Differentiate between Classical and Quantum Mechanics in Detail?

Mechanics :-

It can be defined as a branch of science which deals with the motion of and forces on bodies not in quantum realm.

Mechanics is area of physics concerned with motions of macroscopic objects.

Classical Mechanics :-

It is a branch of physics which deals with motion of objects small as well as larger objects. But it would be valid if objects are :-

$V << C$

Macroscopic

Classical mechanics tells us about state and condition of objects. It has two limitations which are mentioned above

First, the velocity of object should be lower than speed of light.

Secondly, classical mechanics is valid

Scanned with CamScanner
Scanned with CamScanner

for macroscopic system. (larger Objects).
The study of rest or motion state of object is called Classical mechanics It is also known as Newton Mechanics, Hamiltonian Mechanics.

What is it good for? (Classical mechanics

Classical mechanics is helpful in describing the motion of macrosopic solids, fluids under the influences of forces.

e.g. Planets moving around the sun
apples falling on people's heads
Planes flying around.

Invention of Classical Mechanics:-

The Greeks described about it but They were wrong because most of Their ideas were irrelevant.

Later, Galileo Galilei came up with some better scientific idea in (1564-1642).
After his death, Issac Newton came up with calculus laws of motion in

Scanned with CamScanner
Scanned with CamScanner

(1642-1727) which are still in use Today. A few years Later, Joseph Louis Lagrauge (1736-1813) and William Hamiltonian (1805-1865) develop different alternative formulation of Newton's Law to solve complex problem a lot easier.

Boundries:-

Classical mechanics can not describe about :-

Objects moving close to speed of light ($C = 458 \frac{m}{s}$),

Which describes Newton's special relativity

It can not also explain about tiny objects (with noticeable de-Broglie- wavelength- ($\lambda = \frac{h}{p}$).

Which describe Quantaum mechanics.

Classical Mechanics is not 100% accurate and it is not a complete Theory. But it's still useful in its boundries.

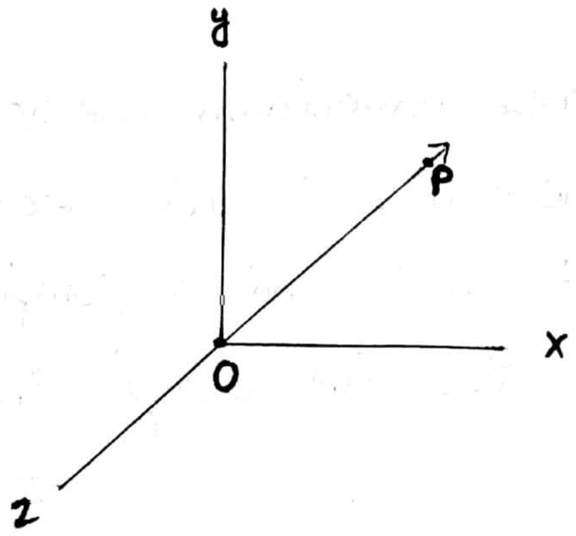
Scanned with CamScanner

Scanned with CamScanner

Some Laws of Classical Mechanics doesn not need any modifications. e.g.
Law of conservation of energy.
Law of Conservation of mass.
Law of conservation of Linear momentum
They are universally accepted.

Frame of Reference :-

The location of any particle or event according to time can be described by three coordinate axis, at a fixed origin. These coordinate axis are known as frame of references. or Cartisian Coordinates.



These axis are mutually perpendicular.

Scanned with CamScanner
Scanned with CamScanner

Frame of reference has Two Types :-

Inertial frame of reference :-

$$a = 0$$

Newton's Law are only valid for inertial frame of reference. Earth is an idealized inertial frame of reference.

Exchange of coordinates does not effect the result, and it is called Newtonian Relativity.

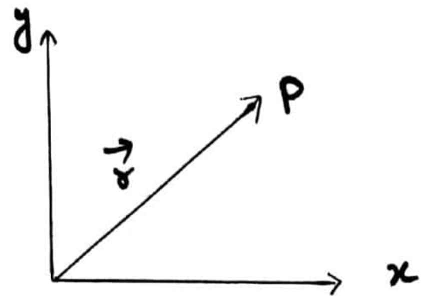
Non- Inertial frame of Reference :-

$$a \neq 0$$

accelaration is not zero in this frame of reference.

Scanned with CamScanner
Scanned with CamScanner

Newtonian Mechanics of a Single Particle



as we know :-

$$\vec{v} = \frac{d\vec{r}}{dt} \rightarrow 1$$

and

$$\vec{a} = \frac{d}{dt}\vec{v}$$

put value of $\vec{v}$

$$= \frac{d}{dt} \cdot \frac{d\vec{r}}{dt}$$

$$\vec{a} = \frac{d^2\vec{r}}{dt^2} \rightarrow 2$$

Newton' Second Law :- (general form)

$$F = \frac{dp}{dt} \rightarrow 3$$

as momentum :-

$$P = mv \rightarrow 4$$

Put value of P :-

$$F = \frac{d\,mv}{dt}$$

Scanned with CamScanner
Scanned with CamScanner

In This frame of reference, Particle is an point mass with a fixed origin. (Inertial frame of reference).

So m is constant.

$$F = m\frac{dv}{dt} \quad \because a = \frac{dv}{dt}$$

So,

$$F = ma \quad (\text{Newton's } 2^{nd} \text{ Law}) \rightarrow \underline{5}$$

Put value of eq (2)

$$F = m\frac{d^2 r}{dt^2} \rightarrow \underline{6}$$

LAW of Conservation of Linear Momentum

As

$$F = \frac{dp}{dt}$$

If system has zero net force.

