chapter

5

Operational Amplifiers

He who will not reason is a bigot; he who cannot is a fool; and he who dares not is a slave.

—Lord Byron

Enhancing Your Career

Career in Electronic Instrumentation

Engineering involves applying physical principles to design devices for the benefit of humanity. But physical principles cannot be understood without measurement. In fact, physicists often say that physics is the science that measures reality. Just as measurements are a tool for understanding the physical world, instruments are tools for measurement. The operational amplifier introduced in this chapter is a building block of modern electronic instrumentation. Therefore, mastery of operational amplifier fundamentals is paramount to any practical application of electronic circuits.

Electronic instruments are used in all fields of science and engineering. They have proliferated in science and technology to the extent that it would be ridiculous to have a scientific or technical education without exposure to electronic instruments. For example, physicists, physiologists, chemists, and biologists must learn to use electronic instruments. For electrical engineering students in particular, the skill in operating digital and analog electronic instruments is crucial. Such instruments include ammeters, voltmeters, ohmmeters, oscilloscopes, spectrum analyzers, and signal generators.

Beyond developing the skill for operating the instruments, some electrical engineers specialize in designing and constructing electronic instruments. These engineers derive pleasure in building their own instruments. Most of them invent and patent their inventions. Specialists in electronic instruments find employment in medical schools, hospitals, research laboratories, aircraft industries, and thousands of other industries where electronic instruments are routinely used.

Electronic Instrumentation used in medical research.

5.1 Introduction

The term operational amplifier was introduced in 1947 by John Ragazzini and his colleagues, in their work on analog computers for the National Defense Research Council after World War II. The first op amps used vacuum tubes rather than transistors.

An op amp may also be regarded as a voltage amplifier with very high gain.

Having learned the basic laws and theorems for circuit analysis, we are now ready to study an active circuit element of paramount importance: the *operational amplifier*, or *op amp* for short. The op amp is a versatile circuit building block.

> The **op amp** is an electronic unit that behaves like a voltage-controlled voltage source.

It can also be used in making a voltage- or current-controlled current source. An op amp can sum signals, amplify a signal, integrate it, or differentiate it. The ability of the op amp to perform these mathematical operations is the reason it is called an *operational amplifier*. It is also the reason for the widespread use of op amps in analog design. Op amps are popular in practical circuit designs because they are versatile, inexpensive, easy to use, and fun to work with.

We begin by discussing the ideal op amp and later consider the nonideal op amp. Using nodal analysis as a tool, we consider ideal op amp circuits such as the inverter, voltage follower, summer, and difference amplifier. We will also analyze op amp circuits with *PSpice*. Finally, we learn how an op amp is used in digital-to-analog converters and instrumentation amplifiers.

5.2 Operational Amplifiers

An operational amplifier is designed so that it performs some mathematical operations when external components, such as resistors and capacitors, are connected to its terminals. Thus,

> An **op amp** is an active circuit element designed to perform mathematical operations of addition, subtraction, multiplication, division, differentiation, and integration.

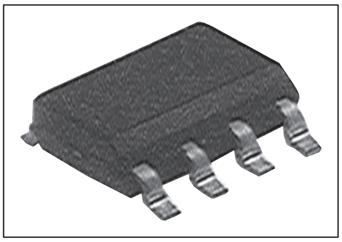

Figure 5.1
A typical operational amplifier.
Courtesy of Tech America.

The pin diagram in Fig. 5.2(a) corresponds to the 741 general-purpose op amp made by Fairchild Semiconductor.

The op amp is an electronic device consisting of a complex arrangement of resistors, transistors, capacitors, and diodes. A full discussion of what is inside the op amp is beyond the scope of this book. It will suffice to treat the op amp as a circuit building block and simply study what takes place at its terminals.

Op amps are commercially available in integrated circuit packages in several forms. Figure 5.1 shows a typical op amp package. A typical one is the eight-pin dual in-line package (or DIP), shown in Fig. 5.2(a). Pin or terminal 8 is unused, and terminals 1 and 5 are of little concern to us. The five important terminals are:

1. The inverting input, pin 2.
2. The noninverting input, pin 3.
3. The output, pin 6.
4. The positive power supply V^+, pin 7.
5. The negative power supply V^-, pin 4.

The circuit symbol for the op amp is the triangle in Fig. 5.2(b); as shown, the op amp has two inputs and one output. The inputs are

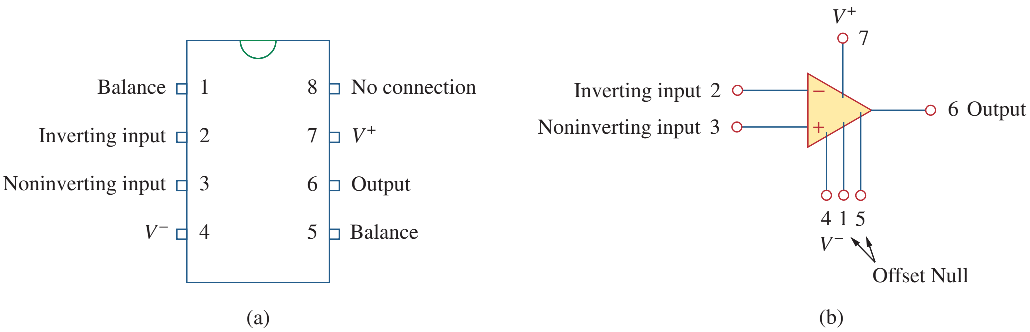

Figure 5.2
A typical op amp: (a) pin configuration, (b) circuit symbol.

marked with minus (−) and plus (+) to specify *inverting* and *noninverting* inputs, respectively. An input applied to the noninverting terminal will appear with the same polarity at the output, while an input applied to the inverting terminal will appear inverted at the output.

