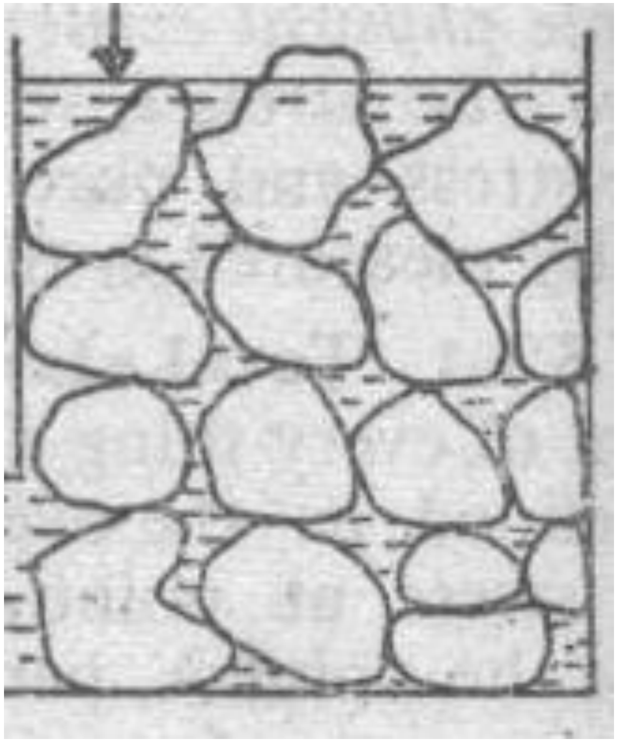
Permeability of soil

- Soils are assemblages of solid particles with interconnected voids where water can flow from a point of **high energy to a point** of **low energy**.



The study of flow water through porous media is important for stability analysis of earth retaining structures subjected to seepage force

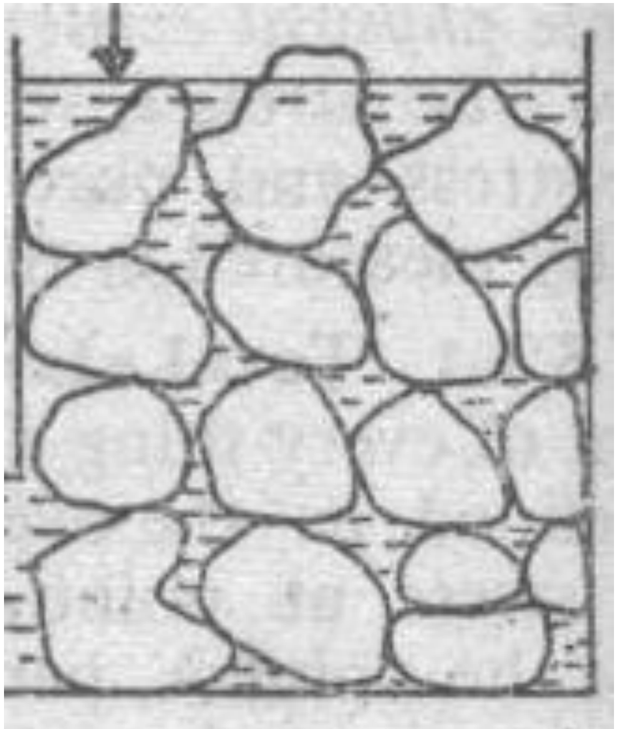
Permeability:

- The property of soils that allows water to pass through them at some rate
- The property is a product of the granular nature of the soil, although it can be affected by other factors (such as water bonding in clays). Different soil has different permeabilities.

Permeability of soil (K)

- The property of soils that allows water to pass through them at some rate

- The ease with which water flows through a soil is quantitatively expressed in terms of permeability (K) of soil .



- Surfaces of solid phase offer resistance to the flow of water
- Narrower and complex is the interconnectivity among the pores low will be the flow of water
- What kind of soil you may perfer?????

The permeability of soils has a decisive effect on the

- Stability of foundations,
- Seepage loss through embankments of reservoirs,
- Drainage of sub grades,
- Rate of flow of water into wells and
- Many others

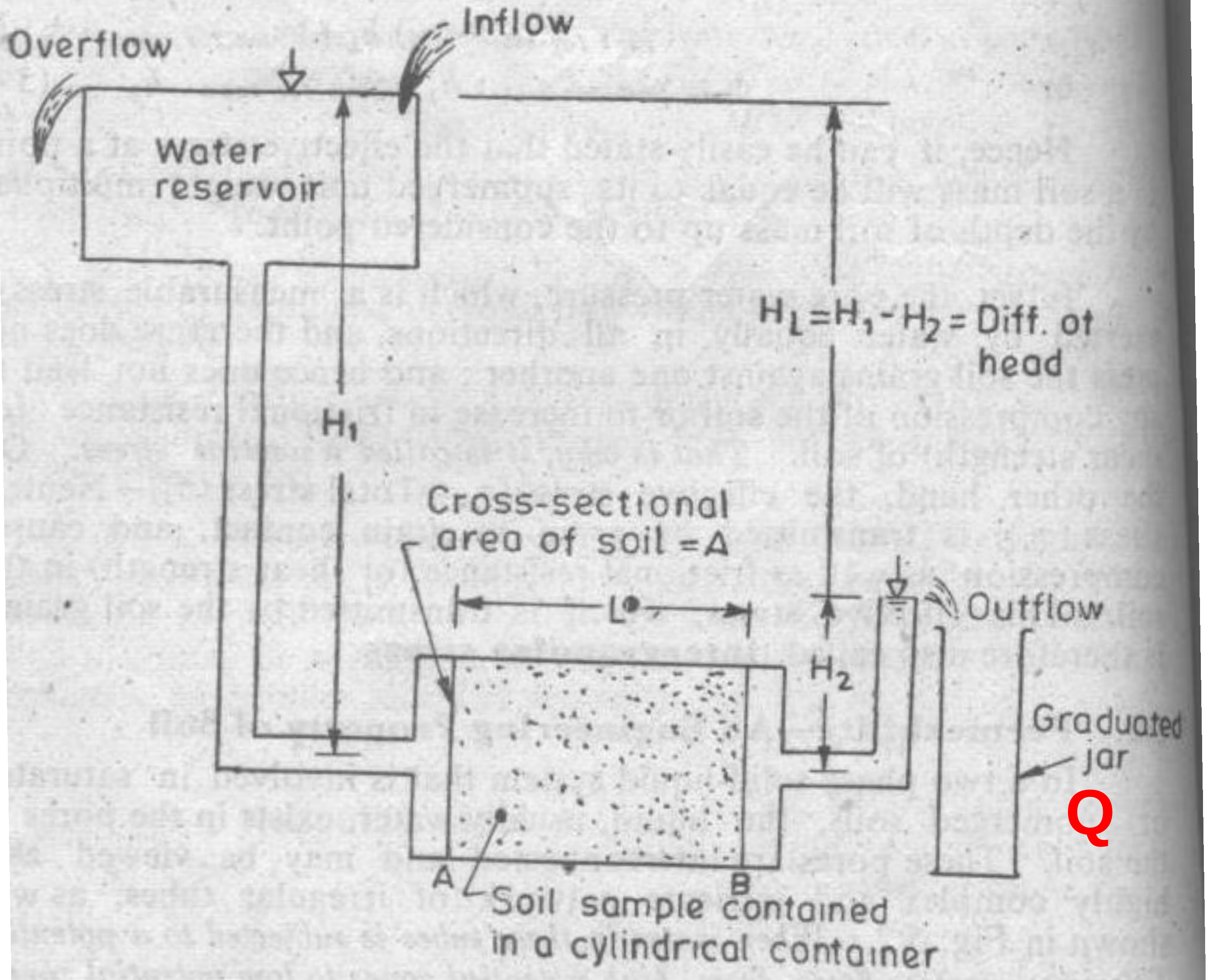


Fig. 5·3. Schematic diagram of one dimensional laminar flow apparatus.

H. Darcy

French scientist

1856

First researcher to study flow of fluids through porous media

$$Q \alpha \frac{H_L}{L} A$$

Based on
Experimental results

$$Q = K \frac{H_L}{L} A$$

Constant K changed when the soil was changed

Thus reflects the property of soil known as Coefficient of permeability

$$Q = KiA$$

$$Q = VA$$

$$V = Ki$$

Darcy’s Law

•V is the discharge velocity

•It is a sort of superficial velocity (Not the actual seepage velocity)

Seepage velocity=v/n

•The permeability of soil then can be viewed as a superficial velocity under a unit hydraulic gradient

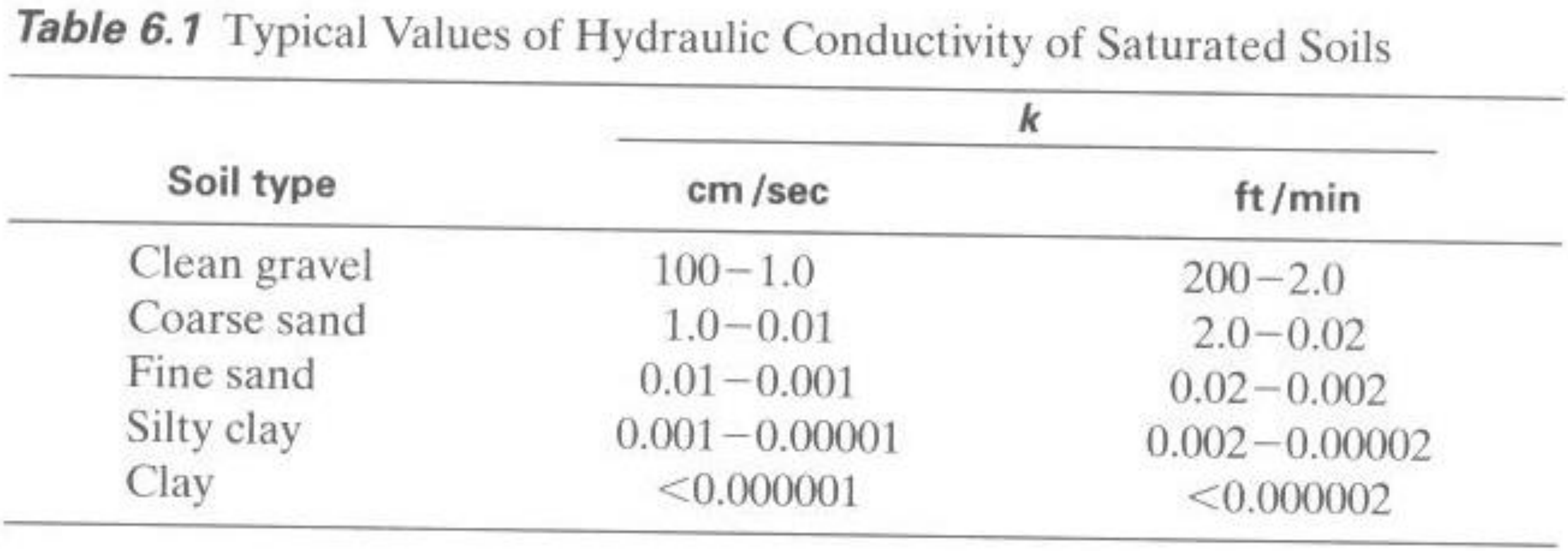


Table 6.1 Typical Values of Hydraulic Conductivity of Saturated Soils

Soil type	k (cm/sec)	k (ft/min)
Clean gravel	100−1.0	200−2.0
Coarse sand	1.0−0.01	2.0−0.02
Fine sand	0.01−0.001	0.02−0.002
Silty clay	0.001−0.00001	0.002−0.00002
Clay	<0.000001	<0.000002

Laboratory Measurement of Permeability

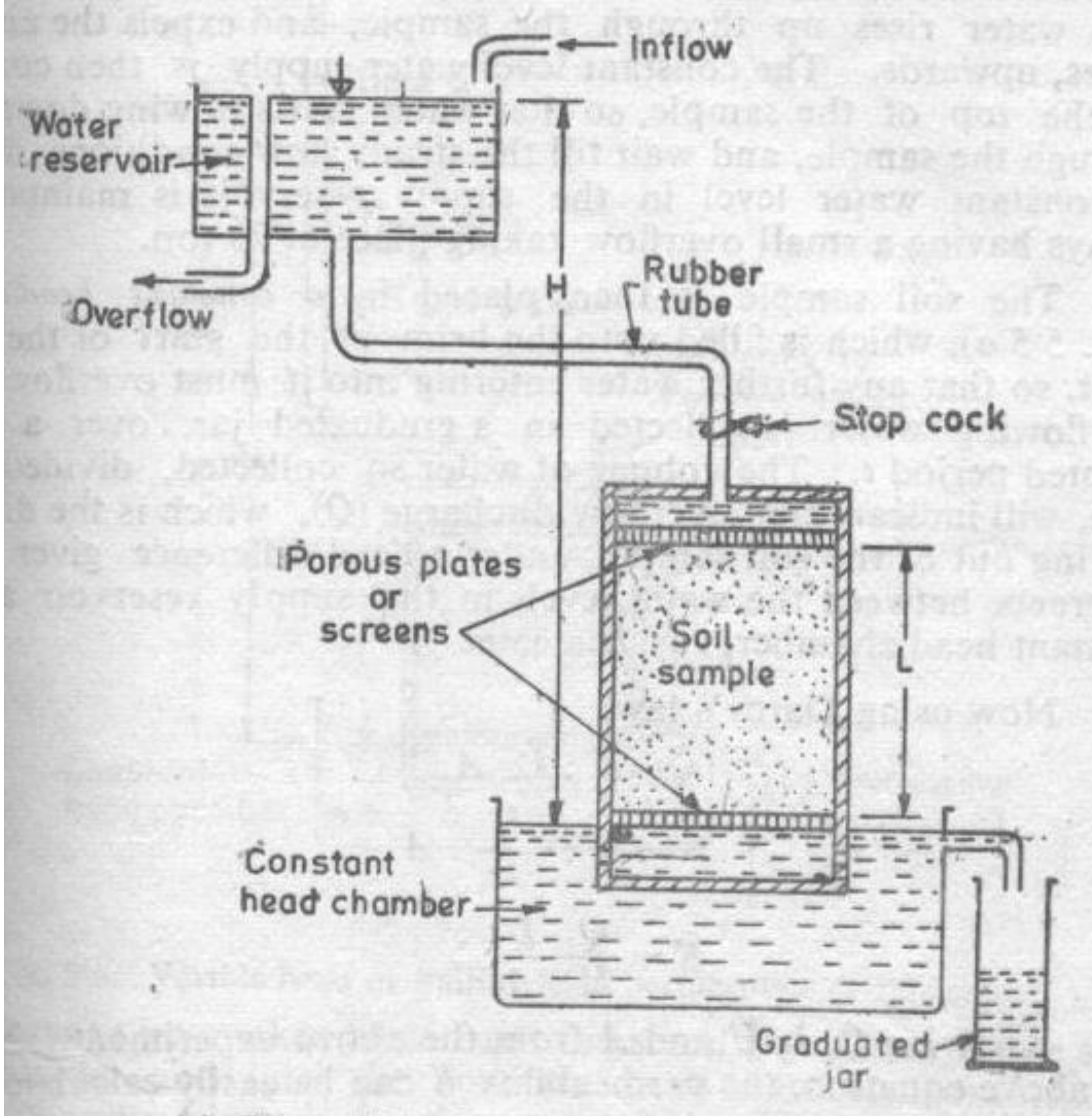
Permeameter

Constant Head Permeameter (Coarse grained soils)

(The head remains constant)

Variable Head Permeameter (Fine grained soils)

(The head is allowed to fall or vary)



(a) Constant head permeameter arrangement.

$$Q = K \frac{H_L}{L} A$$

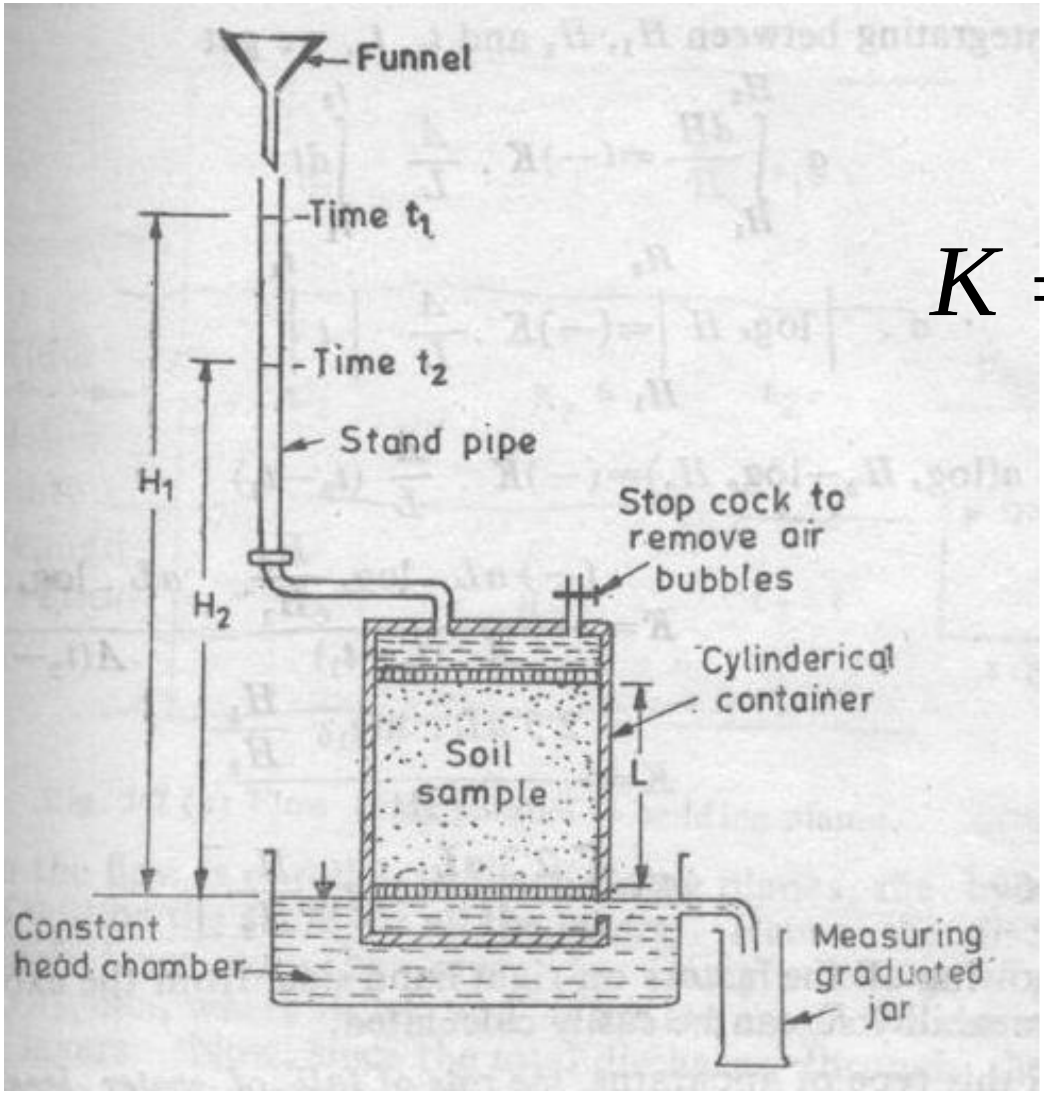
$$K = \frac{QL}{H_L A}$$

$$K = \frac{Q}{Ai}$$

In relatively impervious soils, the quantity of seepage remain very small so this method may be time consuming

Variable head Permeameter

(Falling head Permeameter)



$$K = \left[\frac{2.3aL}{At} \log_{10} \frac{H_1}{H_2} \right]$$

Factors affecting Permeability

Grain Size

Allen Hazen found that Permeability of filter sands can be roughly expressed as

$K=100D^2_{10}$

Properties of Pore fluid

For the same soil, the permeability would depend inversely upon the viscosity of the pore fluid

