced vertical pressure on the clay layer is doubled, what change in thickness of the clay layer will result? Assume: $p_0$ = 1200 lb/ft$^2$ and $\Delta p$ = 600 lb/ft$^2$.

7.20 Settlement analysis for a proposed structure indicates that 2.4 in. of settlement will occur in 4 yr and that the ultimate total settlement will be 9.8 in. The analysis is based on the assumption that the compressible clay layer is drained on both sides. However, it is suspected that there may not be drainage at the bottom surface. For the case of single drainage, estimate the time required for 2.4 in. of settlement.

7.21 The time to reach 60% consolidation is 32.5 sec for a sample 1.27 cm thick tested in a laboratory under conditions of double drainage. How long will the corresponding layer in nature require to reach the same degree of consolidation if it is 4.57 m thick and drained on one side only?

7.22 A certain clay layer 30 ft thick is expected to have an ultimate settlement of 16 in. If the settlement was 4 in. after four years, how much longer will it take to obtain a settlement of 6 in?

7.23 If the coefficient of consolidation of a 3 m thick layer of clay is 0.0003 cm$^2$/sec, what is the average consolidation of that layer of clay (a) in one year with two-way drainage, and (b) the same as above for one-way drainage.

7.24 The average natural moisture content of a deposit is 40%; the specific gravity of the solid matter is 2.8, and the compression index $C_c$ is 0.36. If the clay deposit is 6.1 m thick drained on both sides, calculate the final consolidation settlement $S_t$. Given: $p_0$ = 60 kN/m$^2$ and $\Delta p$ = 30 kN/m$^2$

7.25 A rigid foundation block, circular in plan and 6 m in diameter rests on a bed of compact sand 6 m deep. Below the sand is a 1.6 m thick layer of clay overlying on impervious bed rock. Ground water level is 1.5 m below the surface of the sand. The unit weight of sand above water table is 19.2 kN/m$^3$, the saturated unit weight of sand is 20.80 kN/m$^3$, and the saturated unit weight of the clay is 19.90 kN/m$^3$.

A laboratory consolidation test on an undisturbed sample of the clay, 20 mm thick and drained top and bottom, gave the following results:

| Pressure (kN/m$^2$) | 50 | 100 | 200 | 400 | 800 |
|---|---|---|---|---|---|
| Void ratio | 0.73 | 0.68 | 0.625 | 0.54 | 0.41 |

If the contact pressure at the base of the foundation is 200 kN/m$^2$, and $e_0$ = 0.80, calculate the final average settlement of the foundation assuming 2:1 method for the spread of the load.

7.26 A stratum of clay is 2 m thick and has an initial overburden pressure of 50 kN/m$^2$ at the middle of the clay layer. The clay is overconsolidated with a preconsolidation pressure of 75 kN/m$^2$. The values of the coefficients of recompression and compression indices are 0.05 and 0.25 respectively. Assume the initial void ratio $e_0$ = 1.40. Determine the final settlement due to an increase of pressure of 40 kN/m$^2$ at the middle of the clay layer.

7.27 A clay stratum 5 m thick has the initial void ration of 1.50 and an effective overburden pressure of 120 kN/m$^2$. When the sample is subjected to an increase of pressure of 120 kN/m$^2$, the void ratio reduces to 1.44. Determine the coefficient of volume compressibility and the final settlement of the stratum.

7.28 A 3 m thick clay layer beneath a building is overlain by a permeable stratum and is underlain by an impervious rock. The coefficient of consolidation of the clay was found to be 0.025 cm$^2$/min. The final expected settlement for the layer is 8 cm. Determine (a) how much time will it take for 80 percent of the total settlement, (b) the required time for a settlement of 2.5 cm to occur, and (c) the settlement that would occur in one year.

7.29 An area is underlain by a stratum of clay layer 6 m thick. The layer is doubly drained and has a coefficient of consolidation of 0.3 m$^2$/month. Determine the time required for a surcharge load to cause a settlement of 40 cm if the same load cause a final settlement of 60 cm.

7.30 In an oedometer test, a clay specimen initially 25 mm thick attains 90% consolidation in 10 minutes. In the field, the clay stratum from which the specimen was obtained has a thickness of 6 m and is sandwiched between two sand layers. A structure constructed on this clay experienced an ultimate settlement of 200 mm. Estimate the settlement at the end of 100 days after construction.