B: the card is the jack, queen, or king of diamonds,

C: the card is an ace.

Clearly, the event $A \cap C$ consists of only the two red aces.

Several results that follow from the foregoing definitions, which may easily be verified by means of Venn diagrams, are as follows:

1. $A \cap \phi = \phi$.
2. $A \cup \phi = A$.
3. $A \cap A' = \phi$.
4. $A \cup A' = S$.
5. $S' = \phi$.
6. $\phi' = S$.
7. $(A')' = A$.
8. $(A \cap B)' = A' \cup B'$.
9. $(A \cup B)' = A' \cap B'$.

Exercises

2.1 List the elements of each of the following sample spaces:
(a) the set of integers between 1 and 50 divisible by 8;
(b) the set $S = \{x \mid x^2 + 4x - 5 = 0\}$;
(c) the set of outcomes when a coin is tossed until a tail or three heads appear;
(d) the set $S = \{x \mid x \text{ is a continent}\}$;
(e) the set $S = \{x \mid 2x - 4 \geq 0 \text{ and } x < 1\}$.

2.2 Use the rule method to describe the sample space S consisting of all points in the first quadrant inside a circle of radius 3 with center at the origin.

2.3 Which of the following events are equal?
(a) $A = \{1, 3\}$;
(b) $B = \{x \mid x \text{ is a number on a die}\}$;
(c) $C = \{x \mid x^2 - 4x + 3 = 0\}$;
(d) $D = \{x \mid x \text{ is the number of heads when six coins are tossed}\}$.

2.4 An experiment involves tossing a pair of dice, one green and one red, and recording the numbers that come up. If x equals the outcome on the green die and y the outcome on the red die, describe the sample space S
(a) by listing the elements (x, y);
(b) by using the rule method.

2.5 An experiment consists of tossing a die and then flipping a coin once if the number on the die is even. If the number on the die is odd, the coin is flipped twice. Using the notation $4H$, for example, to denote the outcome that the die comes up 4 and then the coin comes up heads, and $3HT$ to denote the outcome that the die comes up 3 followed by a head and then a tail on the coin, construct a tree diagram to show the 18 elements of the sample space S.

2.6 Two jurors are selected from 4 alternates to serve at a murder trial. Using the notation A_1A_3, for example, to denote the simple event that alternates 1 and 3 are selected, list the 6 elements of the sample space S.

2.7 Four students are selected at random from a chemistry class and classified as male or female. List the elements of the sample space S_1, using the letter M for male and F for female. Define a second sample space S_2 where the elements represent the number of females selected.

2.8 For the sample space of Exercise 2.4,
(a) list the elements corresponding to the event A that the sum is greater than 8;
(b) list the elements corresponding to the event B that a 2 occurs on either die;
(c) list the elements corresponding to the event C that a number greater than 4 comes up on the green die;
(d) list the elements corresponding to the event $A \cap C$;
(e) list the elements corresponding to the event $A \cap B$;
(f) list the elements corresponding to the event $B \cap C$;
(g) construct a Venn diagram to illustrate the intersections and unions of the events A, B, and C.

2.9 For the sample space of Exercise 2.5,
(a) list the elements corresponding to the event A that a number less than 3 occurs on the die;
(b) list the elements corresponding to the event B that two tails occur;
(c) list the elements corresponding to the event A';

(d) list the elements corresponding to the event $A' \cap B$;

(e) list the elements corresponding to the event $A \cup B$.

2.10 An engineering firm is hired to determine if certain waterways in Virginia are safe for fishing. Samples are taken from three rivers.

(a) List the elements of a sample space S, using the letters F for safe to fish and N for not safe to fish.

(b) List the elements of S corresponding to event E that at least two of the rivers are safe for fishing.

(c) Define an event that has as its elements the points

$$\{FFF, NFF, FFN, NFN\}.$$

2.11 The resumés of two male applicants for a college teaching position in chemistry are placed in the same file as the resumés of two female applicants. Two positions become available, and the first, at the rank of assistant professor, is filled by selecting one of the four applicants at random. The second position, at the rank of instructor, is then filled by selecting at random one of the remaining three applicants. Using the notation M_2F_1, for example, to denote the simple event that the first position is filled by the second male applicant and the second position is then filled by the first female applicant,

(a) list the elements of a sample space S;

(b) list the elements of S corresponding to event A that the position of assistant professor is filled by a male applicant;

(c) list the elements of S corresponding to event B that exactly one of the two positions is filled by a male applicant;

(d) list the elements of S corresponding to event C that neither position is filled by a male applicant;

(e) list the elements of S corresponding to the event $A \cap B$;

(f) list the elements of S corresponding to the event $A \cup C$;

(g) construct a Venn diagram to illustrate the intersections and unions of the events A, B, and C.