Void ratio of soil

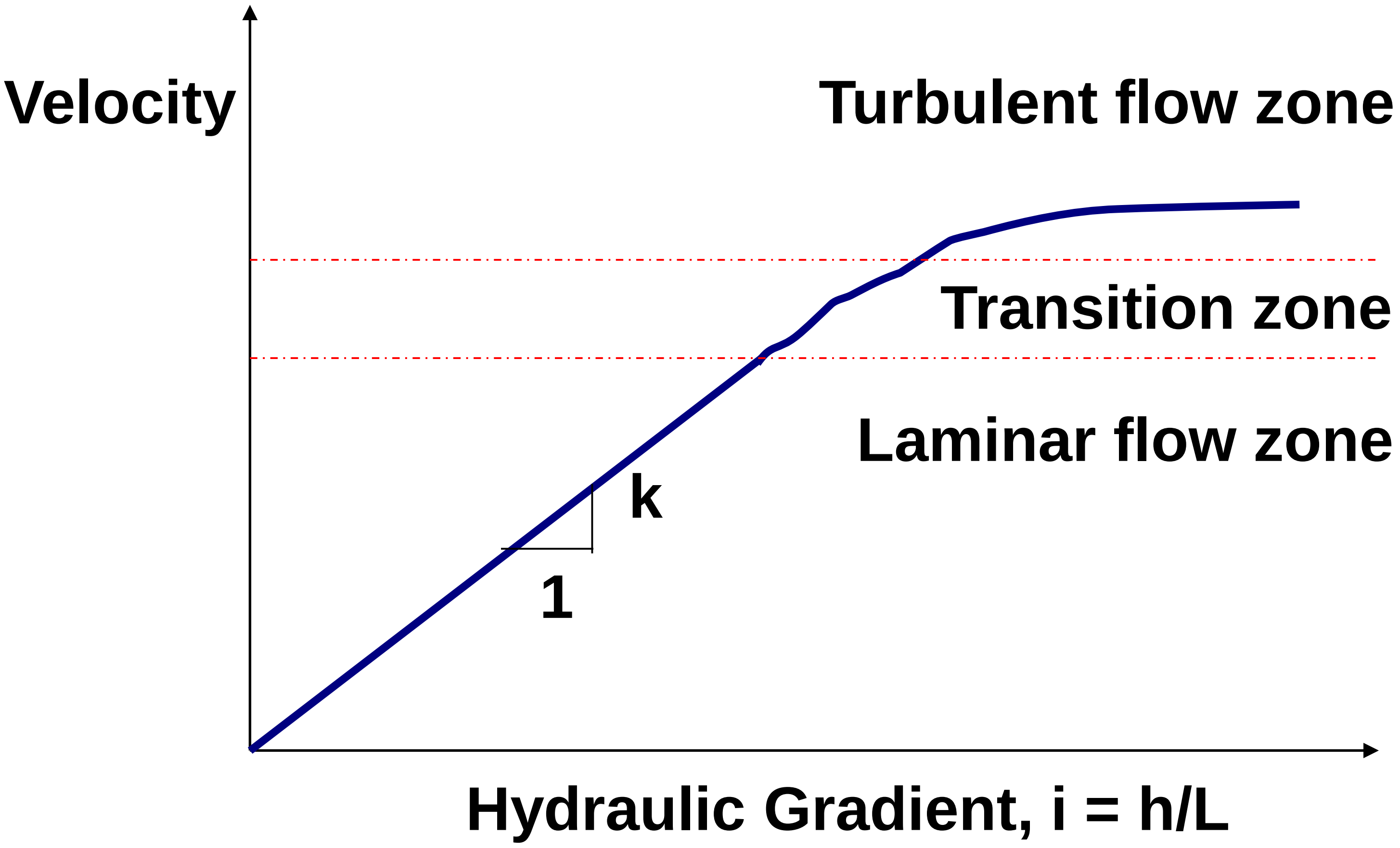
Permeability is directly proportional to the void ratio of soil

For sandy soils K is approximately proportional to e^3

Structural arrangement of soil particles

For the same soil at the same void ratio, the permeability may be different

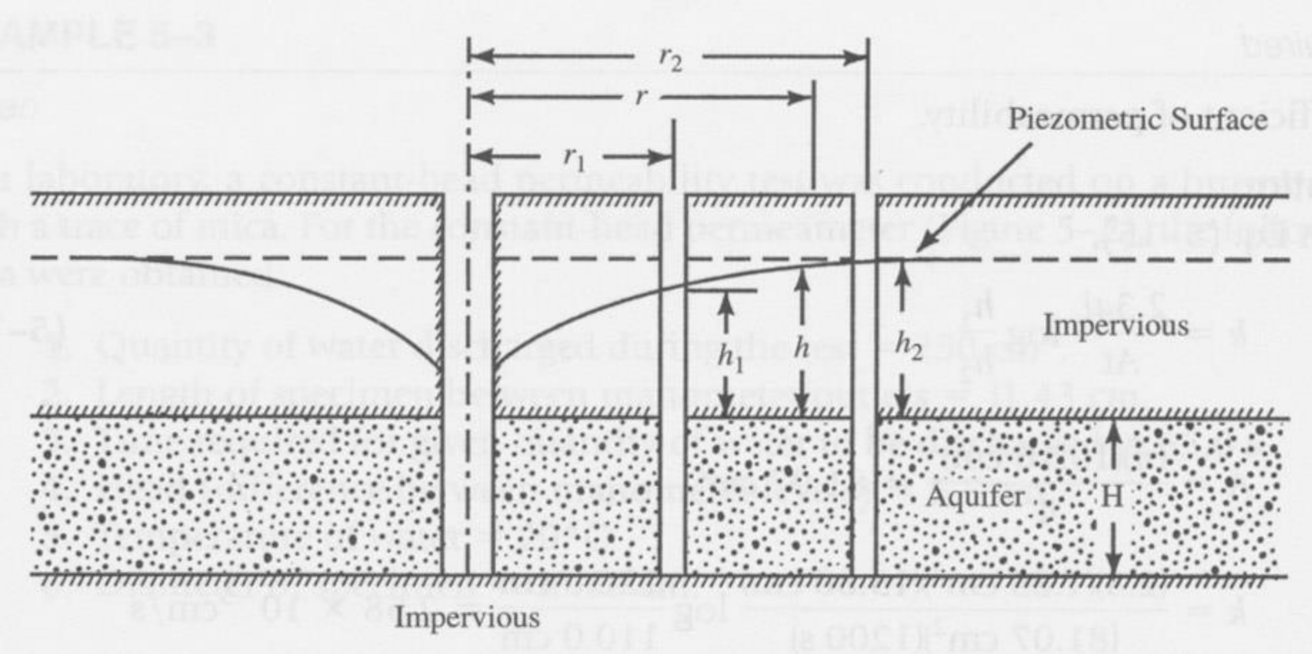
Definition of Permeability (Hydraulic Conductivity)



Coefficient of Permeability

$$k = \frac{q \ln\left(\frac{r_2}{r_1}\right)}{2\pi H\left(h_2 - h_1\right)}$$

: ________________ Confined aquifer



Example

- A pumping test was performed in a well penetrating a confined aquifer to evaluate the coefficient of permeability of the soil in the aquifer.
- When equilibrium flow was reached, the following data were obtained:
  - Equilibrium discharge of water from the well = 200 gal/min.
  - Water levels (h1 and h2) = 15 and 19 ft at distances from the well (r1 and r2) of 60 and 180 ft, respectively.
  - Thickness of aquifer = 20 ft
- Coefficient of permeability of the soil in the aquifer? (1 gal=0.1605 ft^3)

Example

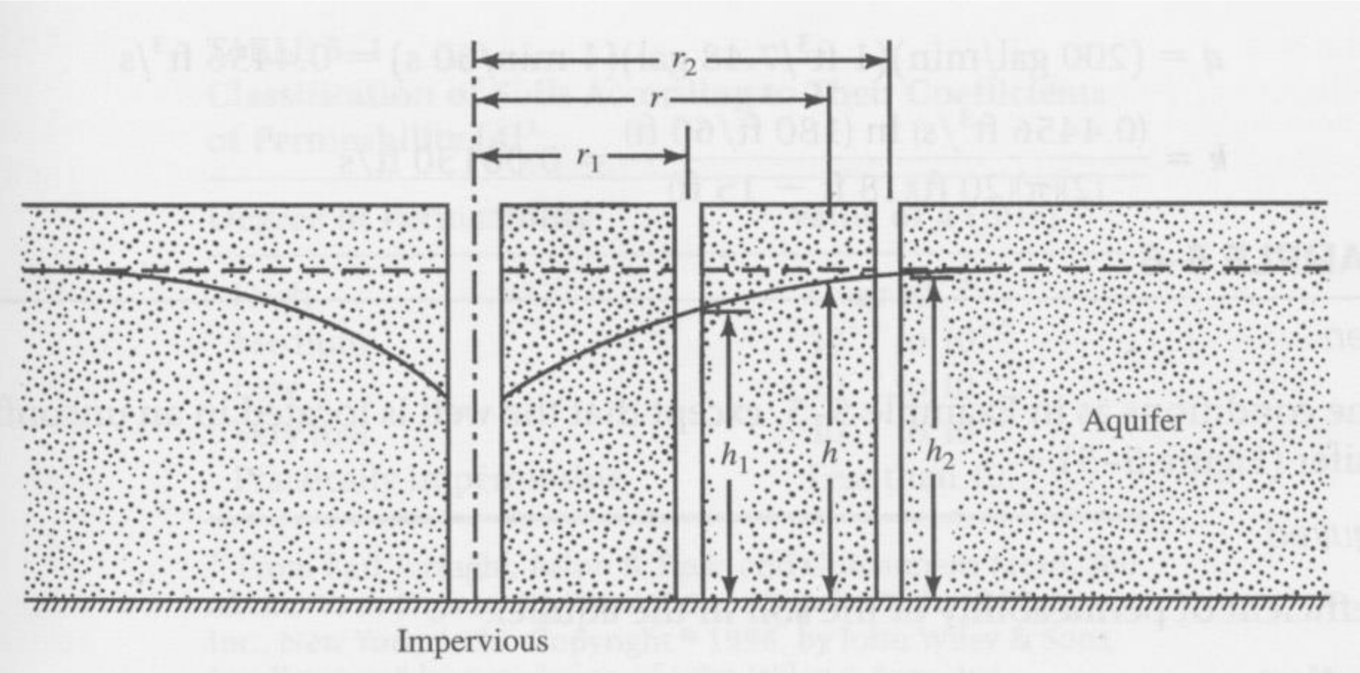
$$q = (200\,gal/\min)\left(\frac{1\,ft^3}{7.48\,gal}\right)\left(\frac{1\min}{60\sec}\right) = 0.4456\,f^3/s$$

$$k = \frac{q\ln(r_2/r_1)}{2\pi H(h_2-h_1)} = \frac{(0.4456\,f^3/s)\ln\left(\frac{180\,ft}{60\,ft}\right)}{2\pi(20\,ft)(18\,ft-15\,ft)} = 0.00130\,ft/s$$

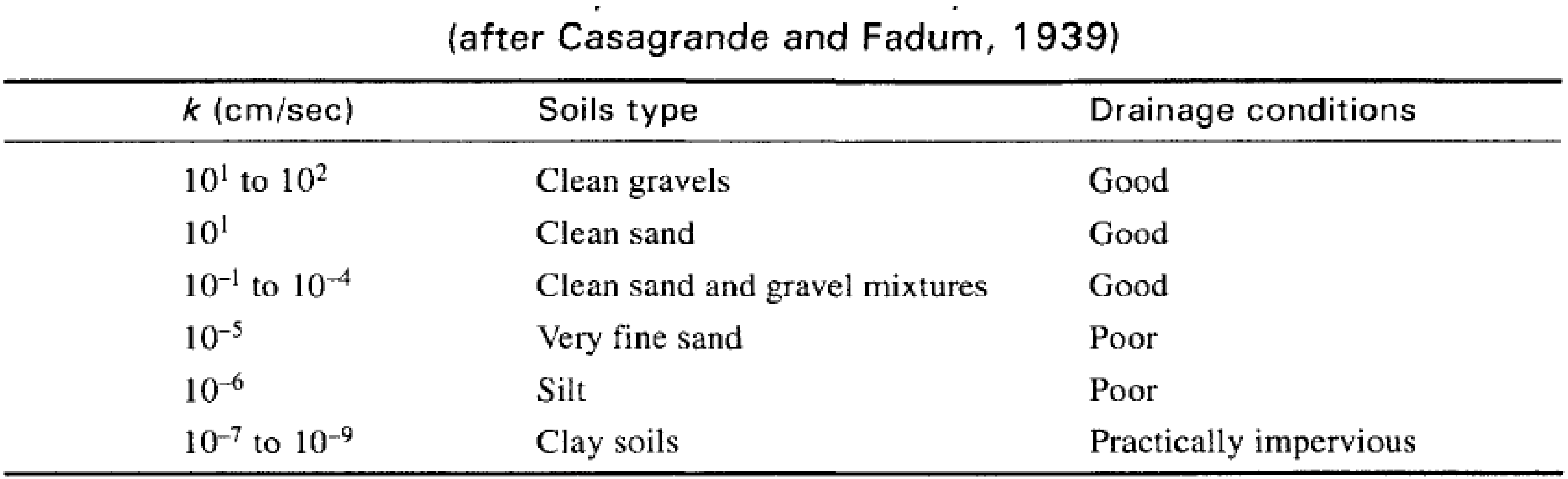
Coefficient of Permeability

$$k = \frac{q \ln\left(\frac{r_2}{r_1}\right)}{\pi\left(h_2^2 - h_1^2\right)}$$

: ____________________Unconfined aquifer

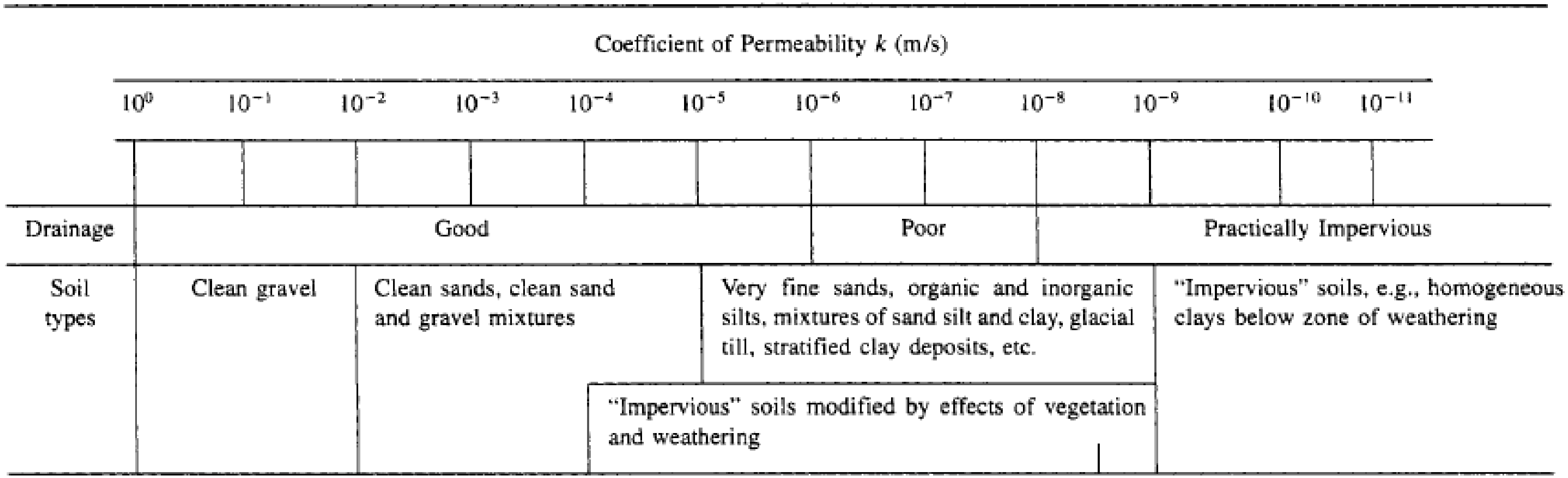


Hydraulic conductivity of some soils



(after Casagrande and Fadum, 1939)

k (cm/sec)	Soils type	Drainage conditions
10^1 to 10^2	Clean gravels	Good
10^1	Clean sand	Good
10^{-1} to 10^{-4}	Clean sand and gravel mixtures	Good
10^{-5}	Very fine sand	Poor
10^{-6}	Silt	Poor
10^{-7} to 10^{-9}	Clay soils	Practically impervious



Coefficient of Permeability k (m/s)

<table>
<tr><th></th><th>10^0</th><th>10^{-1}</th><th>10^{-2}</th><th>10^{-3}</th><th>10^{-4}</th><th>10^{-5}</th><th>10^{-6}</th><th>10^{-7}</th><th>10^{-8}</th><th>10^{-9}</th><th>10^{-10}</th><th>10^{-11}</th></tr>
<tr><td>Drainage</td><td colspan="6">Good</td><td colspan="2">Poor</td><td colspan="4">Practically Impervious</td></tr>
<tr><td rowspan="2">Soil types</td><td colspan="2" rowspan="2">Clean gravel</td><td colspan="3">Clean sands, clean sand and gravel mixtures</td><td colspan="4">Very fine sands, organic and inorganic silts, mixtures of sand silt and clay, glacial till, stratified clay deposits, etc.</td><td colspan="3" rowspan="2">"Impervious" soils, e.g., homogeneous clays below zone of weathering</td></tr>
<tr><td></td><td></td><td colspan="5">"Impervious" soils modified by effects of vegetation and weathering</td></tr>
</table>

Coefficient of Permeability of Common Natural Soil Formations [4][1]

Formation	Value of k (m/s)
River deposits	
Rhône at Genissiat	Up to 4×10^{-3}
Small streams, eastern Alps	2×10^{-4} to 2×10^{-3}
Missouri	2×10^{-4} to 2×10^{-3}
Mississippi	2×10^{-4} to 10^{-3}
Glacial deposits	
Outwash plains	5×10^{-4} to 2×10^{-2}
Esker, Westfield, Mass.	10^{-4} to 10^{-3}
Delta, Chicopee, Mass.	10^{-6} to 1.5×10^{-4}
Till	Less than 10^{-6}
Wind deposits	
Dune sand	10^{-3} to 3×10^{-3}
Loess	$10^{-5} \pm$
Loess loam	$10^{-6} \pm$
Lacustrine and marine offshore deposits	
Very fine uniform sand, $C_U = 5$ to 2	10^{-6} to 6×10^{-5}
Bull's liver, Sixth Ave., N.Y., $C_U = 5$ to 2	10^{-6} to 5×10^{-5}
Bull's liver, Brooklyn, $C_U = 5$	10^{-7} to 10^{-6}
Clay	Less than 10^{-9}

Permeability and Drainage Characteristics of Soils[1] [2][2]

	Coefficient of Permeability *k* (cm/s)(Log Scale)											
	10^{2}	10^{1}	1.0	10^{-1}	10^{-2}	10^{-3}	10^{-4}	10^{-5}	10^{-6}	10^{-7}	10^{-8}	10^{-9}
Drainage	Good						Poor		Practically impervious			
Soil types	Clean gravel		Clean sands, clean sand and gravel mixtures			Very fine sands, organic and inorganic silts, mixtures of sand silt and clay, glacial till, stratified clay deposits, etc.				"Impervious" soils (e.g., homogeneous clays below zone of weathering)		
					"Impervious" soils modified by effects of vegetation and weathering							
Direct determination of k	Direct testing of soil in its original position—pumping tests; reliable if properly conducted; considerable experience required											
	Constant-head permeameter; little experience required											
Indirect determination of k			Falling-head permeameter; reliable; little experience required			Falling-head permeameter; unreliable; much experience required			Falling-head permeameter; fairly reliable; considerable experience necessary			
	Computation from grain-size distribution; applicable only to clean cohesionless sands and gravels									Computation based on results of consolidation tests; reliable; considerable experience required		

HYDRAULIC CONDUCTIVITY IN STRATIFIED LAYERS OF SOILS

- Hydraulic conductivity of a disturbed sample may be different from that of the undisturbed sample even though the void ratio is the same.