As an active element, the op amp must be powered by a voltage supply as typically shown in Fig. 5.3. Although the power supplies are often ignored in op amp circuit diagrams for the sake of simplicity, the power supply currents must not be overlooked. By KCL,

$$i_o = i_1 + i_2 + i_+ + i_- \tag{5.1}$$

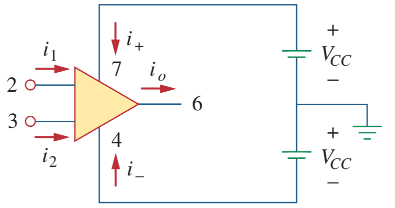

Figure 5.3
Powering the op amp.

The equivalent circuit model of an op amp is shown in Fig. 5.4. The output section consists of a voltage-controlled source in series with the output resistance R_o. It is evident from Fig. 5.4 that the input resistance R_i is the Thevenin equivalent resistance seen at the input terminals, while the output resistance R_o is the Thevenin equivalent resistance seen at the output. The differential input voltage v_d is given by

$$v_d = v_2 - v_1 \tag{5.2}$$

where v_1 is the voltage between the inverting terminal and ground and v_2 is the voltage between the noninverting terminal and ground. The op amp senses the difference between the two inputs, multiplies it by the gain A, and causes the resulting voltage to appear at the output. Thus, the output v_o is given by

$$\boxed{v_o = Av_d = A(v_2 - v_1)} \tag{5.3}$$

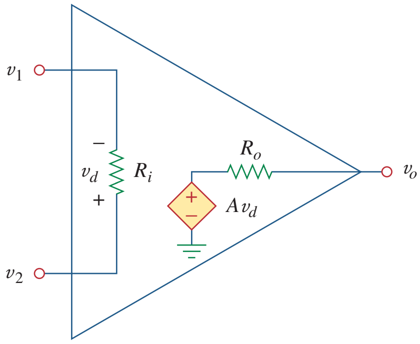

Figure 5.4
The equivalent circuit of the nonideal op amp.

A is called the *open-loop voltage gain* because it is the gain of the op amp without any external feedback from output to input. Table 5.1

Sometimes, voltage gain is expressed in decibels (dB), as discussed in Chapter 14.

$$A \text{ dB} = 20 \log_{10} A$$

TABLE 5.1

Typical ranges for op amp parameters.

Parameter	Typical range	Ideal values
Open-loop gain, A	10^5 to 10^8	∞
Input resistance, R_i	10^5 to 10^{13} Ω	∞ Ω
Output resistance, R_o	10 to 100 Ω	0 Ω
Supply voltage, V_{CC}	5 to 24 V	

shows typical values of voltage gain A, input resistance R_i, output resistance R_o, and supply voltage V_{CC}.

The concept of feedback is crucial to our understanding of op amp circuits. A negative feedback is achieved when the output is fed back to the inverting terminal of the op amp. As Example 5.1 shows, when there is a feedback path from output to input, the ratio of the output voltage to the input voltage is called the *closed-loop gain*. As a result of the negative feedback, it can be shown that the closed-loop gain is almost insensitive to the open-loop gain A of the op amp. For this reason, op amps are used in circuits with feedback paths.

A practical limitation of the op amp is that the magnitude of its output voltage cannot exceed $|V_{CC}|$. In other words, the output voltage is dependent on and is limited by the power supply voltage. Figure 5.5 illustrates that the op amp can operate in three modes, depending on the differential input voltage v_d:

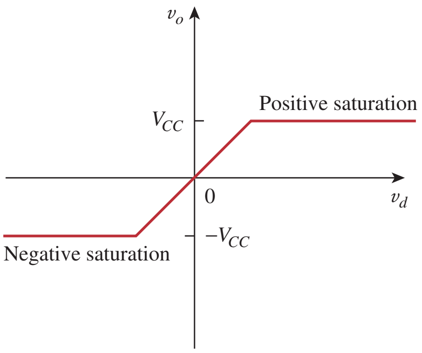

Figure 5.5
Op amp output voltage v_o as a function of the differential input voltage v_d.

1. Positive saturation, $v_o = V_{CC}$.
2. Linear region, $-V_{CC} \le v_o = Av_d \le V_{CC}$.
3. Negative saturation, $v_o = -V_{CC}$.

If we attempt to increase v_d beyond the linear range, the op amp becomes saturated and yields $v_o = V_{CC}$ or $v_o = -V_{CC}$. Throughout this book, we will assume that our op amps operate in the linear mode. This means that the output voltage is restricted by

$$-V_{CC} \le v_o \le V_{CC} \tag{5.4}$$

Throughout this book, we assume that an op amp operates in the linear range. Keep in mind the voltage constraint on the op amp in this mode.

Although we shall always operate the op amp in the linear region, the possibility of saturation must be borne in mind when one designs with op amps, to avoid designing op amp circuits that will not work in the laboratory.