2.12 Exercise and diet are being studied as possible substitutes for medication to lower blood pressure. Three groups of subjects will be used to study the effect of exercise. Group 1 is sedentary, while group 2 walks and group 3 swims for 1 hour a day. Half of each of the three exercise groups will be on a salt-free diet. An additional group of subjects will not exercise or restrict their salt, but will take the standard medication. Use Z for sedentary, W for walker, S for swimmer, Y for salt, N for no salt, M for medication, and F for medication free.

(a) Show all of the elements of the sample space S.

(b) Given that A is the set of nonmedicated subjects and B is the set of walkers, list the elements of $A \cup B$.

(c) List the elements of $A \cap B$.

2.13 Construct a Venn diagram to illustrate the possible intersections and unions for the following events relative to the sample space consisting of all automobiles made in the United States.

$F:$ Four door, $S:$ Sun roof, $P:$ Power steering.

2.14 If $S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ and $A = \{0, 2, 4, 6, 8\}$, $B = \{1, 3, 5, 7, 9\}$, $C = \{2, 3, 4, 5\}$, and $D = \{1, 6, 7\}$, list the elements of the sets corresponding to the following events:

(a) $A \cup C$;

(b) $A \cap B$;

(c) C';

(d) $(C' \cap D) \cup B$;

(e) $(S \cap C)'$;

(f) $A \cap C \cap D'$.

2.15 Consider the sample space $S = \{$copper, sodium, nitrogen, potassium, uranium, oxygen, zinc$\}$ and the events

$$A = \{\text{copper, sodium, zinc}\},$$
$$B = \{\text{sodium, nitrogen, potassium}\},$$
$$C = \{\text{oxygen}\}.$$

List the elements of the sets corresponding to the following events:

(a) A';

(b) $A \cup C$;

(c) $(A \cap B') \cup C'$;

(d) $B' \cap C'$;

(e) $A \cap B \cap C$;

(f) $(A' \cup B') \cap (A' \cap C)$.

2.16 If $S = \{x \mid 0 < x < 12\}$, $M = \{x \mid 1 < x < 9\}$, and $N = \{x \mid 0 < x < 5\}$, find

(a) $M \cup N$;

(b) $M \cap N$;

(c) $M' \cap N'$.

2.17 Let A, B, and C be events relative to the sample space S. Using Venn diagrams, shade the areas representing the following events:

(a) $(A \cap B)'$;

(b) $(A \cup B)'$;

(c) $(A \cap C) \cup B$.

2.18 Which of the following pairs of events are mutually exclusive?

(a) A golfer scoring the lowest 18-hole round in a 72-hole tournament and losing the tournament.

(b) A poker player getting a flush (all cards in the same suit) and 3 of a kind on the same 5-card hand.

(c) A mother giving birth to a baby girl and a set of twin daughters on the same day.

(d) A chess player losing the last game and winning the match.

2.19 Suppose that a family is leaving on a summer vacation in their camper and that M is the event that they will experience mechanical problems, T is the event that they will receive a ticket for committing a traffic violation, and V is the event that they will arrive at a campsite with no vacancies. Referring to the Venn diagram of Figure 2.5, state in words the events represented by the following regions:

(a) region 5;

(b) region 3;

(c) regions 1 and 2 together;

(d) regions 4 and 7 together;

(e) regions 3, 6, 7, and 8 together.

2.20 Referring to Exercise 2.19 and the Venn diagram of Figure 2.5, list the numbers of the regions that represent the following events:

(a) The family will experience no mechanical problems and will not receive a ticket for a traffic violation but will arrive at a campsite with no vacancies.

(b) The family will experience both mechanical problems and trouble in locating a campsite with a vacancy but will not receive a ticket for a traffic violation.

(c) The family will either have mechanical trouble or arrive at a campsite with no vacancies but will not receive a ticket for a traffic violation.

(d) The family will not arrive at a campsite with no vacancies.

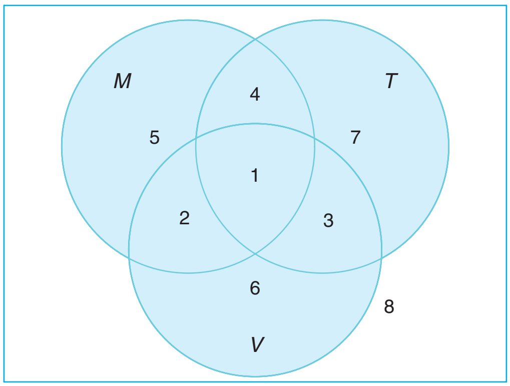

Figure 2.5: Venn diagram for Exercises 2.19 and 2.20.

2.3 Counting Sample Points

One of the problems that the statistician must consider and attempt to evaluate is the element of chance associated with the occurrence of certain events when an experiment is performed. These problems belong in the field of probability, a subject to be introduced in Section 2.4. In many cases, we shall be able to solve a probability problem by counting the number of points in the sample space without actually listing each element. The fundamental principle of counting, often referred to as the **multiplication rule**, is stated in Rule 2.1.