- This may be due to a change in the structure or due to the stratification of the undisturbed soil or a combination of both of these factors.

HYDRAULIC CONDUCTIVITY IN STRATIFIED LAYERS OF SOILS

- Two fine-grained soils at the same void ratio, one dispersed and the other flocculated, will exhibit different permeabilities.

- The average permeability of stratified soil can be computed if the permeabilities of each layer are determined in the laboratory.

Permeability in Stratified Soils

$$v_y = k_y i = k_1 i_1 = k_2 i_1 = \cdots = k_n i_n$$

$\because$ Assume the velocity of each layer is equal

$$iH = i_1 H_1 + i_2 H_2 + \cdots + i_n H_n$$

$\because$ Head loss is the sum of head losses in all layers

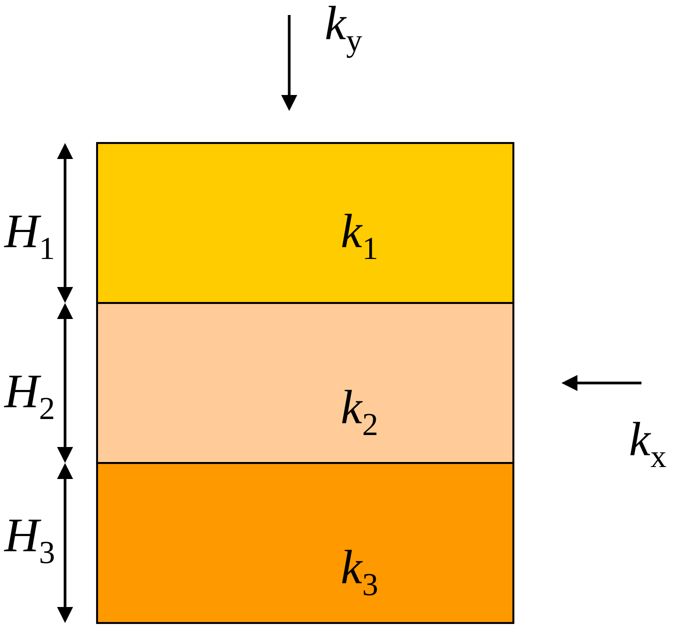
$$i = \frac{i_1 H_1 + i_2 H_2 + \cdots + i_n H_n}{H}$$

$$i_1 = \frac{k_y i}{k_1} \quad i_2 = \frac{k_y i}{k_2} \quad i_3 = \frac{k_y i}{k_3} \quad \cdots$$

$$i = \frac{\left(\frac{k_y i}{k_1}\right) H_1 + \left(\frac{k_y i}{k_2}\right) H_2 + \cdots + \left(\frac{k_y i}{k_n}\right) H_n}{H}$$

$$i = \frac{(k_y i)\left(\frac{H_1}{k_1} + \frac{H_2}{k_2} + \cdots + \frac{H_n}{k_n}\right)}{H}$$

$$k_y = \frac{H}{\frac{H_1}{k_1} + \frac{H_2}{k_2} + \cdots + \frac{H_n}{k_n}}$$

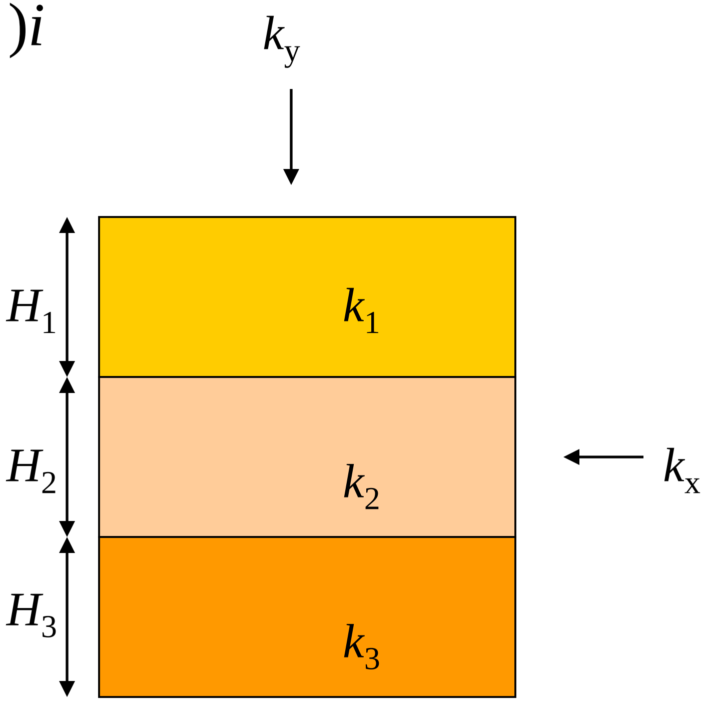


Permeability in Stratified Soils

$$q = kiA$$

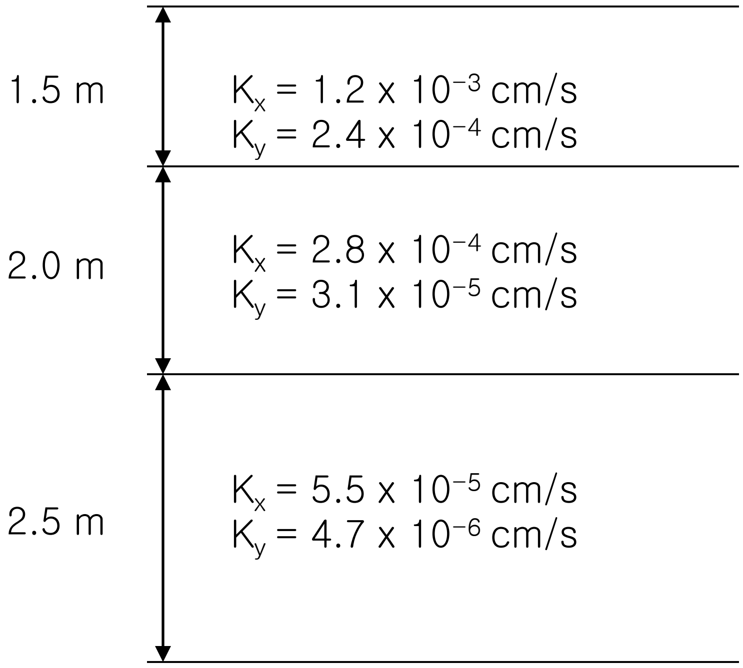
$$q = k_x iH = (k_1 H_1 + k_2 H_2 + \cdots k_n H_n)i$$

$$k_x = \frac{k_1 H_1 + k_2 H_2 + \cdots k_n H_n}{H}$$



Example

- A non-homogeneous soil consisting of layers of soil with different permeabilities.
- Average coefficient of permeability in the horizontal direction and vertical direction



1.5 m

$K_x = 1.2 \times 10^{-3}$ cm/s

$K_y = 2.4 \times 10^{-4}$ cm/s

2.0 m

$K_x = 2.8 \times 10^{-4}$ cm/s

$K_y = 3.1 \times 10^{-5}$ cm/s

2.5 m

$K_x = 5.5 \times 10^{-5}$ cm/s

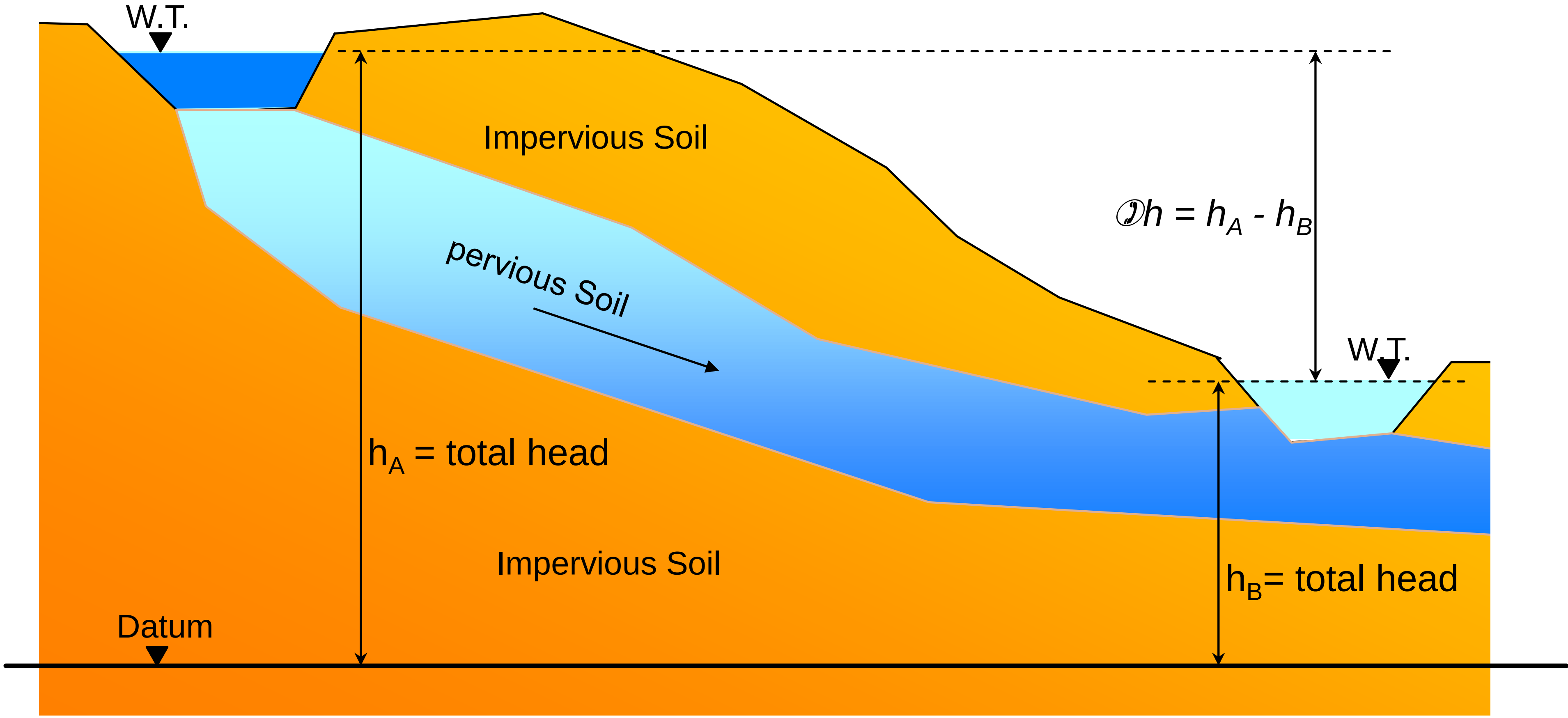
$K_y = 4.7 \times 10^{-6}$ cm/s

Example

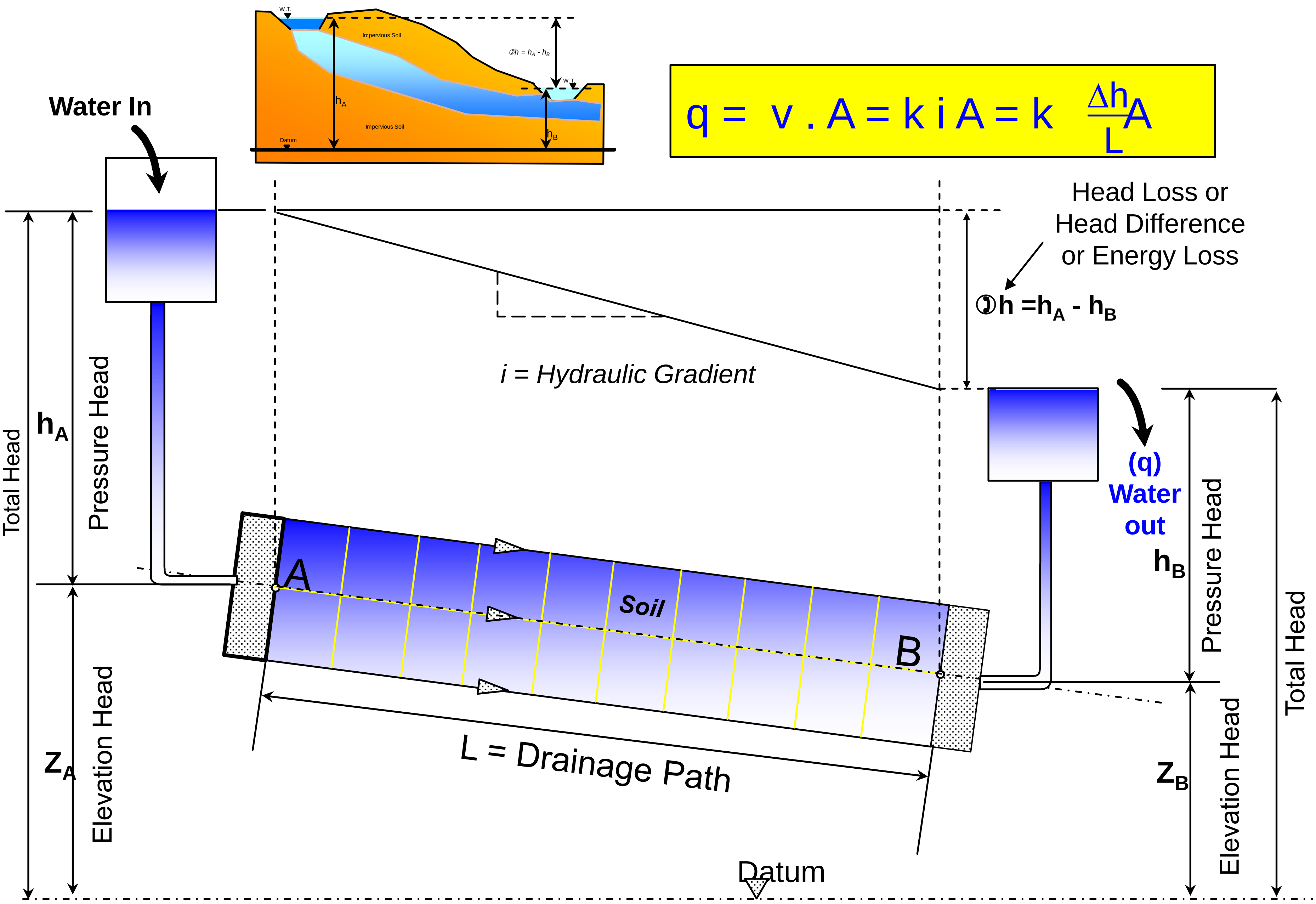
$$
\begin{aligned}
k_x &= \frac{k_1 H_1 + k_2 H_2 + k_3 H_3}{H} \\
&= \frac{(1.2\times10^{-3} cm/s)(1.5m) + (2.8\times10^{-4} cm/s)(2.0m) + (5.5\times10^{-5} cm/s)(2.5m)}{1.5m + 2.0m + 2.5m} \\
&= 4.16\times10^{-4} cm/s
\end{aligned}
$$

$$
\begin{aligned}
k_y &= \frac{H}{(H_1/k_1) + (H_2/k_2) + (H_3/k_3)} \\
&= \frac{1.5m + 2.0m + 2.5m}{\left(\dfrac{1.5m}{2.4\times10^{-4} cm/s}\right) + \left(\dfrac{2.0m}{3.1\times10^{-5} cm/s}\right) + \left(\dfrac{2.5m}{4.7\times10^{-6} cm/s}\right)} \\
&= 9.96\times10^{-6} cm/s
\end{aligned}
$$

Seepage Through Porous Media



$$q = v \, . \, A = k \, i \, A = k \, \frac{\Delta h}{L} A$$



Bernouli′s Equation:

Total Energy = Elevation Energy + Pressure Energy + Velocity Energy

or

Total Head = Elevation Head + Pressure Head + Velocity Head

$$h_{total} = Z + \frac{P}{\gamma} + \frac{V^2}{2\,g}$$

Darcy′s Law:

v 🔔 i v = discharge velocity & i = hydraulic gradient

v = k i k = coefficient of permeability

v = k ✆h/L

To determine the rate of flow, two parameters are needed

* k = coefficient of permeability
* i = hydraulic gradient

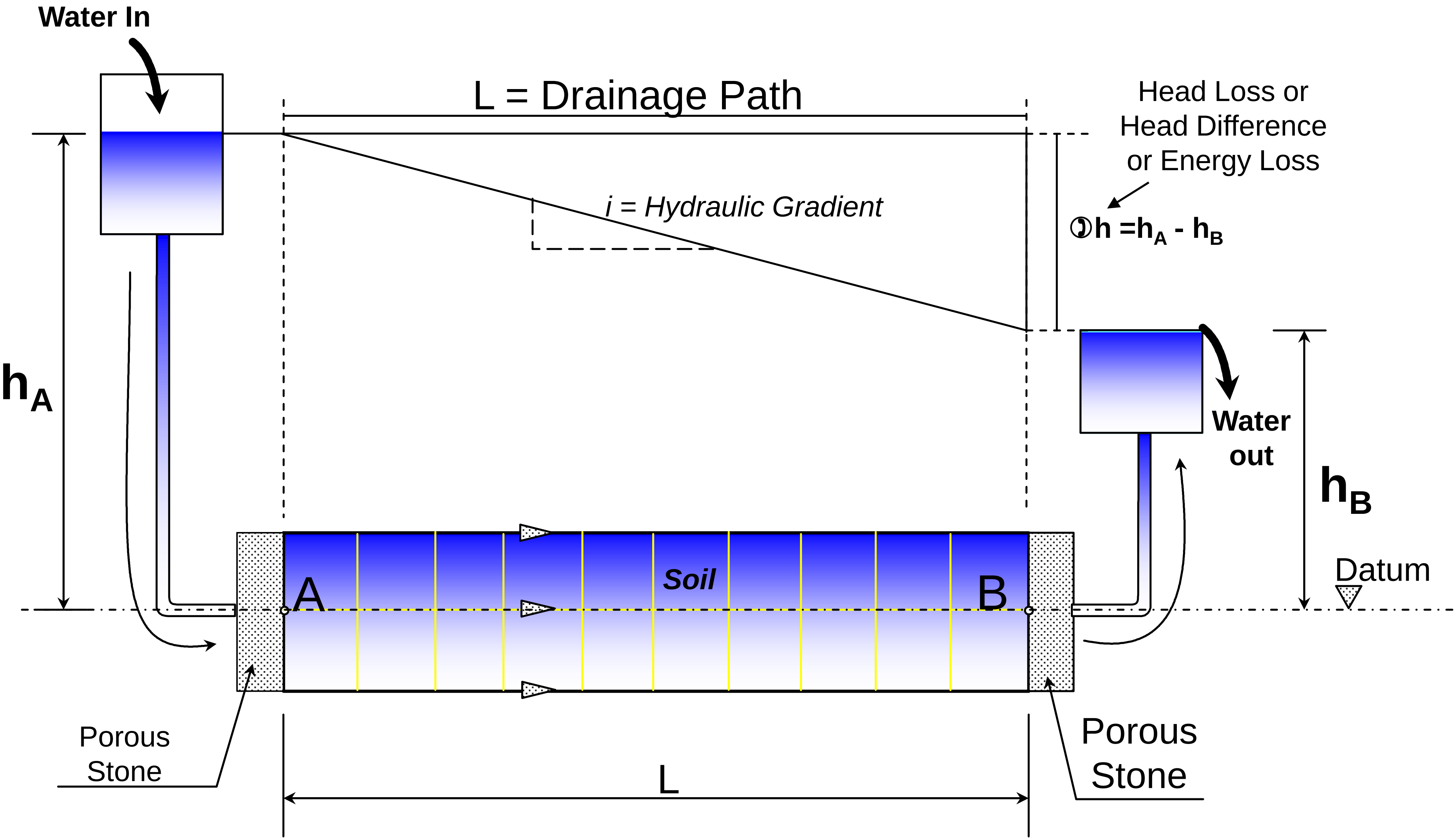
***k* can be determined using**

1- Laboratory Testing → [constant head test & falling head test]
2- Field Testing → [pumping from wells]
3- Empirical Equations