Example 5.1

A 741 op amp has an open-loop voltage gain of 2×10^5, input resistance of 2 MΩ, and output resistance of 50 Ω. The op amp is used in the circuit of Fig. 5.6(a). Find the closed-loop gain v_o/v_s. Determine current i when $v_s = 2$ V.

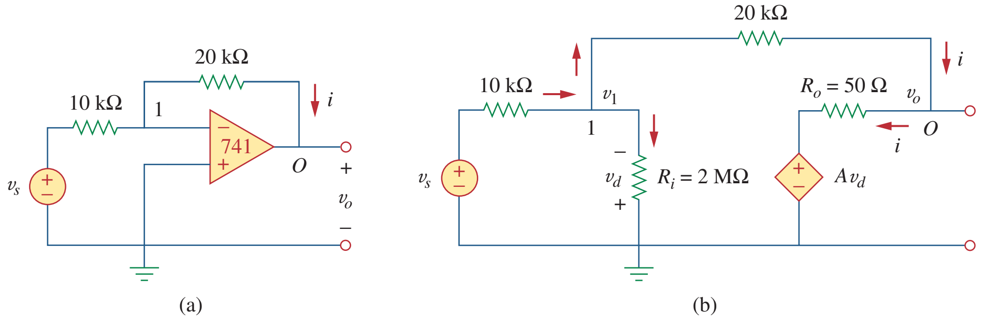

Figure 5.6
For Example 5.1: (a) original circuit, (b) the equivalent circuit.

Solution:

Using the op amp model in Fig. 5.4, we obtain the equivalent circuit of Fig. 5.6(a) as shown in Fig. 5.6(b). We now solve the circuit in Fig. 5.6(b) by using nodal analysis. At node 1, KCL gives

$$\frac{v_s - v_1}{10 \times 10^3} = \frac{v_1}{2000 \times 10^3} + \frac{v_1 - v_o}{20 \times 10^3}$$

Multiplying through by 2000×10^3, we obtain

$$200v_s = 301v_1 - 100v_o$$

or

$$2v_s \simeq 3v_1 - v_o \qquad \Rightarrow \qquad v_1 = \frac{2v_s + v_o}{3} \tag{5.1.1}$$

At node O,

$$\frac{v_1 - v_o}{20 \times 10^3} = \frac{v_o - Av_d}{50}$$

But $v_d = -v_1$ and $A = 200{,}000$. Then

$$v_1 - v_o = 400(v_o + 200{,}000v_1) \tag{5.1.2}$$

Substituting v_1 from Eq. (5.1.1) into Eq. (5.1.2) gives

$$0 \simeq 26{,}667{,}067v_o + 53{,}333{,}333v_s \qquad \Rightarrow \qquad \frac{v_o}{v_s} = -1.9999699$$

This is closed-loop gain, because the 20-kΩ feedback resistor closes the loop between the output and input terminals. When $v_s = 2$ V, $v_o = -3.9999398$ V. From Eq. (5.1.1), we obtain $v_1 = 20.066667\ \mu$V. Thus,

$$i = \frac{v_1 - v_o}{20 \times 10^3} = 0.19999 \text{ mA}$$

It is evident that working with a nonideal op amp is tedious, as we are dealing with very large numbers.

Practice Problem 5.1

If the same 741 op amp in Example 5.1 is used in the circuit of Fig. 5.7, calculate the closed-loop gain v_o/v_s. Find i_o when $v_s = 1$ V.

Answer: 9.00041, 657 μA.

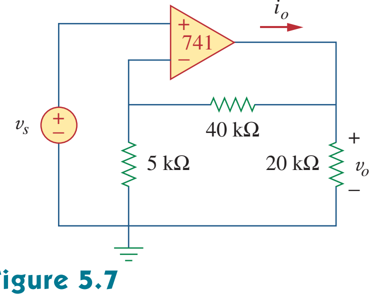

Figure 5.7
For Practice Prob. 5.1.

5.3 Ideal Op Amp

To facilitate the understanding of op amp circuits, we will assume ideal op amps. An op amp is ideal if it has the following characteristics:

1. Infinite open-loop gain, $A \simeq \infty$.
2. Infinite input resistance, $R_i \simeq \infty$.
3. Zero output resistance, $R_o \simeq 0$.

An **ideal op amp** is an amplifier with infinite open-loop gain, infinite input resistance, and zero output resistance.

Although assuming an ideal op amp provides only an approximate analysis, most modern amplifiers have such large gains and input impedances that the approximate analysis is a good one. Unless stated otherwise, we will assume from now on that every op amp is ideal.

For circuit analysis, the ideal op amp is illustrated in Fig. 5.8, which is derived from the nonideal model in Fig. 5.4. Two important characteristics of the ideal op amp are:

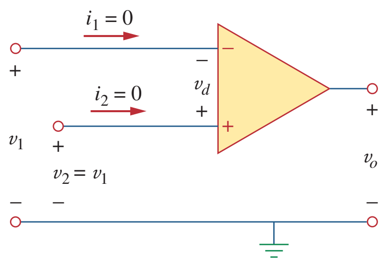

Figure 5.8
Ideal op amp model.

1. The currents into both input terminals are zero:

$$\boxed{i_1 = 0, \qquad i_2 = 0} \tag{5.5}$$

This is due to infinite input resistance. An infinite resistance between the input terminals implies that an open circuit exists there and current cannot enter the op amp. But the output current is not necessarily zero according to Eq. (5.1).

