Define operator and how the operator classified?

Operator:-

An operator is a set of mathmatical symbols that can convert one function into another function such as,

$$\frac{d}{dx} x^3 = 3x^2$$

Here, $\frac{d}{dx} x^3$ is a function and

$3x^2$ is a new function.

Classification of operator:-

These are classified into

- Linear operator
- vector operator
- Laplacian operator
- Differential operator
- Hamiltonian operator

Linear operator:-

If the result of operatorion of an operator on a function is equal to the sum of operation on an individual function Then operator is known as "**Linear operator**."

For example:-

$$\frac{d}{dx}x^3 = 3x^2 , \frac{d}{dx}x^2 = 2x . \frac{d}{dx}x = 1$$

$$\frac{d}{dx}(x^3+x^2+x) = 3x^2+2x+1$$

In this example $\frac{d}{dx}$ is a linear operator
Another example is

$$\hat{A}(g+h) = \hat{A}g + \hat{A}h$$

$$\sqrt{g+h} = \sqrt{g} + \sqrt{h}.$$

vector operator:-

vector is a differential operator.
vector operator are defined in term of del, and include the gradient

divergence and curl.

$$\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$

$$\nabla\cdot\nabla = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)\left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)$$

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \qquad \because \hat{i}\cdot\hat{i} = 1,\ \hat{j}\,\hat{j} = 1,\ \hat{k}\,\hat{k} = 1$$

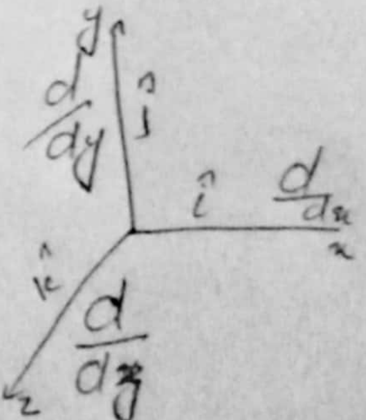
Here $\hat{i}, \hat{j}, \hat{k}$ are unit vector.

→ Laplacian operator

It is a differential operator given by the divergance of the gradient of (the) a function on Euclidean space.

It is usually denoted by $\nabla.$ or Δ

$$\nabla = \hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}$$



Here ~~x~~, y and z are axies and $\frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z}$ are differential and $\hat{i}, \hat{j}, \hat{k}$ are vector units.

Differential operator:-

It is defiend as or operator defined as a function of the differentiation operator

Hamiltonion operator

In quantum mechanics, a Hamiltonian is an operator corresponding to the sum of kinetic energies plus the potentical energies for all the particals in system It is usually denoted by H. but also $\check{H}$ or $\hat{H}$ to highlights it's function as operator

The Hamiltonian is named after William Rowan Hamilton, who created a revolutionary reformulation of Newtonian mechanics, now called Hamiltonian mechanics, which is important in quantum mechanics.

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$\hat{H}$ = kinetic energy + potential energy

$= T + V$

$= \frac{1}{2} mv^2 + V$

By multiplying and divided by m

$\hat{H} = \frac{1}{2} m.v^2 \cdot \frac{m}{m} + V$

$\hat{H} = \frac{m^2 v^2}{2m} + V$

But momentum $p = mv$

$p^2 = m^2 v^2$

$\hat{H} = P_{xyz}^2 + V$

$P_{xyz}^2 = p_x^2 + P_y^2 + P_z^2$

the momentum p_{xyz} for small bodies in quantum mechanics is defiend as

$P_x = \hbar i$

For x $P_x = \hbar i \frac{\partial}{\partial x}$

For y

$P_y = \hbar i \frac{\partial}{\partial y}$

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Similarly for z

$P_z = \hbar i \frac{\partial}{\partial z}$

$\because \hbar = \frac{h}{2\pi}$

So, $\frac{ih}{2\pi} \cdot \frac{\partial}{\partial x}$

$P_x^2 = \frac{i^2 h^2}{4\pi} \cdot \frac{\partial^2}{\partial x^2}$

$P_y^2 = \frac{ih^2}{4\pi^2} \cdot \frac{\partial^2}{\partial y^2}$

$P_z^2 = \frac{i^2 h^2}{4\pi^2} \cdot \frac{\partial^2}{\partial z^2}$

$= \frac{-1h^2}{4\pi^2} \cdot \frac{\partial^2}{\partial x^2} - \frac{h^2}{4\pi^2} \frac{\partial^2}{\partial y^2} - \frac{h^2}{4\pi^2} \frac{\partial^2}{\partial z^2}$

$= \frac{-h^2}{4\pi^2}\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right)$

$P^2_{xyz} = \frac{-h^2}{4\pi^2} \nabla^2$

put in $\hat{H} = \frac{P_{xyz}}{2m} + V$

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$$\hat{H} = \frac{-h^2}{4\pi^2} \frac{\nabla^2}{2m} + V$$

$$\hat{H} = \frac{-h^2 \nabla^2}{8m\pi^2} + V \rightarrow \text{Himiltonion operator.}$$

→ Eigen Function

The Function which we operate the result is Same operator with same numerical value. is called Eigen Function.

→ Eigen value:-

The value obtained from eigen function is called eigen value.

if

$$\frac{d}{dx}(e^{ax}) = \frac{1}{a} e^{ax} \rightarrow \text{eigen function}$$

↳ eigenvalue.

In quantum mechanics n, l, m, all these are eigen values.

→ Eigen theorem:-

In quantum mechanics if an operator "A" operate on a function then the result obtained in term of eigen value are represented as,

$$\hat{A}\psi = \lambda\psi$$

If the result of an operator is proportional to the orignal function then the function is known as eigen function and proportional constant is know as Eigen value.

Expectation Theorem:-

When some measurments are made on a function then the expectation value in terms of eigen are calculated as,

$$\hat{A}\psi = \lambda\psi$$

$$\lambda = \frac{\hat{A}\psi}{\psi}$$

$$\lambda = \frac{\int_{-\infty}^{+\infty} \hat{A}\psi\psi^*}{\int_{-\infty}^{+\infty} \psi\psi^*}$$ here ψ^* is derivative.