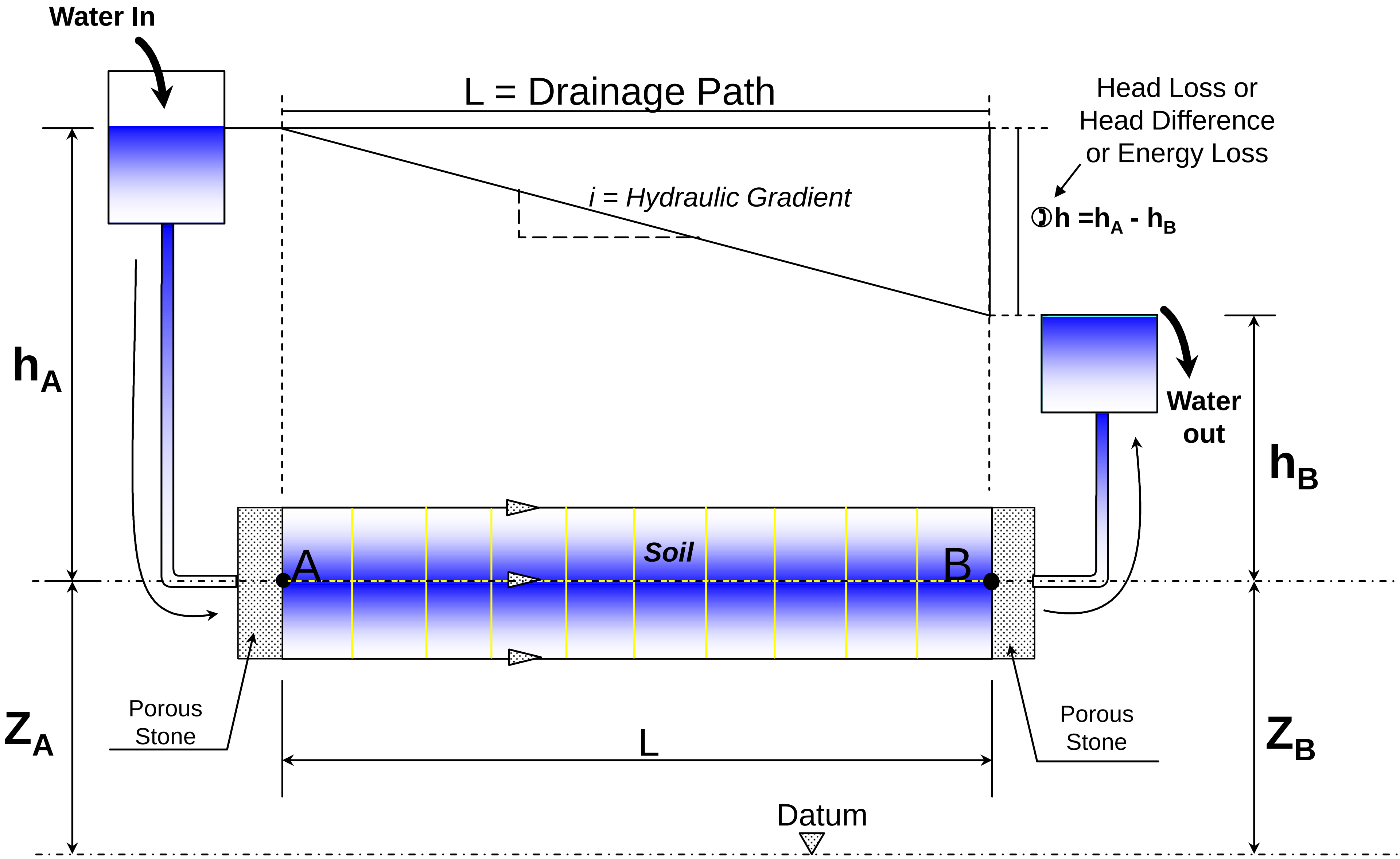
***i* can be determined**

1- from the head loss
2- flow net

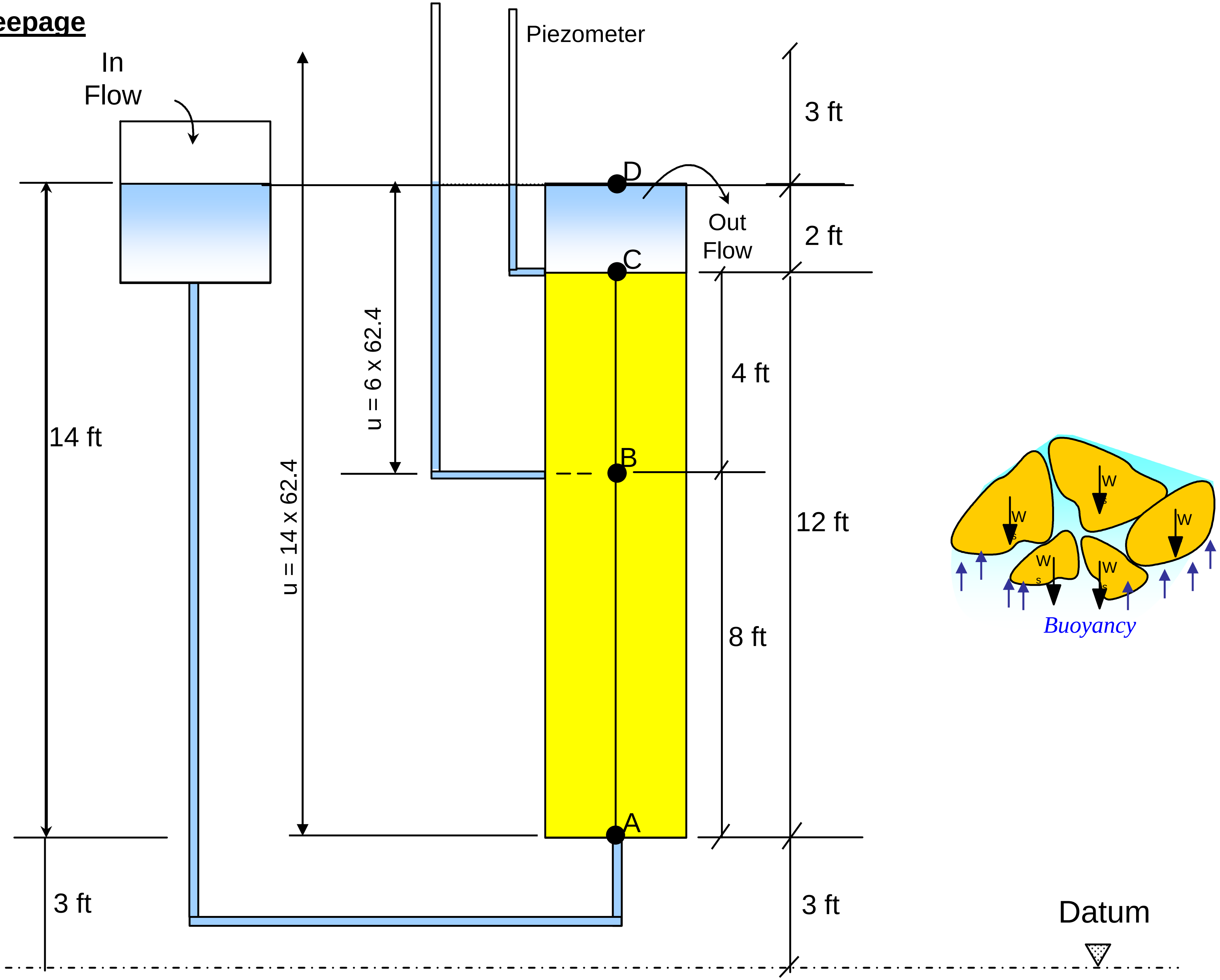
Seepage Through Porous Media



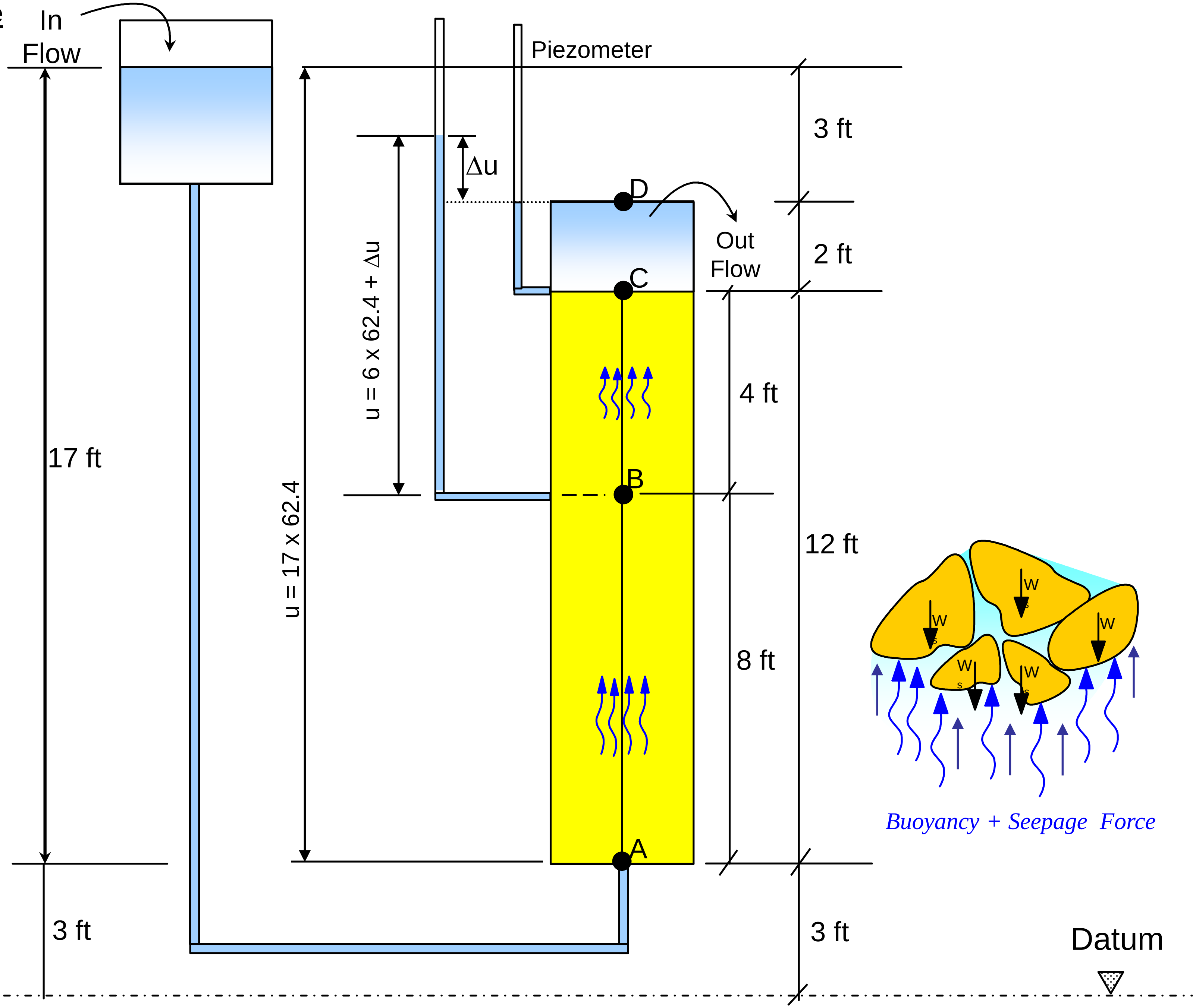
Seepage Through Porous Media



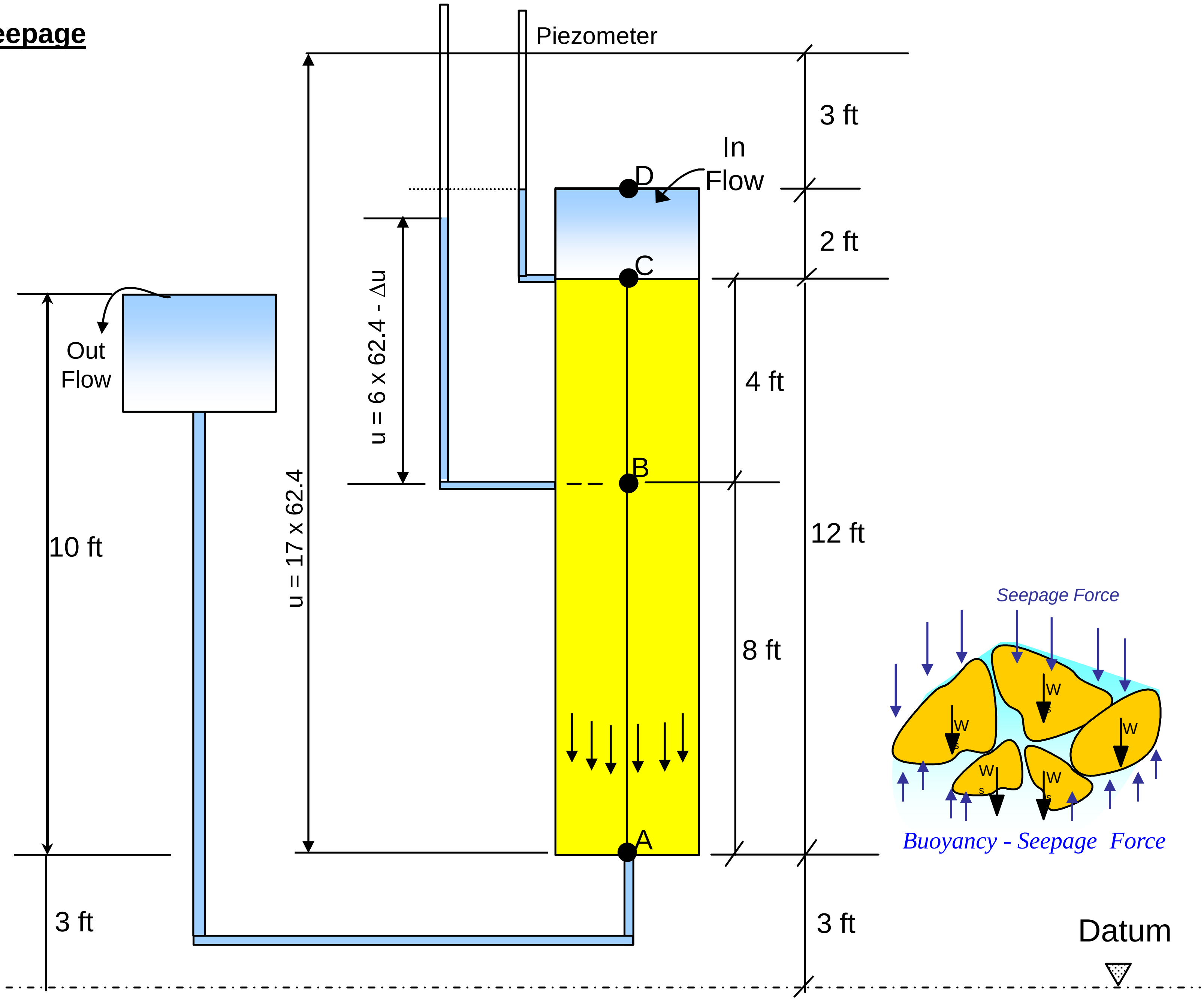
No Seepage


In Flow
Piezometer
D
C
B
A
Out Flow
14 ft
3 ft
u = 14 x 62.4
u = 6 x 62.4
3 ft
2 ft
12 ft
3 ft
4 ft
8 ft
W
s
W
s
W
s
W
s
W
s
W
Buoyancy
Datum