2. The voltage across the input terminals is equal to zero; i.e.,

$$v_d = v_2 - v_1 = 0 \tag{5.6}$$

or

$$\boxed{v_1 = v_2} \tag{5.7}$$

Thus, an ideal op amp has zero current into its two input terminals and the voltage between the two input terminals is equal to zero. Equations (5.5) and (5.7) are extremely important and should be regarded as the key handles to analyzing op amp circuits.

The two characteristics can be exploited by noting that for voltage calculations the input port behaves as a short circuit, while for current calculations the input port behaves as an open circuit.

Example 5.2

Rework Practice Prob. 5.1 using the ideal op amp model.

Solution:

We may replace the op amp in Fig. 5.7 by its equivalent model in Fig. 5.9 as we did in Example 5.1. But we do not really need to do this. We just need to keep Eqs. (5.5) and (5.7) in mind as we analyze the circuit in Fig. 5.7. Thus, the Fig. 5.7 circuit is presented as in Fig. 5.9. Notice that

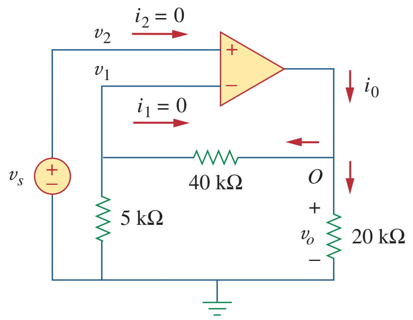

Figure 5.9
For Example 5.2.

$$v_2 = v_s \tag{5.2.1}$$

Since $i_1 = 0$, the 40-kΩ and 5-kΩ resistors are in series; the same current flows through them. v_1 is the voltage across the 5-kΩ resistor. Hence, using the voltage division principle,

$$v_1 = \frac{5}{5 + 40} v_o = \frac{v_o}{9} \tag{5.2.2}$$

According to Eq. (5.7),

$$v_2 = v_1 \tag{5.2.3}$$

Substituting Eqs. (5.2.1) and (5.2.2) into Eq. (5.2.3) yields the closed-loop gain,

$$v_s = \frac{v_o}{9} \quad \Rightarrow \quad \frac{v_o}{v_s} = 9 \tag{5.2.4}$$

which is very close to the value of 9.00041 obtained with the nonideal model in Practice Prob. 5.1. This shows that negligibly small error results from assuming ideal op amp characteristics.

At node O,

$$i_o = \frac{v_o}{40 + 5} + \frac{v_o}{20} \text{mA} \tag{5.2.5}$$

From Eq. (5.2.4), when $v_s = 1$ V, $v_o = 9$ V. Substituting for $v_o = 9$ V in Eq. (5.2.5) produces

$$i_o = 0.2 + 0.45 = 0.65 \text{ mA}$$

This, again, is close to the value of 0.657 mA obtained in Practice Prob. 5.1 with the nonideal model.

Practice Problem 5.2

Repeat Example 5.1 using the ideal op amp model.

Answer: -2, 200 μA.

5.4 Inverting Amplifier

In this and the following sections, we consider some useful op amp circuits that often serve as modules for designing more complex circuits. The first of such op amp circuits is the inverting amplifier shown in Fig. 5.10. In this circuit, the noninverting input is grounded, v_i is connected to the inverting input through R_1, and the feedback resistor R_f is connected between the inverting input and output. Our goal is to obtain the relationship between the input voltage v_i and the output voltage v_o. Applying KCL at node 1,

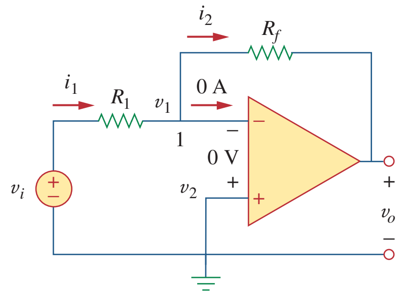

Figure 5.10
The inverting amplifier.

A key feature of the inverting amplifier is that both the input signal and the feedback are applied at the inverting terminal of the op amp.

$$i_1 = i_2 \quad \Rightarrow \quad \frac{v_i - v_1}{R_1} = \frac{v_1 - v_o}{R_f} \tag{5.8}$$

But $v_1 = v_2 = 0$ for an ideal op amp, since the noninverting terminal is grounded. Hence,

$$\frac{v_i}{R_1} = -\frac{v_o}{R_f}$$

or

$$\boxed{v_o = -\frac{R_f}{R_1} v_i} \tag{5.9}$$

Note there are two types of gains: The one here is the closed-loop voltage gain A_v, while the op amp itself has an open-loop voltage gain A.

The voltage gain is $A_v = v_o/v_i = -R_f/R_1$. The designation of the circuit in Fig. 5.10 as an *inverter* arises from the negative sign. Thus,

> An inverting amplifier reverses the polarity of the input signal while amplifying it.

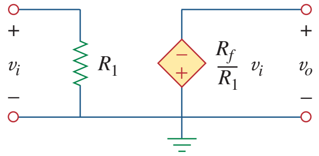

Figure 5.11
An equivalent circuit for the inverter in Fig. 5.10.