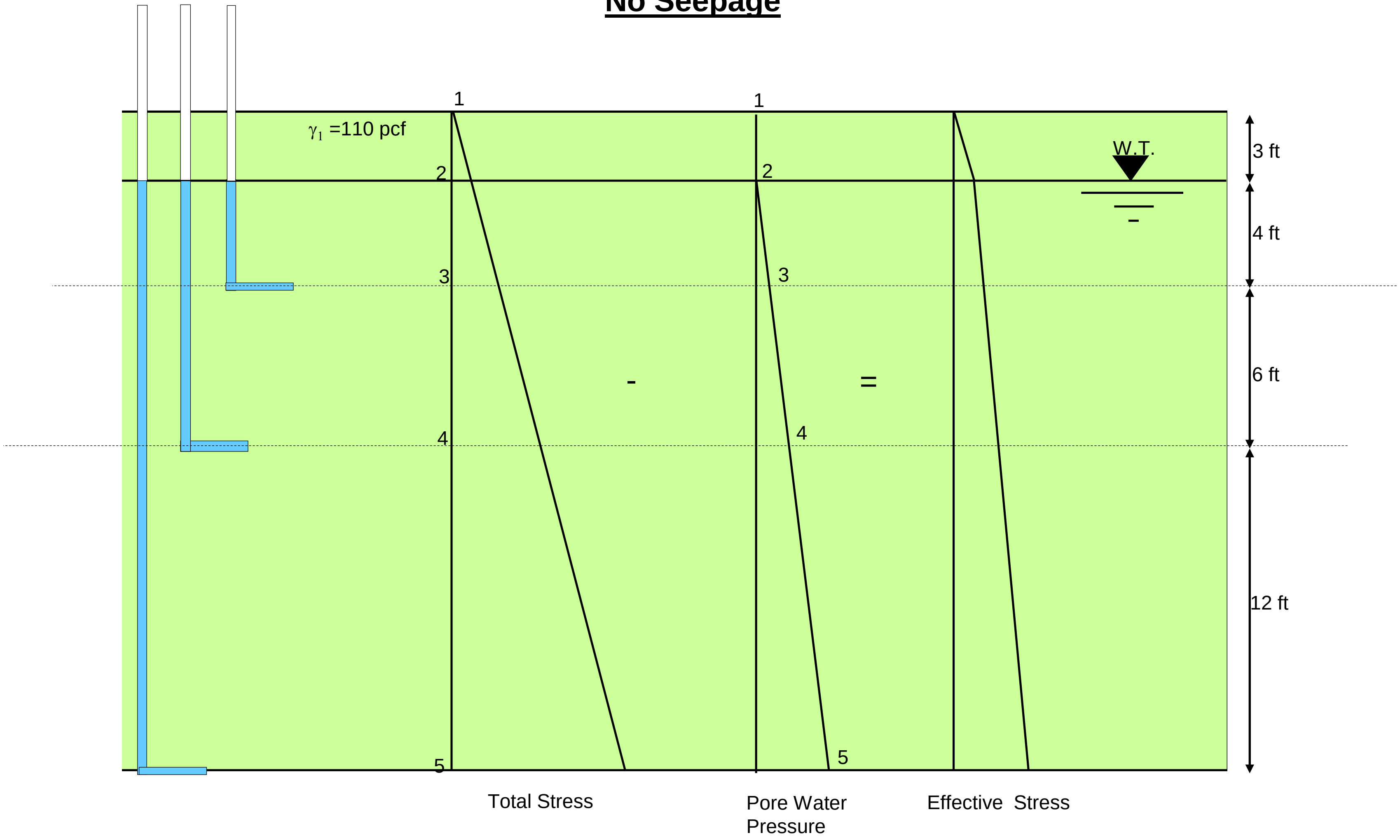
Upward Seepage



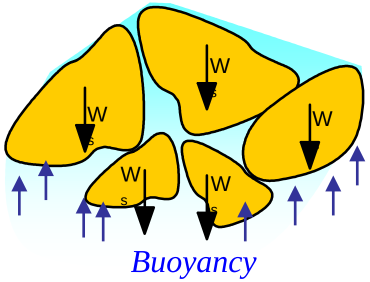
Downward Seepage



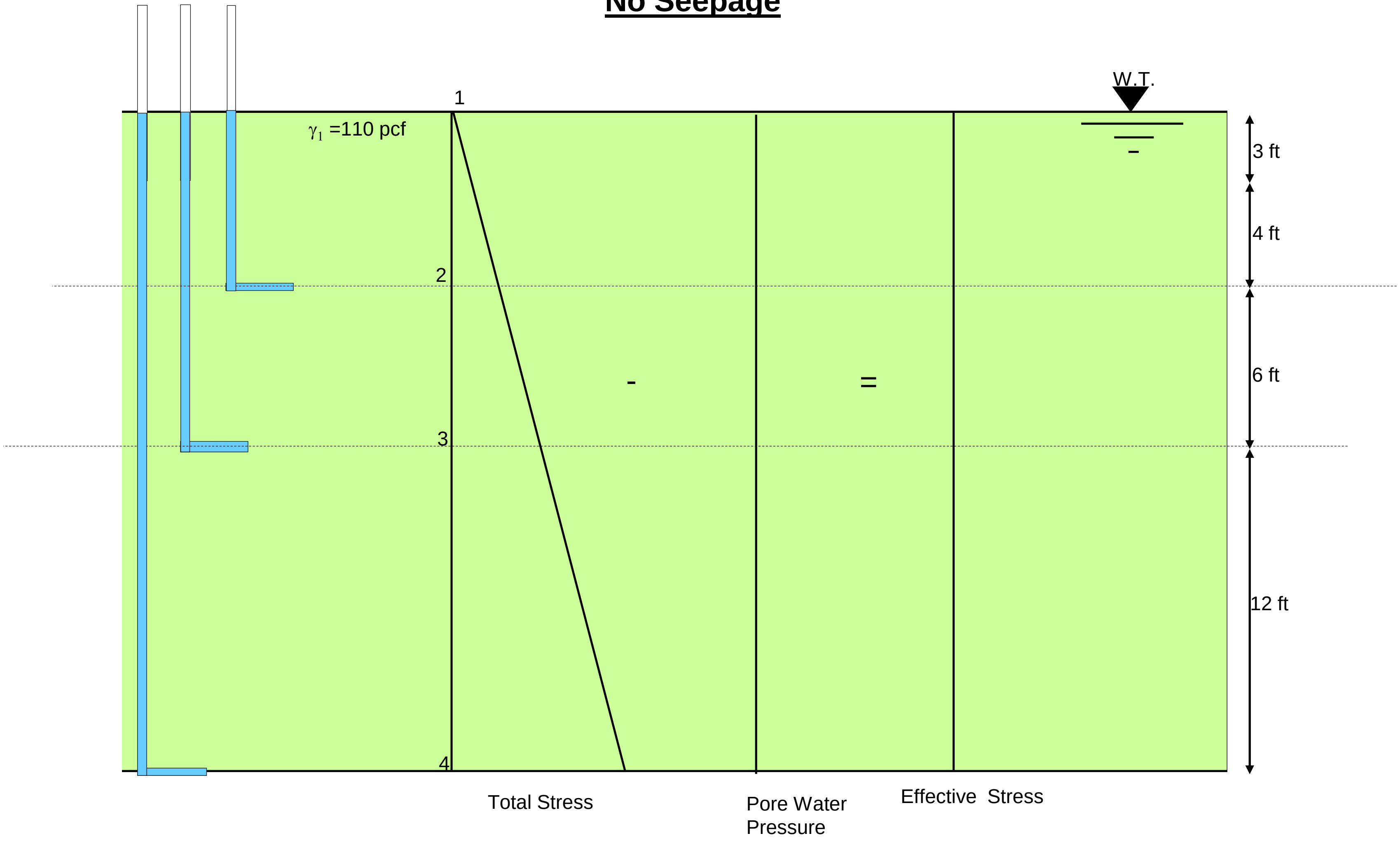
No Seepage

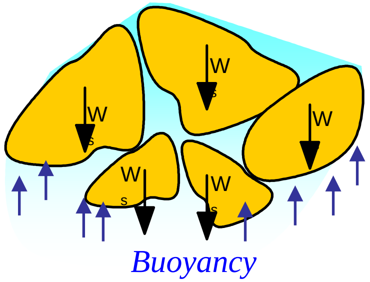


Total Stress	**Pore Water Pressure**	**Effective Stress**
σ1 =	u1 =	σ1 =
σ2 =	u2 =	σ2 =
σ3 =	u3 =	σ3 =
σ4 =	u4 =	σ4 =
σ5 =	u5 =	σ5 =

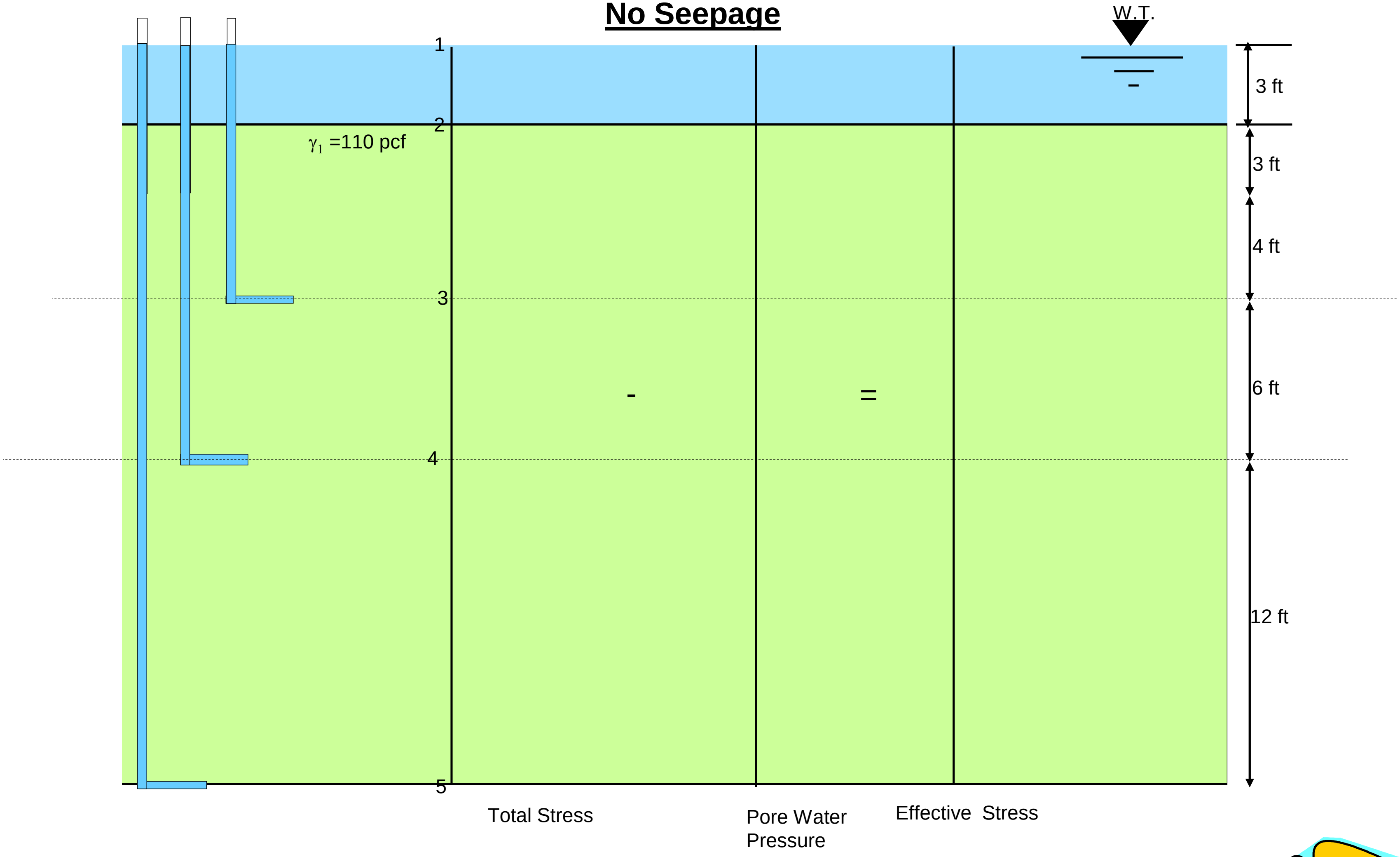


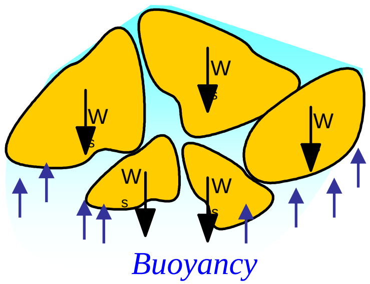
No Seepage


W.T.
1
γ1 =110 pcf
2
3
4
-
=
3 ft
4 ft
6 ft
12 ft
Total Stress
Pore Water Pressure
Effective Stress

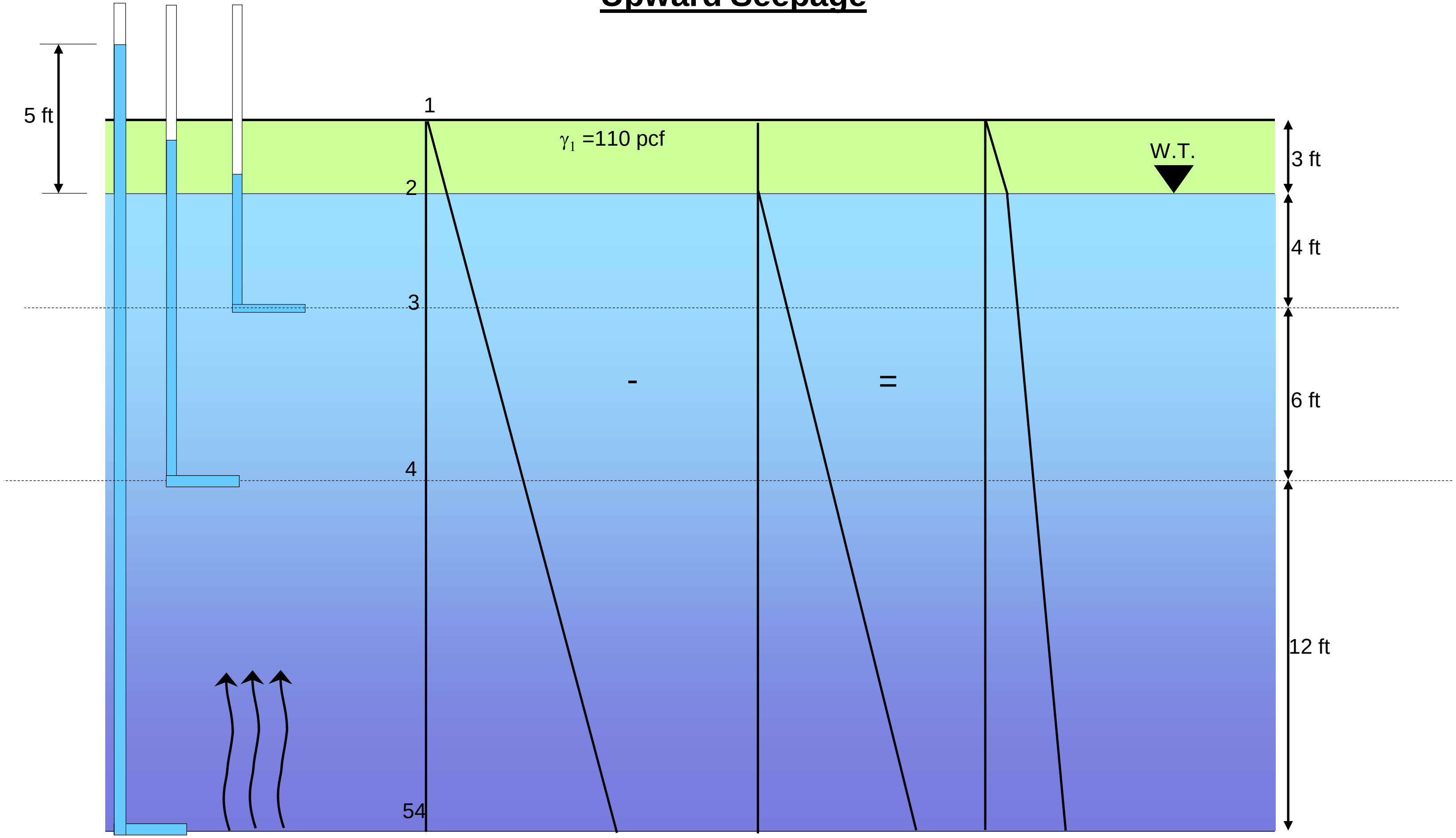

W s
W s
W s
W s
W s
W
Buoyancy

No Seepage


W.T.
1
2
γ_1 =110 pcf
3
4
5
-
=
3 ft
3 ft
4 ft
6 ft
12 ft
Total Stress
Pore Water Pressure
Effective Stress


W s
W s
W s
W s
W s
Buoyancy

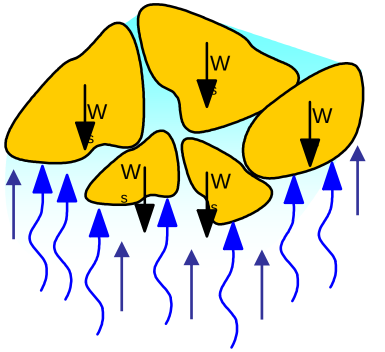
Upward Seepage



Total Stress

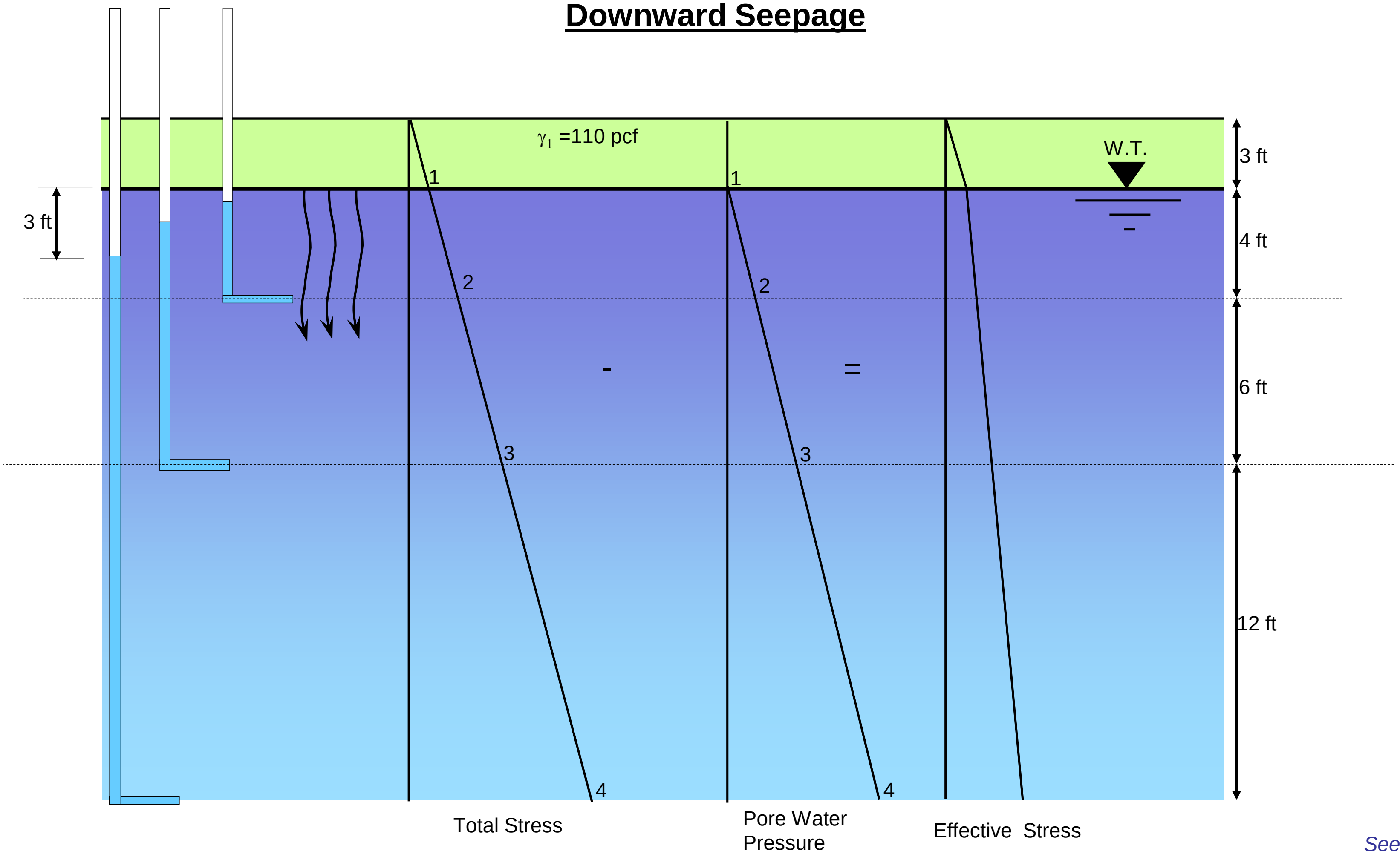
Pore Water Pressure

Effective Stress



Buoyancy + Seepage Force

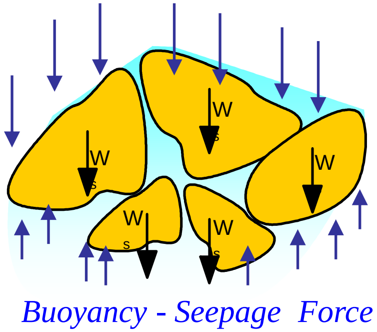
Downward Seepage

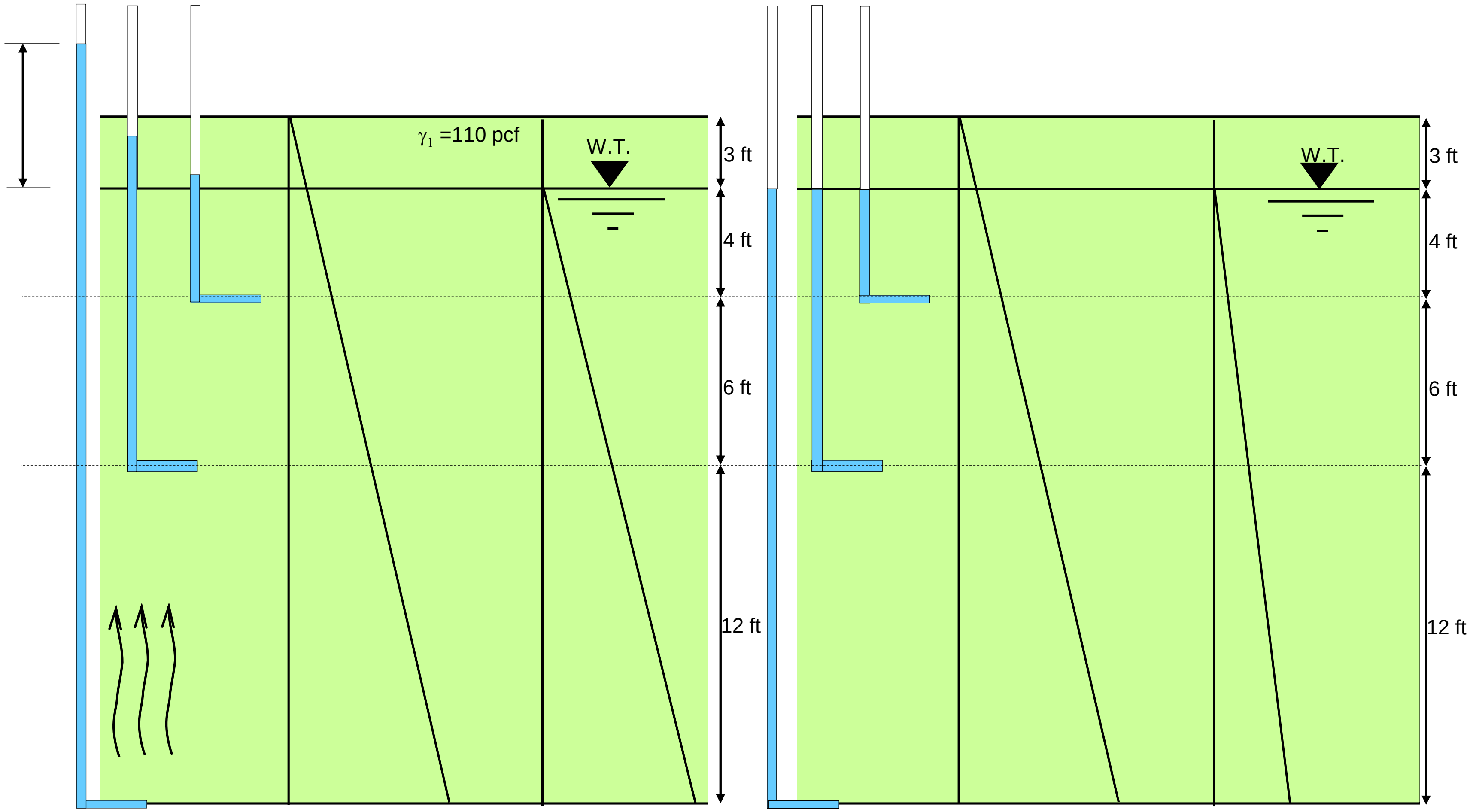


Total Stress

Pore Water Pressure

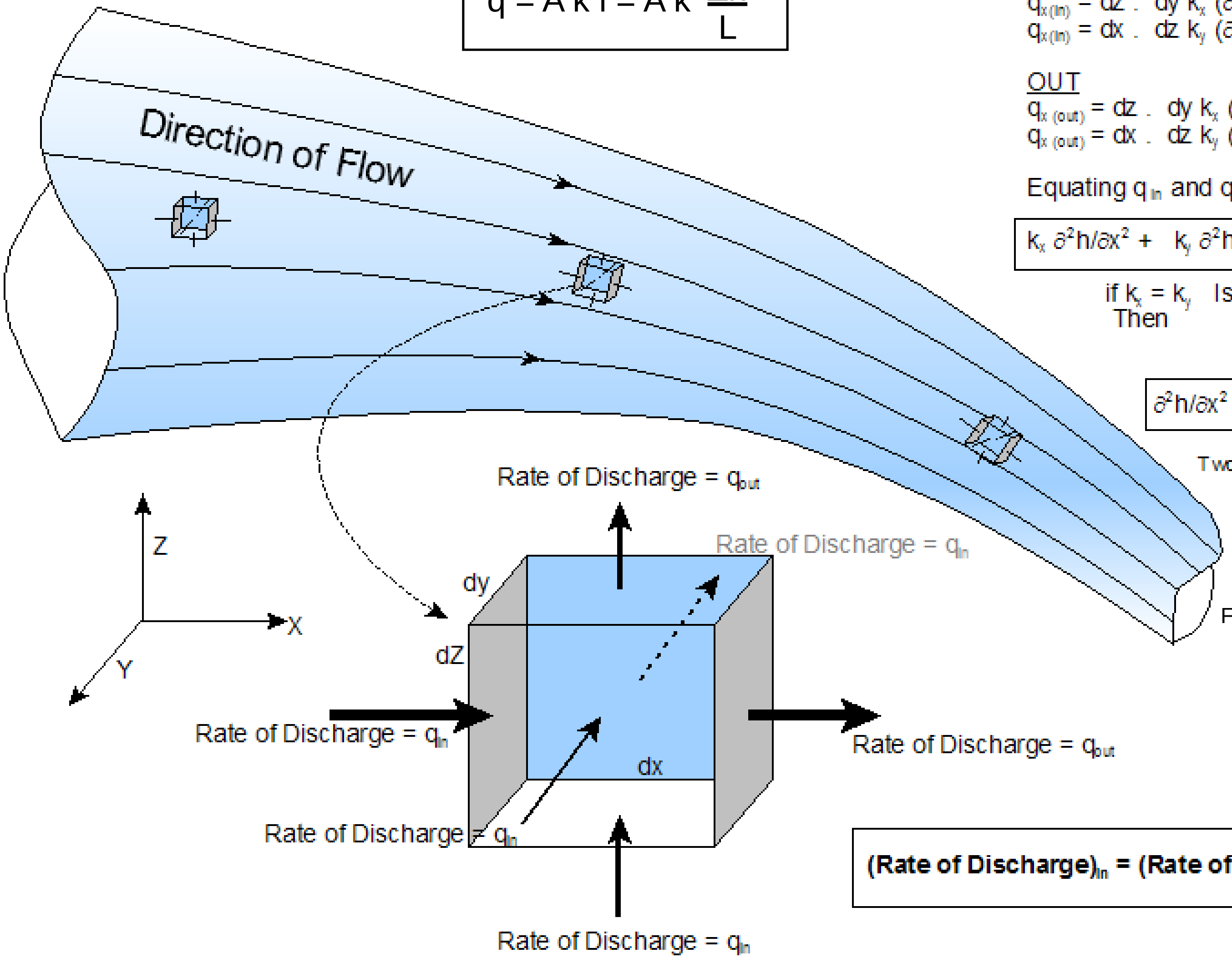
Effective Stress




γ₁ =110 pcf
W.T.
3 ft
4 ft
6 ft
12 ft
W.T.
3 ft
4 ft
6 ft
12 ft

Seepage Through Porous Media

$$\boxed{q = A\,k\,i = A\,k\,\frac{\Delta h}{L}}$$



IN

$q_{x(in)} = dz\ .\ dy\ k_x\ (\partial h/\partial x)$

$q_{x(in)} = dx\ .\ dz\ k_y\ (\partial h/\partial y)$

OUT

$q_{x\ (out)} = dz\ .\ dy\ k_x\ (\partial h/\partial x + \partial^2 h/\partial x^2\ dx\ )$

$q_{x\ (out)} = dx\ .\ dz\ k_y\ (\partial h/\partial y + \partial^2 h/\partial y^2\ dy\ )$

Equating q_{in} and q_{out}

$$\boxed{k_x\ \partial^2 h/\partial x^2 + k_y\ \partial^2 h/\partial y^2 = 0}$$

if $k_x = k_y$ Isotropic Flow
Then

$$\boxed{\partial^2 h/\partial x^2 + \partial^2 h/\partial y^2 = 0}$$

Two sets of curves

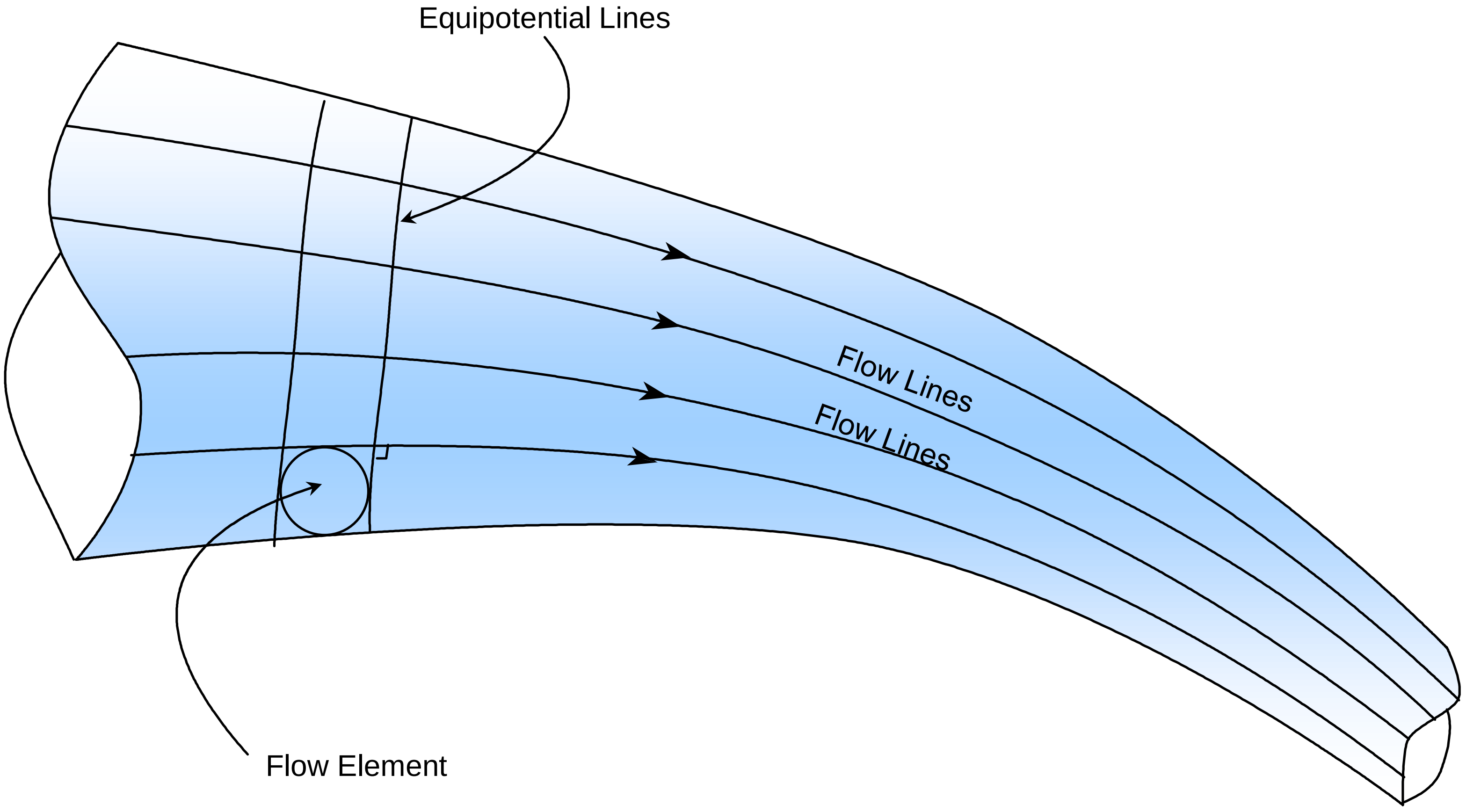
Flow Lines

Equipotential Lines

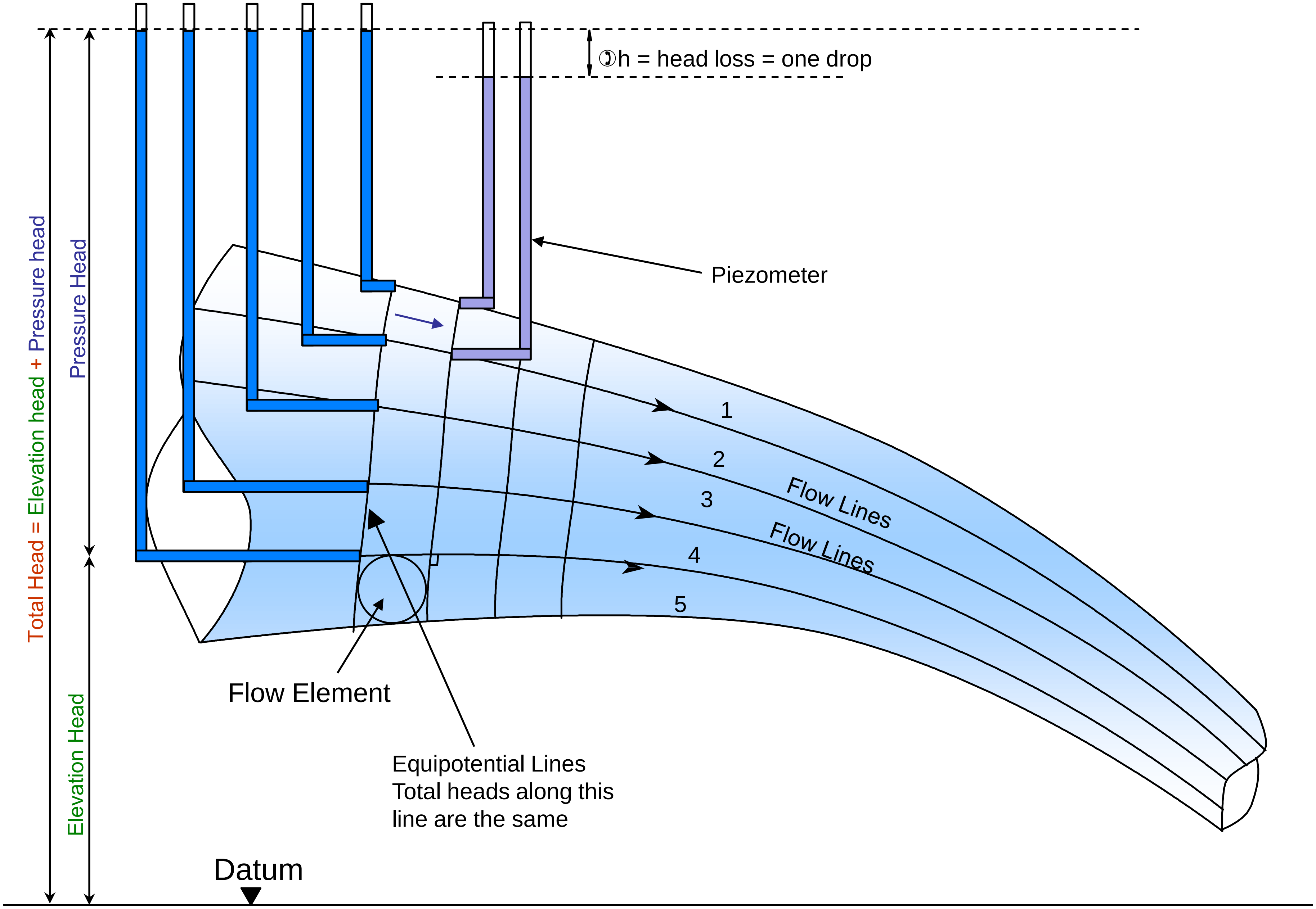
$$\boxed{(\text{Rate of Discharge})_{in} = (\text{Rate of Discharge})_{out}}$$

By Kamal Tawfiq, Ph.D., P.E.

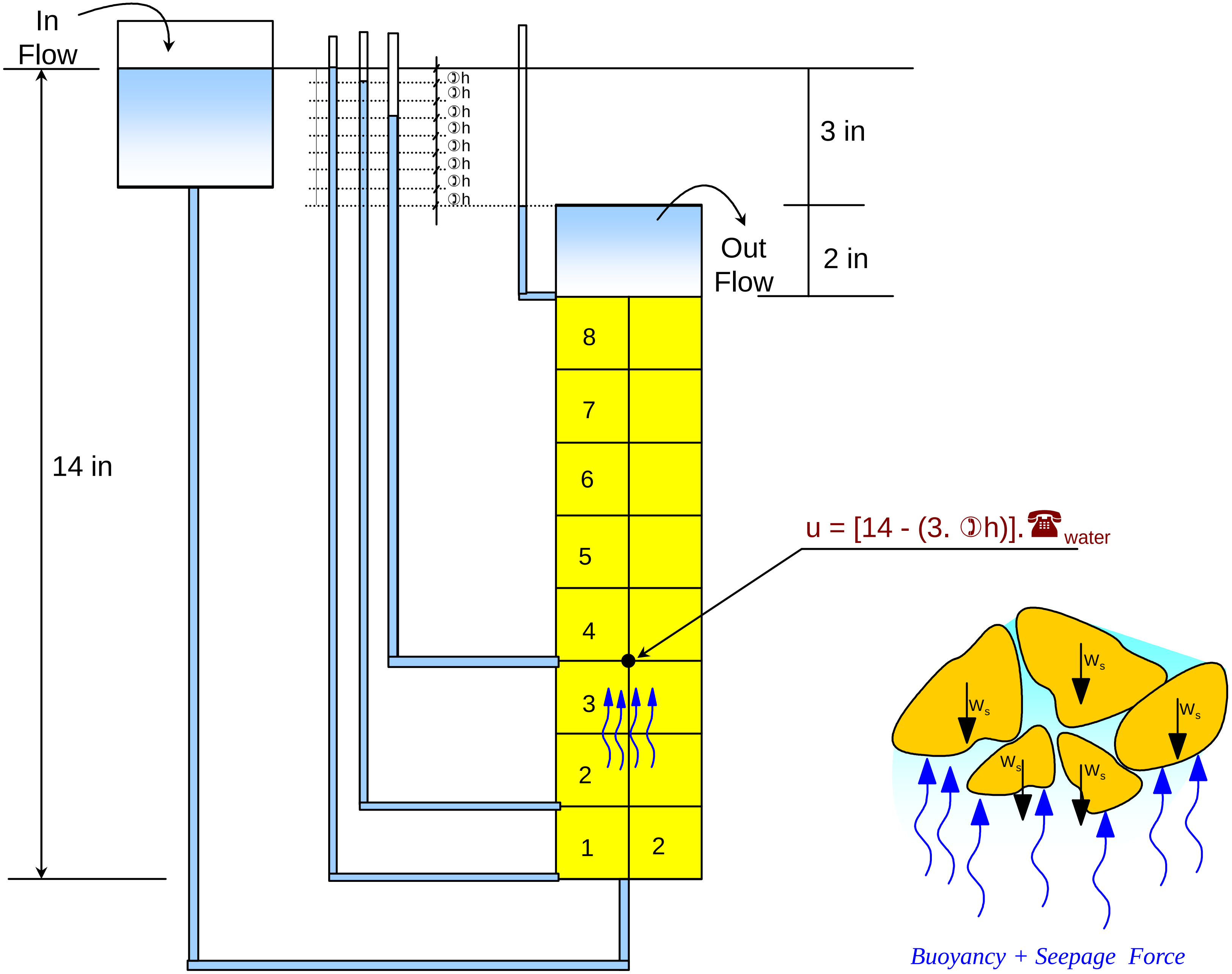
Principles of the Flow Net



Principles of the Flow Net



Equipotential Lines
Total heads along this
line are the same



Buoyancy + Seepage Force

Seepage Through Porous Media

IN

$q_{x(in)} = dz\ .\ dy\ k_x\ (\partial h/\partial x)$

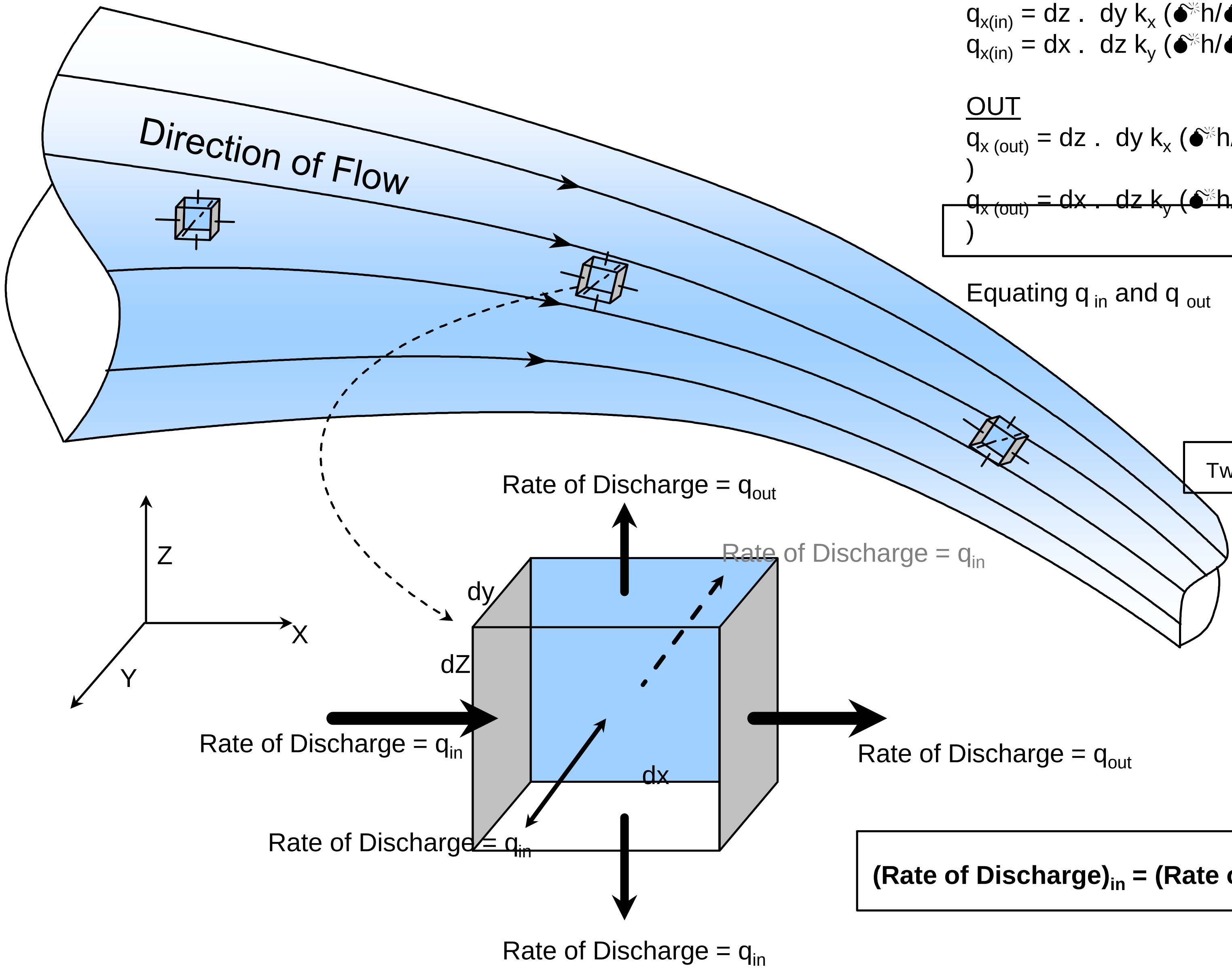
$q_{x(in)} = dx\ .\ dz\ k_y\ (\partial h/\partial y)$