Notice that the gain is the feedback resistance divided by the input resistance which means that the gain depends only on the external elements connected to the op amp. In view of Eq. (5.9), an equivalent circuit for the inverting amplifier is shown in Fig. 5.11. The inverting amplifier is used, for example, in a current-to-voltage converter.

Example 5.3

Refer to the op amp in Fig. 5.12. If $v_i = 0.5$ V, calculate: (a) the output voltage v_o, and (b) the current in the 10-kΩ resistor.

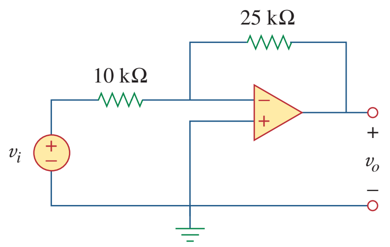

Figure 5.12
For Example 5.3.

Solution:
(a) Using Eq. (5.9),

$$\frac{v_o}{v_i} = -\frac{R_f}{R_1} = -\frac{25}{10} = -2.5$$

$$v_o = -2.5v_i = -2.5(0.5) = -1.25 \text{ V}$$

(b) The current through the 10-kΩ resistor is

$$i = \frac{v_i - 0}{R_1} = \frac{0.5 - 0}{10 \times 10^3} = 50\ \mu\text{A}$$

Practice Problem 5.3

Find the output of the op amp circuit shown in Fig. 5.13. Calculate the current through the feedback resistor.

Answer: -3.15 V, 26.25 μA.

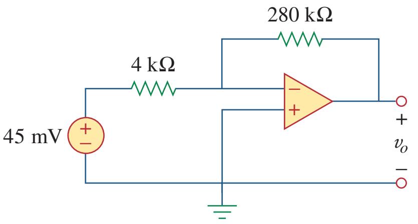

Figure 5.13
For Practice Prob. 5.3.

Example 5.4

Determine v_o in the op amp circuit shown in Fig. 5.14.

Solution:
Applying KCL at node a,

$$\frac{v_a - v_o}{40\text{ k}\Omega} = \frac{6 - v_a}{20\text{ k}\Omega}$$

$$v_a - v_o = 12 - 2v_a \quad \Rightarrow \quad v_o = 3v_a - 12$$

But $v_a = v_b = 2$ V for an ideal op amp, because of the zero voltage drop across the input terminals of the op amp. Hence,

$$v_o = 6 - 12 = -6\text{ V}$$

Notice that if $v_b = 0 = v_a$, then $v_o = -12$, as expected from Eq. (5.9).

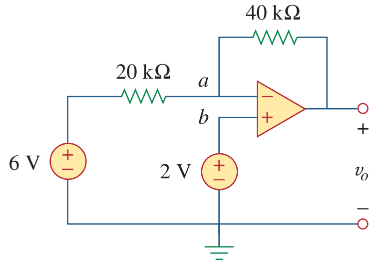

Figure 5.14
For Example 5.4.

Practice Problem 5.4

Two kinds of current-to-voltage converters (also known as *transresistance amplifiers*) are shown in Fig. 5.15.

(a) Show that for the converter in Fig. 5.15(a),

$$\frac{v_o}{i_s} = -R$$

(b) Show that for the converter in Fig. 5.15(b),

$$\frac{v_o}{i_s} = -R_1\left(1 + \frac{R_3}{R_1} + \frac{R_3}{R_2}\right)$$

Answer: Proof.

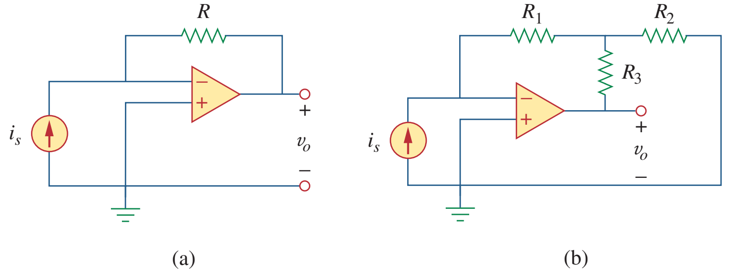

Figure 5.15
For Practice Prob. 5.4.

5.5 Noninverting Amplifier

Another important application of the op amp is the noninverting amplifier shown in Fig. 5.16. In this case, the input voltage v_i is applied directly at the noninverting input terminal, and resistor R_1 is connected

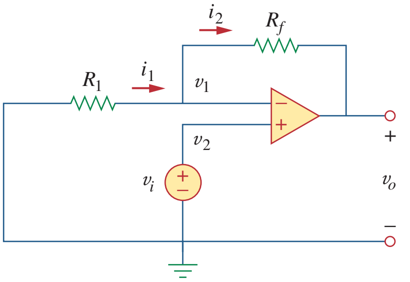

Figure 5.16
The noninverting amplifier.

between the ground and the inverting terminal. We are interested in the output voltage and the voltage gain. Application of KCL at the inverting terminal gives

$$i_1 = i_2 \quad \Rightarrow \quad \frac{0 - v_1}{R_1} = \frac{v_1 - v_o}{R_f} \tag{5.10}$$

But $v_1 = v_2 = v_i$. Equation (5.10) becomes

$$\frac{-v_i}{R_1} = \frac{v_i - v_o}{R_f}$$

or

$$\boxed{v_o = \left(1 + \frac{R_f}{R_1}\right)v_i} \tag{5.11}$$

The voltage gain is $A_v = v_o/v_i = 1 + R_f/R_1$, which does not have a negative sign. Thus, the output has the same polarity as the input.

A noninverting amplifier is an op amp circuit designed to provide a positive voltage gain.

Again we notice that the gain depends only on the external resistors.

Notice that if feedback resistor $R_f = 0$ (short circuit) or $R_1 = \infty$ (open circuit) or both, the gain becomes 1. Under these conditions ($R_f = 0$ and $R_1 = \infty$), the circuit in Fig. 5.16 becomes that shown in Fig. 5.17, which is called a *voltage follower* (or *unity gain amplifier*) because the output follows the input. Thus, for a voltage follower

$$\boxed{v_o = v_i} \tag{5.12}$$

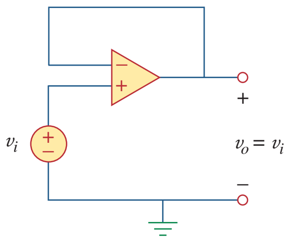

Figure 5.17
The voltage follower.

Such a circuit has a very high input impedance and is therefore useful as an intermediate-stage (or buffer) amplifier to isolate one circuit from another, as portrayed in Fig. 5.18. The voltage follower minimizes interaction between the two stages and eliminates interstage loading.

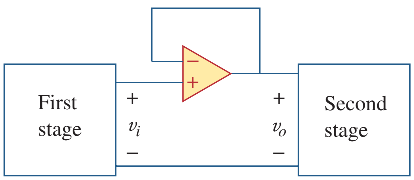

Figure 5.18
A voltage follower used to isolate two cascaded stages of a circuit.

Example 5.5

For the op amp circuit in Fig. 5.19, calculate the output voltage v_o.

Solution:
We may solve this in two ways: using superposition and using nodal analysis.

■ **METHOD 1** Using superposition, we let

$$v_o = v_{o1} + v_{o2}$$

where v_{o1} is due to the 6-V voltage source, and v_{o2} is due to the 4-V input. To get v_{o1}, we set the 4-V source equal to zero. Under this condition, the circuit becomes an inverter. Hence Eq. (5.9) gives

$$v_{o1} = -\frac{10}{4}(6) = -15 \text{ V}$$

To get v_{o2}, we set the 6-V source equal to zero. The circuit becomes a noninverting amplifier so that Eq. (5.11) applies.

$$v_{o2} = \left(1 + \frac{10}{4}\right)4 = 14 \text{ V}$$

Thus,

$$v_o = v_{o1} + v_{o2} = -15 + 14 = -1 \text{ V}$$

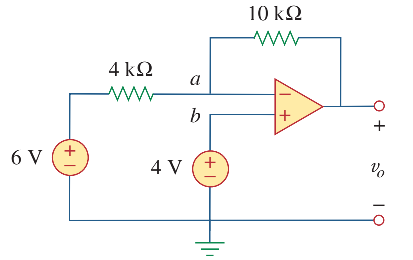

Figure 5.19
For Example 5.5.

METHOD 2 Applying KCL at node a,

$$\frac{6 - v_a}{4} = \frac{v_a - v_o}{10}$$

But $v_a = v_b = 4$, and so

$$\frac{6 - 4}{4} = \frac{4 - v_o}{10} \quad \Rightarrow \quad 5 = 4 - v_o$$

or $v_o = -1$ V, as before.

Practice Problem 5.5

Calculate v_o in the circuit of Fig. 5.20.

Answer: 7 V.

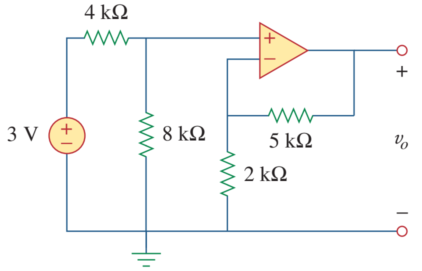

Figure 5.20
For Practice Prob. 5.5.

5.6 Summing Amplifier

Besides amplification, the op amp can perform addition and subtraction. The addition is performed by the summing amplifier covered in this section; the subtraction is performed by the difference amplifier covered in the next section.

> A summing amplifier is an op amp circuit that combines several inputs and produces an output that is the weighted sum of the inputs.

The summing amplifier, shown in Fig. 5.21, is a variation of the inverting amplifier. It takes advantage of the fact that the inverting configuration can handle many inputs at the same time. We keep in mind

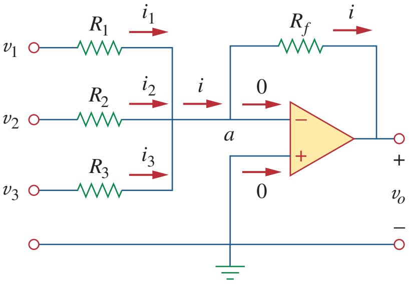

Figure 5.21
The summing amplifier.

that the current entering each op amp input is zero. Applying KCL at node a gives

$$i = i_1 + i_2 + i_3 \tag{5.13}$$

But

$$i_1 = \frac{v_1 - v_a}{R_1}, \quad i_2 = \frac{v_2 - v_a}{R_2}$$
$$i_3 = \frac{v_3 - v_a}{R_3}, \quad i = \frac{v_a - v_o}{R_f} \tag{5.14}$$

We note that $v_a = 0$ and substitute Eq. (5.14) into Eq. (5.13). We get

$$\boxed{v_o = -\left(\frac{R_f}{R_1}v_1 + \frac{R_f}{R_2}v_2 + \frac{R_f}{R_3}v_3\right)} \tag{5.15}$$

indicating that the output voltage is a weighted sum of the inputs. For this reason, the circuit in Fig. 5.21 is called a *summer*. Needless to say, the summer can have more than three inputs.