OUT

$q_{x\ (out)} = dz\ .\ dy\ k_x\ (\partial h/\partial x + \partial^2 h/\partial x^2\ dx\ )$

$q_{x\ (out)} = dx\ .\ dz\ k_y\ (\partial h/\partial y + \partial^2 h/\partial y^2\ dy\ )$

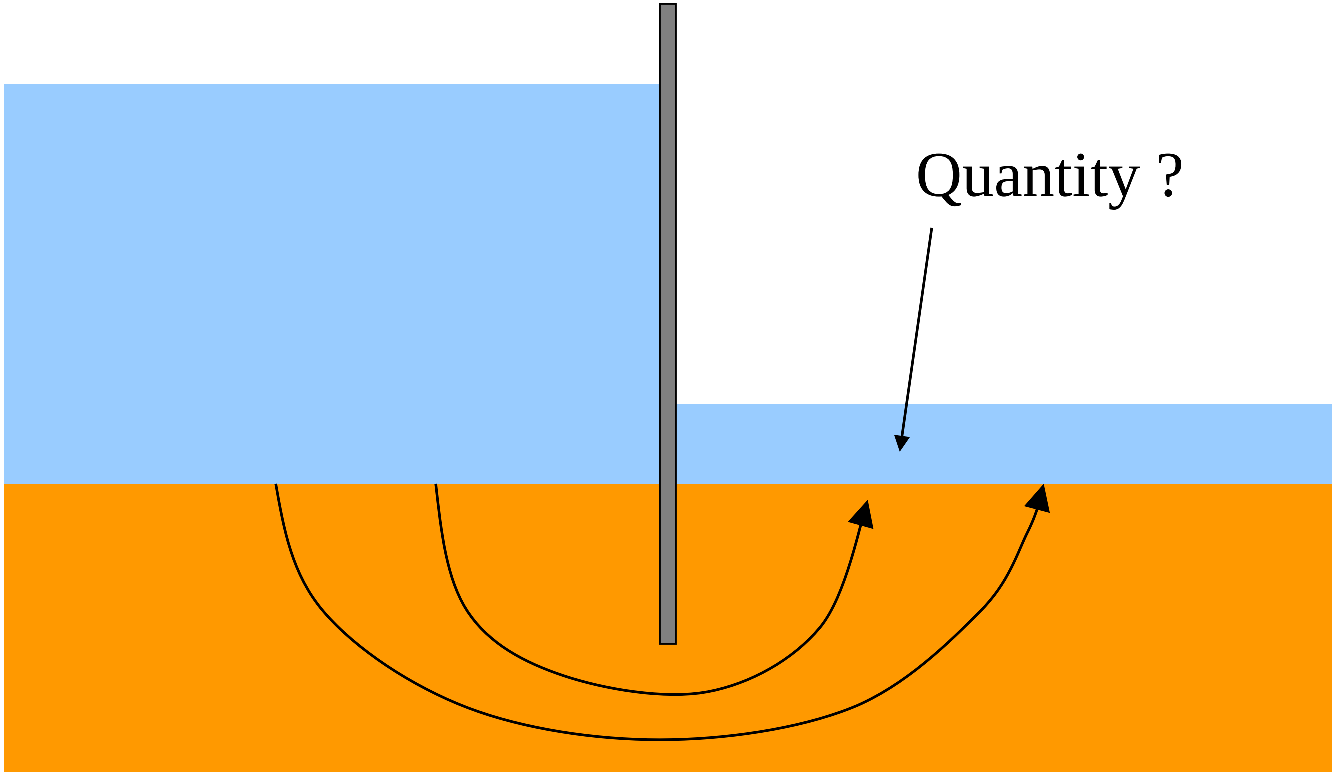
Equating q in and q out



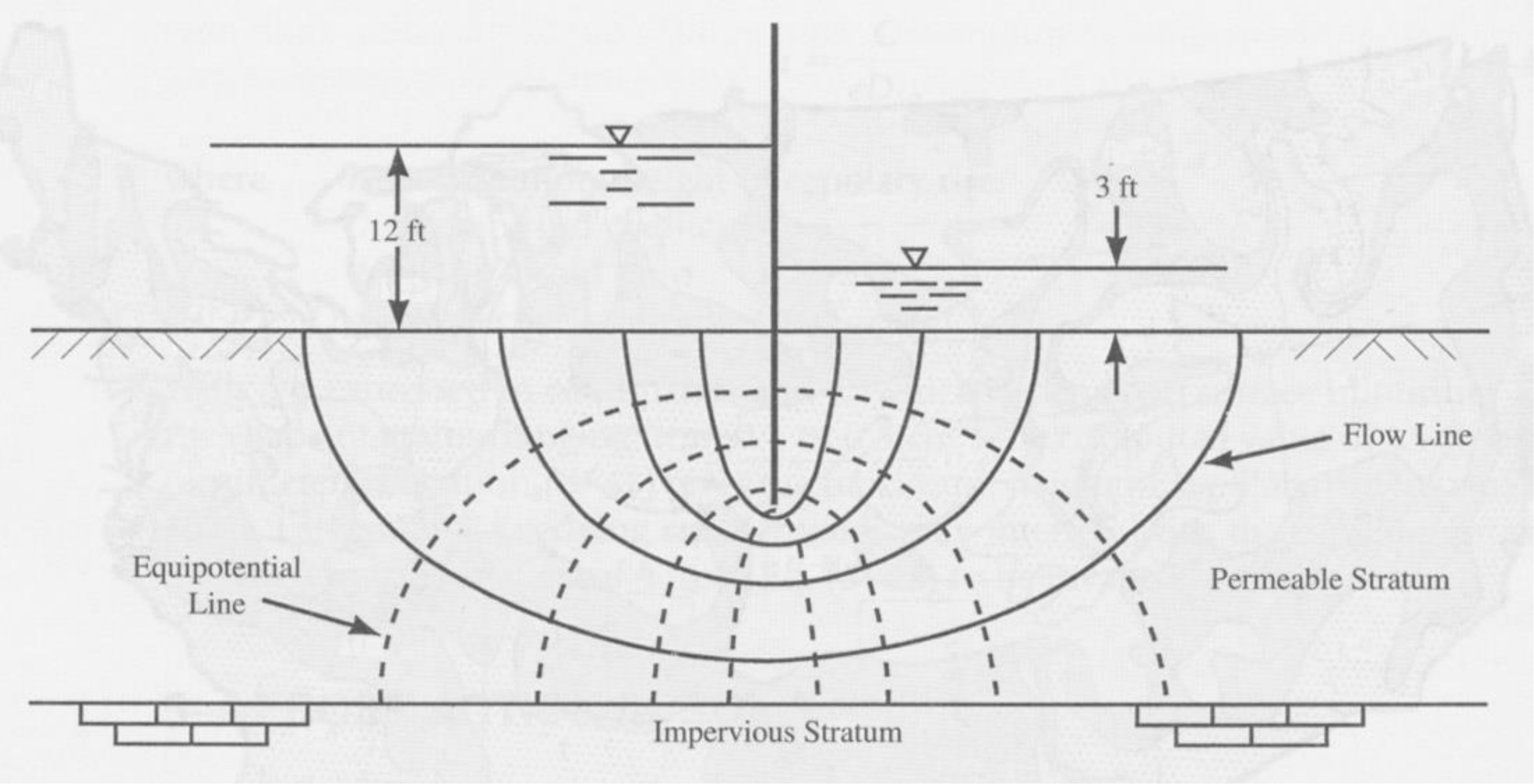
Two sets of curves

$(\text{Rate of Discharge})_{in} = (\text{Rate of Discharge})_{out}$

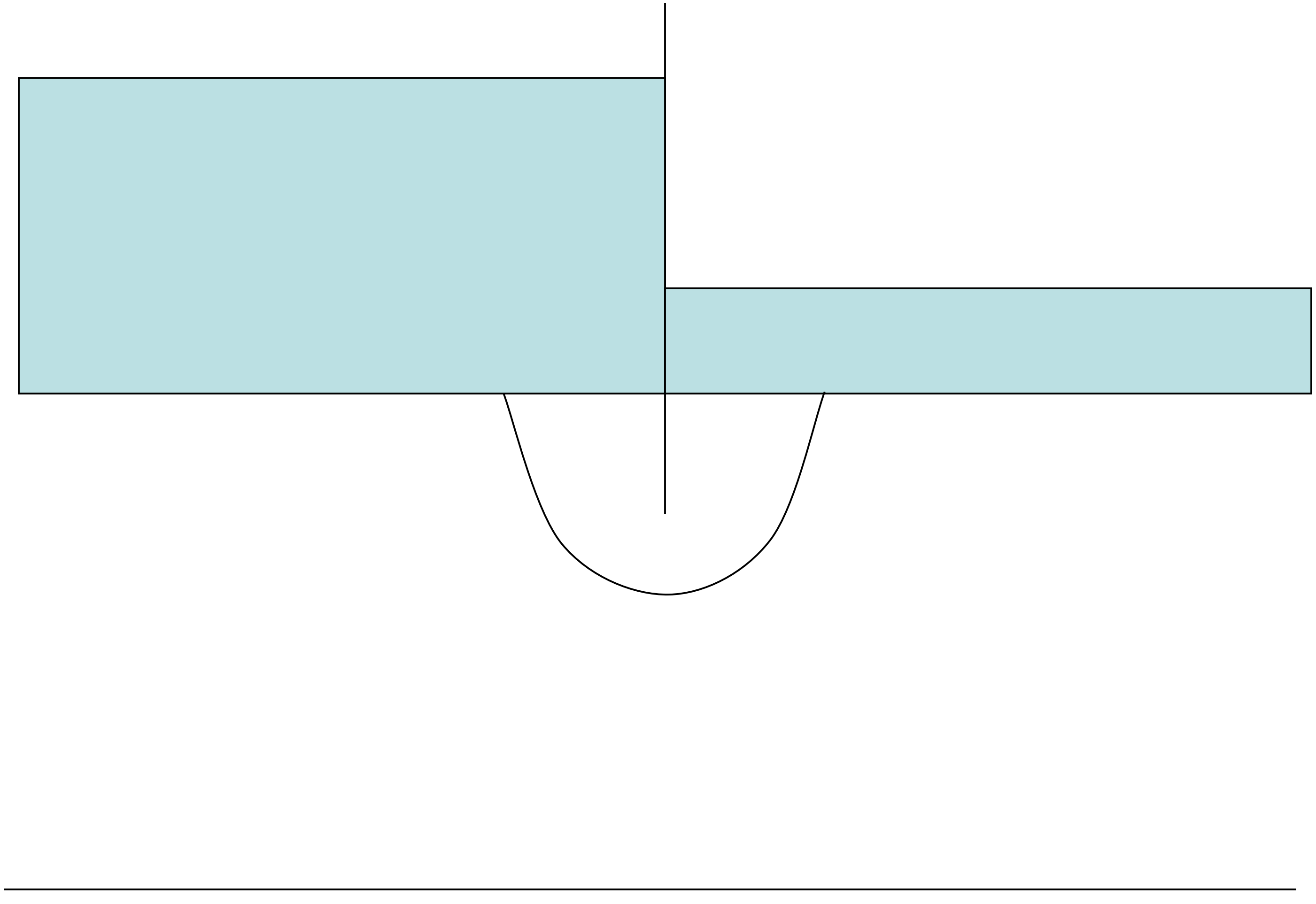
Question



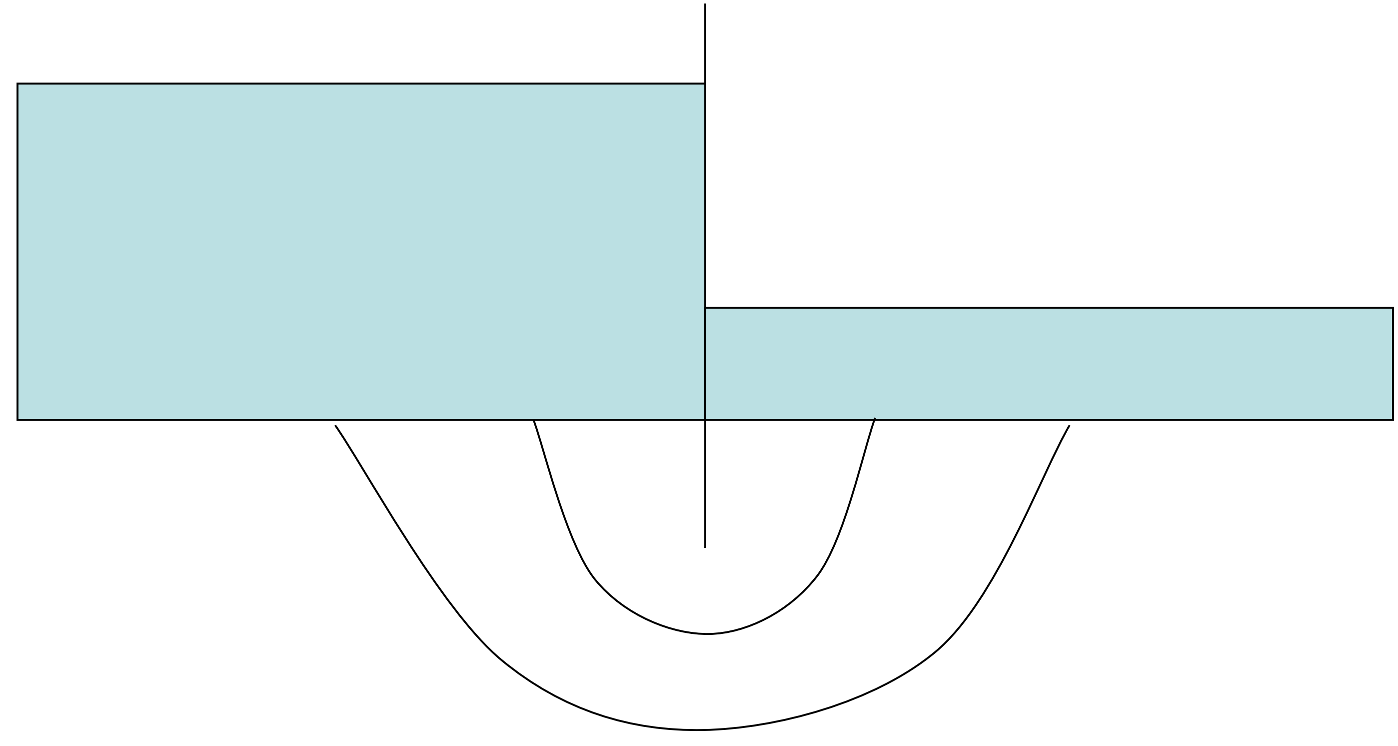
Flow Nets and Seepage



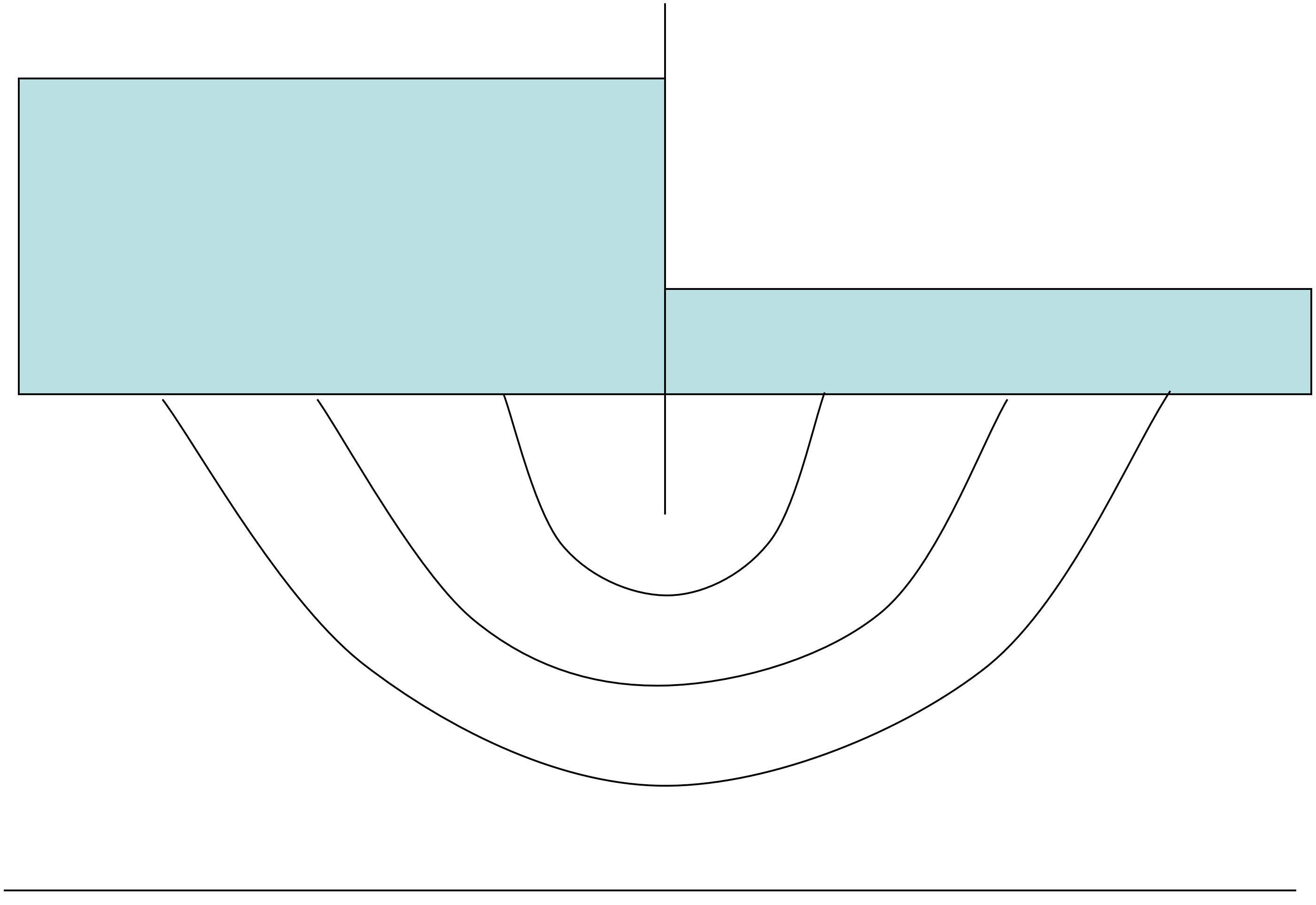
Flow Nets



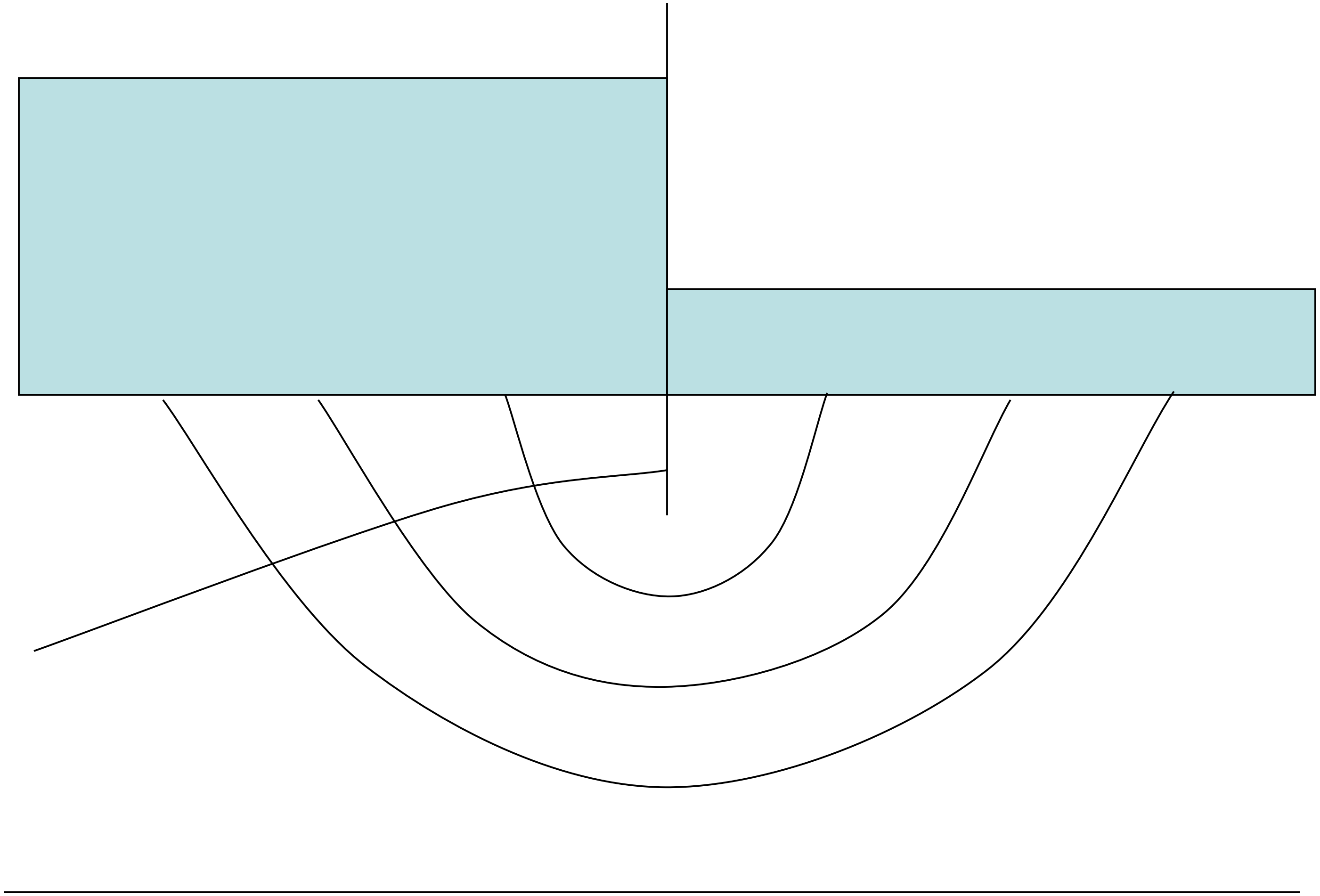
Flow Nets



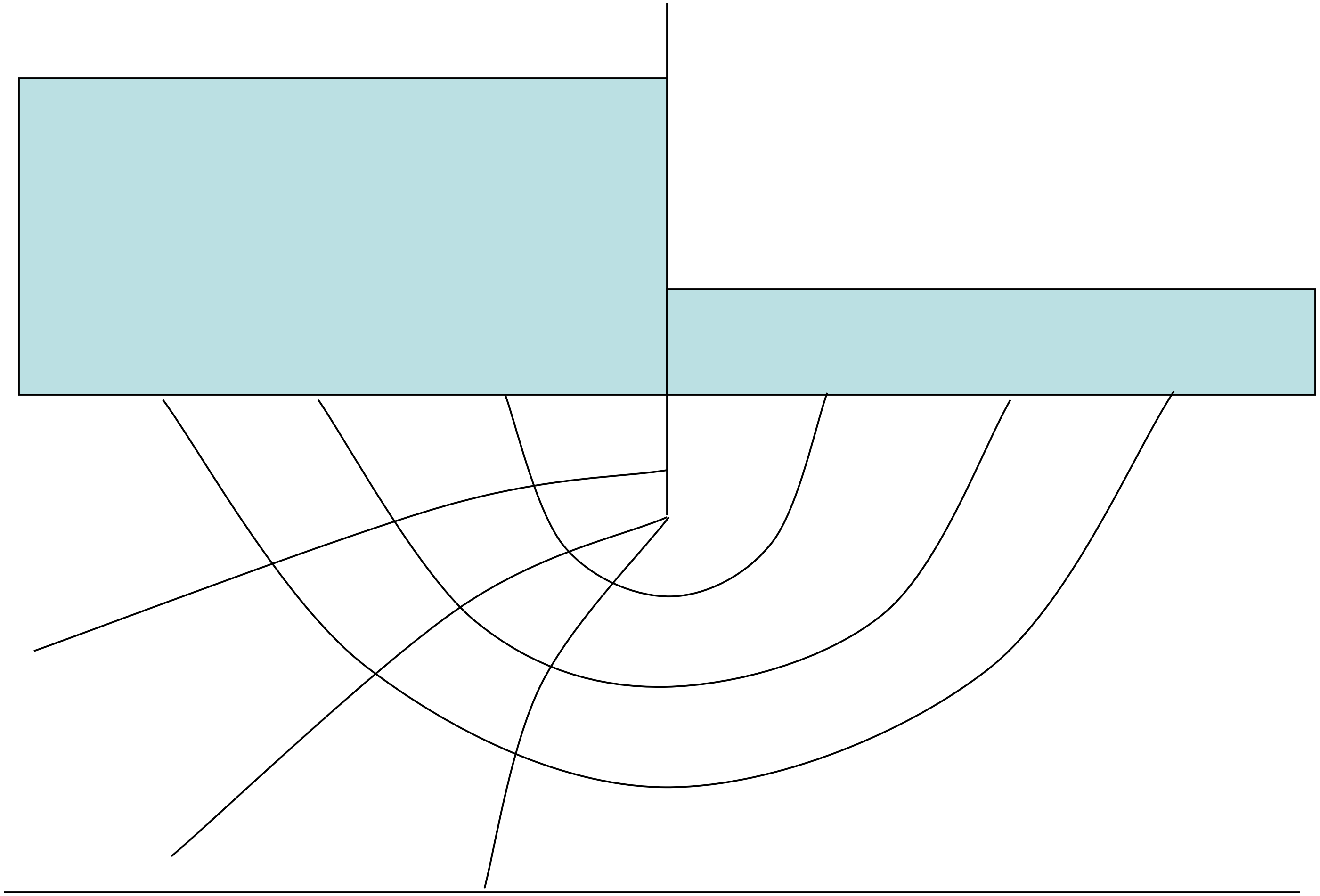
Flow Nets



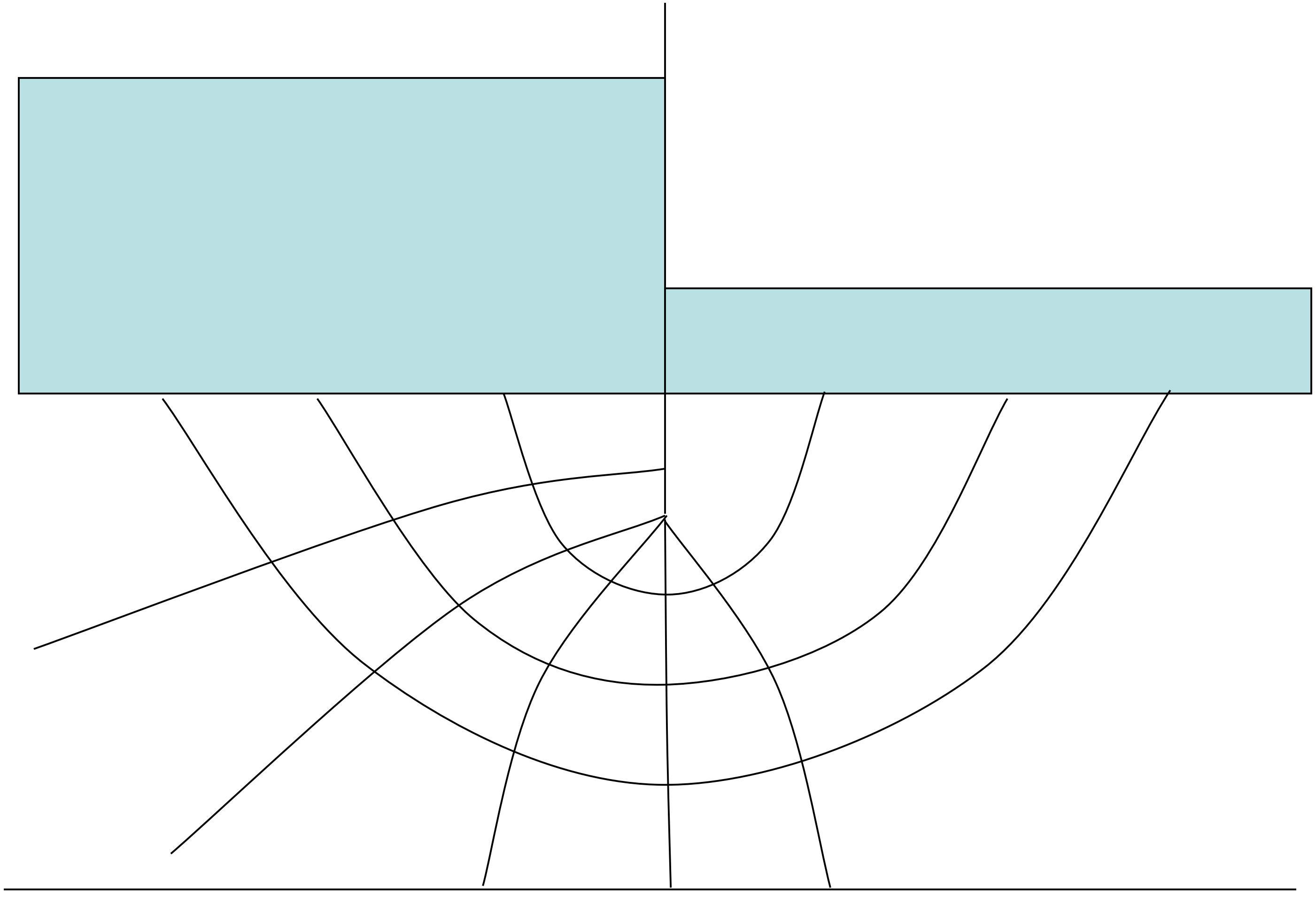
Flow Nets



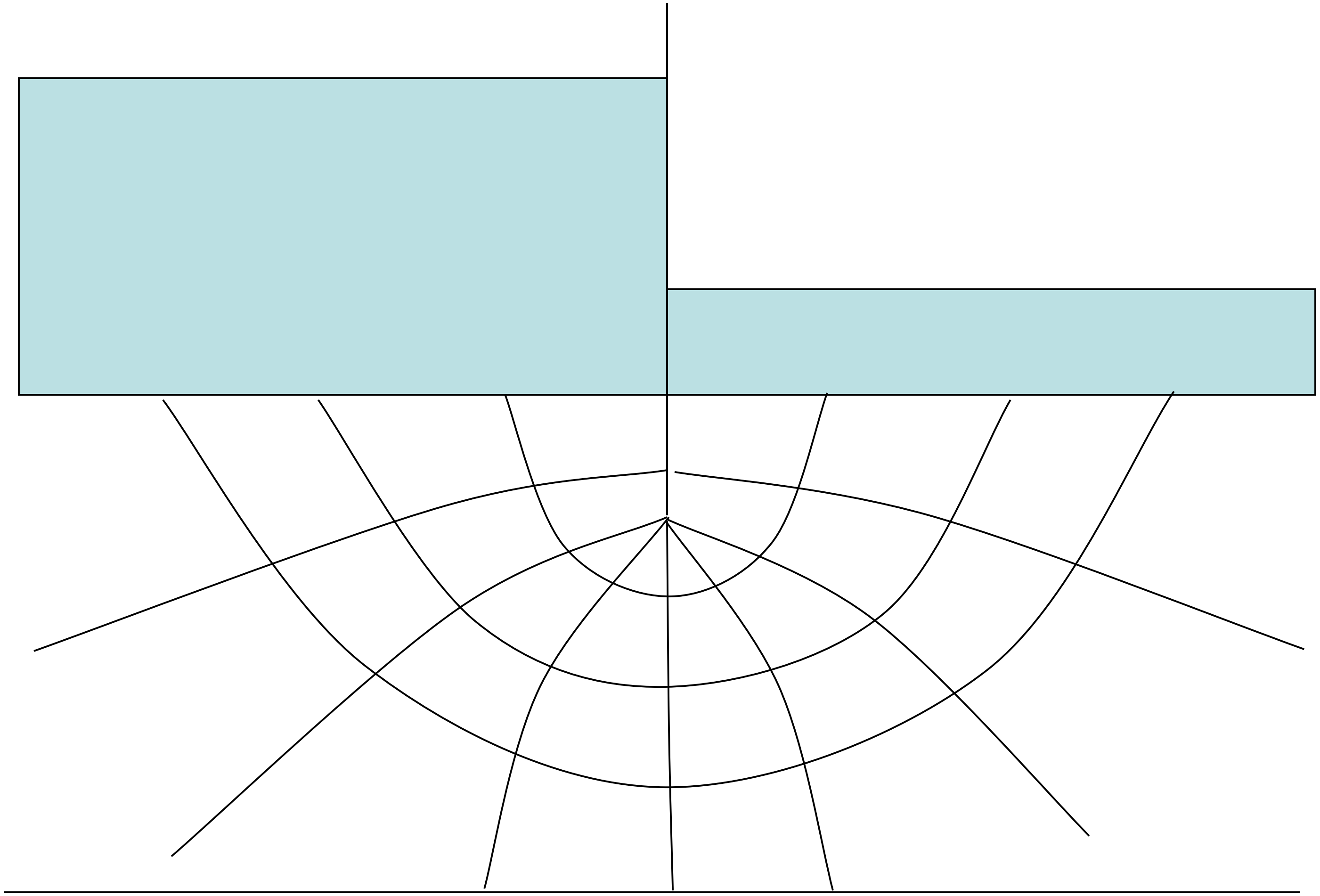
Flow Nets



Flow Nets



Flow Nets



Flow Nets and Seepage

$$q = kiA$$

$$\Delta h = \text{drop in head}$$

$$\Delta q = \text{flow rate}$$

$$\Delta q = k\frac{\Delta h}{x}y = k\Delta h \quad \because \frac{y}{x} = 1$$

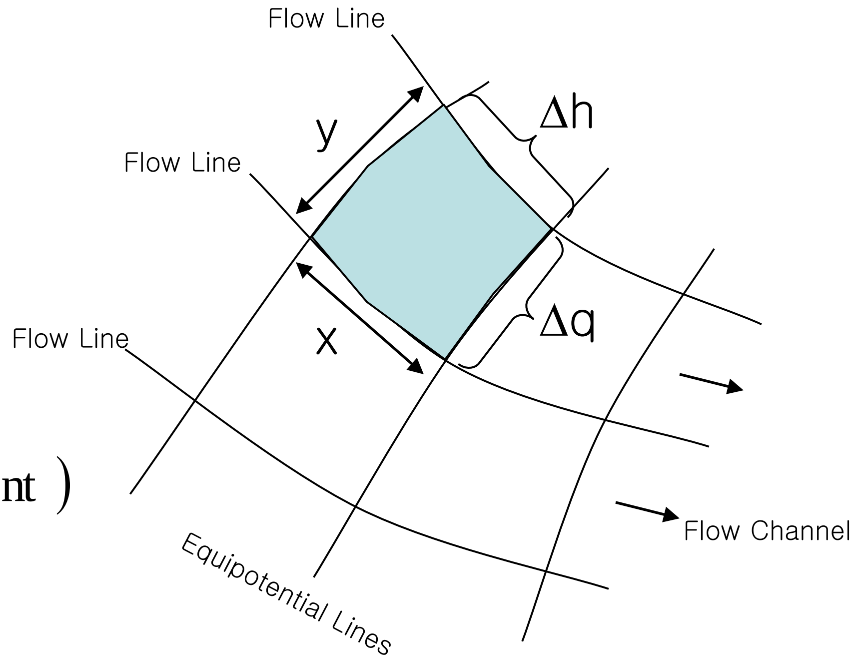
$$\Delta h = \frac{h}{N_d}$$

$$\left(N_d \text{ is number of equpotenti al increment}\right)$$

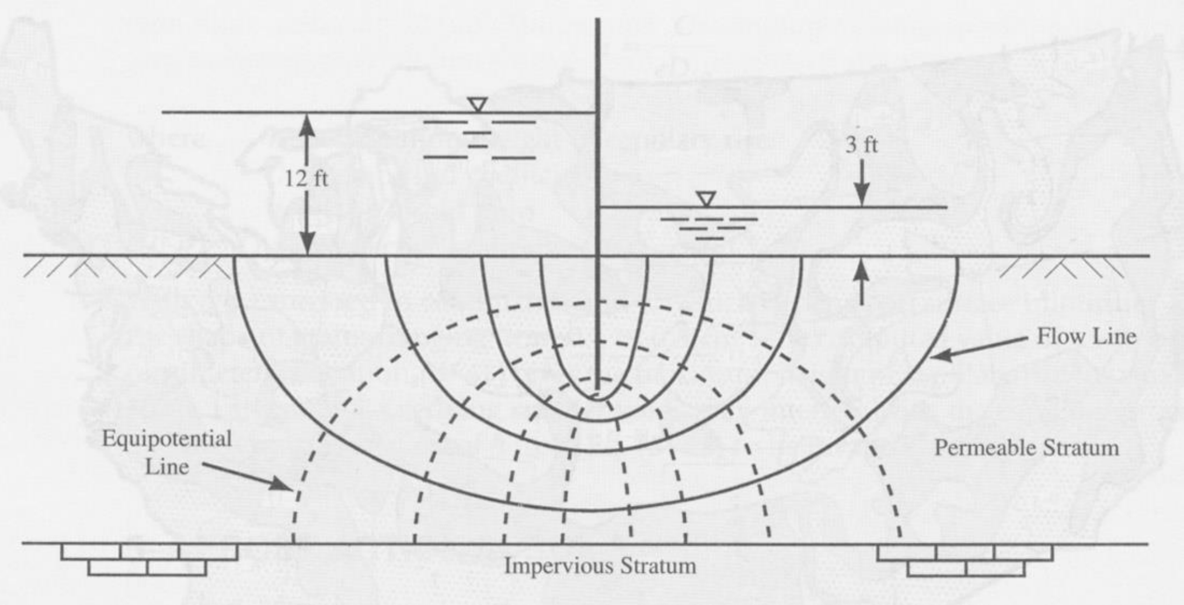
$$\Delta q = k\Delta h = k\frac{h}{N_d}$$

$$\Delta q = \frac{q}{N_f}$$

$$\frac{q}{N_f} = \frac{kh}{N_d} \quad \rightarrow \quad q = \frac{khN_f}{N_d}$$



Example



$$q = \frac{khN_f}{N_d}$$

$$k = (4.8\times10^{-3}\,cm/s)(1in/2.54cm)(1\,ft/12in) = 1.57\times10^{-4}\ ft/s$$

$$h = 12\,ft - 3\,ft = 9\,ft$$

$$N_f = 5$$

$$N_d = 9$$

$$q = \frac{(1.57\times10^{-4}\ ft/s)(9\,ft)(5)}{9} = 7.85\times10^{-4}\ ft^3/s \text{ perfoot of sheet pile}$$