Example 5.6

Calculate v_o and i_o in the op amp circuit in Fig. 5.22.

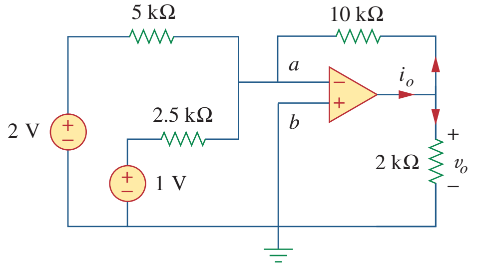

Figure 5.22
For Example 5.6.

Solution:
This is a summer with two inputs. Using Eq. (5.15) gives

$$v_o = -\left[\frac{10}{5}(2) + \frac{10}{2.5}(1)\right] = -(4 + 4) = -8 \text{ V}$$

The current i_o is the sum of the currents through the 10-kΩ and 2-kΩ resistors. Both of these resistors have voltage $v_o = -8$ V across them, since $v_a = v_b = 0$. Hence,

$$i_o = \frac{v_o - 0}{10} + \frac{v_o - 0}{2} \text{ mA} = -0.8 - 4 = -4.8 \text{ mA}$$

Practice Problem 5.6

Find v_o and i_o in the op amp circuit shown in Fig. 5.23.

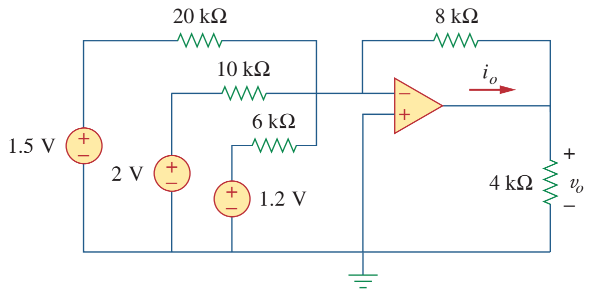

Figure 5.23
For Practice Prob. 5.6.

Answer: −3.8 V, −1.425 mA.

5.7 Difference Amplifier

Difference (or differential) amplifiers are used in various applications where there is a need to amplify the difference between two input signals. They are first cousins of the *instrumentation amplifier*, the most useful and popular amplifier, which we will discuss in Section 5.10.

The difference amplifier is also known as the *subtractor*, for reasons to be shown later.

A difference amplifier is a device that amplifies the difference between two inputs but rejects any signals common to the two inputs.

Consider the op amp circuit shown in Fig. 5.24. Keep in mind that zero currents enter the op amp terminals. Applying KCL to node a,

$$\frac{v_1 - v_a}{R_1} = \frac{v_a - v_o}{R_2}$$

or

$$v_o = \left(\frac{R_2}{R_1} + 1\right)v_a - \frac{R_2}{R_1}v_1 \qquad \textbf{(5.16)}$$

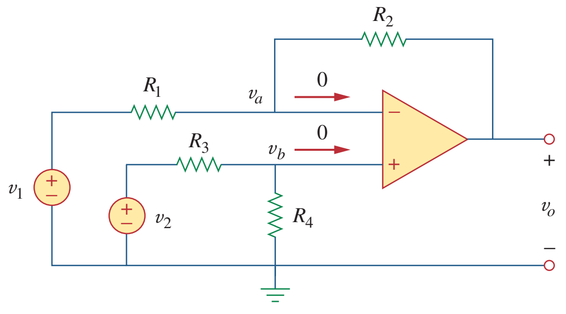

Figure 5.24
Difference amplifier.

Applying KCL to node b,

$$\frac{v_2 - v_b}{R_3} = \frac{v_b - 0}{R_4}$$

or

$$v_b = \frac{R_4}{R_3 + R_4} v_2 \tag{5.17}$$

But $v_a = v_b$. Substituting Eq. (5.17) into Eq. (5.16) yields

$$v_o = \left(\frac{R_2}{R_1} + 1\right)\frac{R_4}{R_3 + R_4} v_2 - \frac{R_2}{R_1} v_1$$

or

$$\boxed{v_o = \frac{R_2(1 + R_1/R_2)}{R_1(1 + R_3/R_4)} v_2 - \frac{R_2}{R_1} v_1} \tag{5.18}$$

Since a difference amplifier must reject a signal common to the two inputs, the amplifier must have the property that $v_o = 0$ when $v_1 = v_2$. This property exists when

$$\frac{R_1}{R_2} = \frac{R_3}{R_4} \tag{5.19}$$

Thus, when the op amp circuit is a difference amplifier, Eq. (5.18) becomes

$$v_o = \frac{R_2}{R_1}(v_2 - v_1) \tag{5.20}$$

If $R_2 = R_1$ and $R_3 = R_4$, the difference amplifier becomes a *subtractor*, with the output

$$v_o = v_2 - v_1 \tag{5.21}$$

Example 5.7

Design an op amp circuit with inputs v_1 and v_2 such that $v_o = -5v_1 + 3v_2$.

Solution:
The circuit requires that

$$v_o = 3v_2 - 5v_1 \tag{5.7.1}$$

This circuit can be realized in two ways.

Design 1 If we desire to use only one op amp, we can use the op amp circuit of Fig. 5.24. Comparing Eq. (5.7.1) with Eq. (5.18), we see

$$\frac{R_2}{R_1} = 5 \qquad \Rightarrow \qquad R_2 = 5R_1 \tag{5.7.2}$$

Also,

$$5\frac{(1 + R_1/R_2)}{(1 + R_3/R_4)} = 3 \quad \Rightarrow \quad \frac{\frac{6}{5}}{1 + R_3/R_4} = \frac{3}{5}$$

or

$$2 = 1 + \frac{R_3}{R_4} \quad \Rightarrow \quad R_3 = R_4 \tag{5.7.3}$$

If we choose $R_1 = 10\ \text{k}\Omega$ and $R_3 = 20\ \text{k}\Omega$, then $R_2 = 50\ \text{k}\Omega$ and $R_4 = 20\ \text{k}\Omega$.

Design 2 If we desire to use more than one op amp, we may cascade an inverting amplifier and a two-input inverting summer, as shown in Fig. 5.25. For the summer,

$$v_o = -v_a - 5v_1 \tag{5.7.4}$$

and for the inverter,

$$v_a = -3v_2 \tag{5.7.5}$$

Combining Eqs. (5.7.4) and (5.7.5) gives

$$v_o = 3v_2 - 5v_1$$

which is the desired result. In Fig. 5.25, we may select $R_1 = 10\ \text{k}\Omega$ and $R_3 = 20\ \text{k}\Omega$ or $R_1 = R_3 = 10\ \text{k}\Omega$.

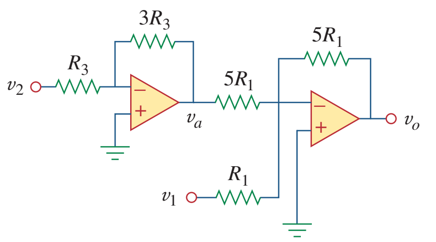

Figure 5.25
For Example 5.7.

Practice Problem 5.7

Design a difference amplifier with gain 7.5.

Answer: Typical: $R_1 = R_3 = 20\text{k}\Omega$, $R_2 = R_4 = 150\ \text{k}\Omega$.

Example 5.8

An *instrumentation amplifier* shown in Fig. 5.26 is an amplifier of low-level signals used in process control or measurement applications and commercially available in single-package units. Show that

$$v_o = \frac{R_2}{R_1}\left(1 + \frac{2R_3}{R_4}\right)(v_2 - v_1)$$

Solution:

We recognize that the amplifier A_3 in Fig. 5.26 is a difference amplifier. Thus, from Eq. (5.20),

$$v_o = \frac{R_2}{R_1}(v_{o2} - v_{o1}) \tag{5.8.1}$$

Since the op amps A_1 and A_2 draw no current, current i flows through the three resistors as though they were in series. Hence,

$$v_{o1} - v_{o2} = i(R_3 + R_4 + R_3) = i(2R_3 + R_4) \tag{5.8.2}$$

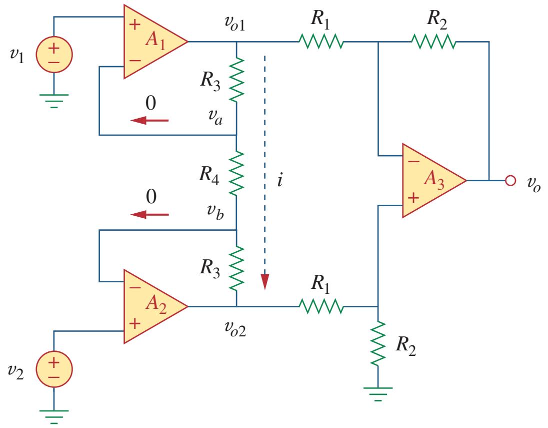

Figure 5.26
Instrumentation amplifier; for Example 5.8.

But

$$i = \frac{v_a - v_b}{R_4}$$

and $v_a = v_1$, $v_b = v_2$. Therefore,

$$i = \frac{v_1 - v_2}{R_4} \tag{5.8.3}$$

Inserting Eqs. (5.8.2) and (5.8.3) into Eq. (5.8.1) gives

$$v_o = \frac{R_2}{R_1}\left(1 + \frac{2R_3}{R_4}\right)(v_2 - v_1)$$

as required. We will discuss the instrumentation amplifier in detail in Section 5.10.

Practice Problem 5.8

Obtain i_o in the instrumentation amplifier circuit of Fig. 5.27.

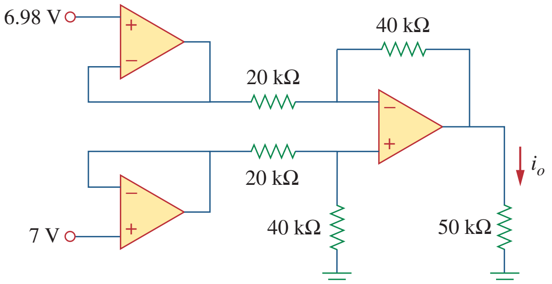

Figure 5.27
Instrumentation amplifier; for Practice Prob. 5.8.

Answer: $-800\ \mu$A.