Then,

$$\frac{dp}{dt} = F_{net}$$

Let's suppose :-

$$F_{net} = 0$$

$$\frac{dp}{dt} = 0$$

Scanned with CamScanner
Scanned with CamScanner

Integrate :-

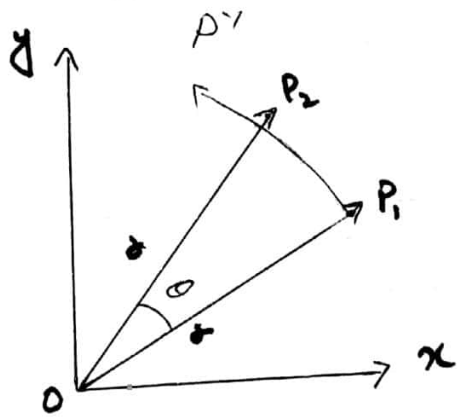
$$\int \frac{dp}{dt} = \int 0$$

So,

$$P = \text{constant}$$

if net force is zero, then Linear momentum will be constant.

LAW of Conservation of Angular Momentum



Moment of Linear Momentum with fixed origin is known as Angular Momentum.

$$L = r \times P \rightarrow 7$$

Taking derivative both sides with respect to time:-

$$\frac{d}{dt} L = \frac{d}{dt} (r \times P)$$

Scanned with CamScanner
Scanned with CamScanner

Sum Theorm :-

$$\frac{d}{dt} L = \frac{dr}{dt} \times P + r \times \frac{dP}{dt}$$

as we know :- $P = mv$, $\frac{dr}{dt} = V$

So, and $\frac{dP}{dt} = F$

$$= V \times mV + r \times F$$

According To vector Calculus rule, Cross product of Two same vector is zero. So,

$$V \times V = 0 \rightarrow V \times mV = 0$$

$$= 0 + r \times F$$

$$r \times F = J$$

$$\cancel{dt} \quad \frac{dL}{dt} = 0 + J$$

$$\frac{dL}{dt} = J \rightarrow \underline{8}$$

And

$$\frac{dP}{dt} = F$$

F produce Linear motion and J produce angular motion.

if $J = 0$

Scanned with CamScanner
Scanned with CamScanner

$$\frac{dL}{dt} = 0$$

Integrate :-

$$\int \frac{dL}{dt} = \int 0$$

$$L = \text{Constant}$$

LAW of Conservation of Energy

Work Energy Principle

$$W_{12} = \int_1^2 F \cdot ds \rightarrow \underline{9}$$

As we know :-

$$F = ma$$

or $$F = m\frac{dv}{dt} \rightarrow (a)$$

And

$$S = vt$$

or $$ds = vdt \rightarrow (b)$$

put value of a , b in eq (9).

$$W_{12} = \int_1^2 m\frac{dv}{dt} \cdot vdt$$

Simplify :-

Scanned with CamScanner

Scanned with CamScanner

$$W_{12} = m\int_1^2 dv\ v$$

$$= m\int_1^2 v \cdot dv$$

$$= m\int_1^2 v\,dv$$

Integerate:-

$$W_{1.2} = \frac{m}{2} v^2 \Big|_1^2$$

$$= \frac{m}{2}\left[v_2^2 - v_1^2\right]$$

$$= \frac{1}{2} m v_2^2 - \frac{1}{2} m v_1^2$$

$$W_{12} = T_2 - T_1 \longrightarrow \underline{10}$$

Work on P.E:-

$$\Delta U = -W_{12}$$

or $$U_2 - U_1 = -W_{12}$$

Simplify:-

$$W_{12} = U_1 - U_2 \longrightarrow 11$$

Compare equ $\underline{11}$ and $\underline{10}$:-

$$T_2 - T_1 = U_1 - U_2$$

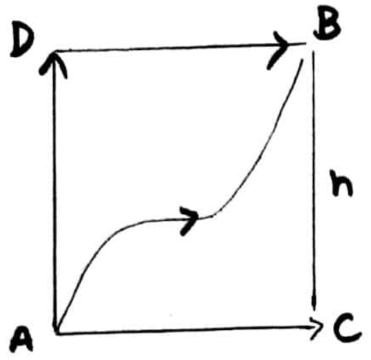
$$T_2 + U_2 = U_1 + T_1$$

Rearrange :-

Scanned with CamScanner
Scanned with CamScanner

$$T_1 + U_1 = T_2 + U_2$$

Conservative force :-



If work done by a force is independent of path, so force is known as conservative force.

e.g gravitational force.

$$\int F \cdot ds = 0$$

QUANTUM Mechanics

Definition:-

The branch of mechanics that deals with mathematical description of motion and interaction of subatomic particles.

It is incorporating the concepts of quantization of energy,

Scanned with CamScanner
Scanned with CamScanner

wave-particle duality, The uncertainty principle, and correspondence principle

Postulates of Quantum Mechanics:-

Postulate 1 :-

The state of a System is completely described by Hermitian Linear operators.

Postulate 2 :-

All measurable quantities are described by a wave function $\psi(r,t)$.

Postulat 3 :-

The only values That are obtained in a measurement of an observable "A" are eigen values "a_n" of corresponding operator "A". The measurement Changes The state of system To eigen function of "A" with eigen value "a_n".

Scanned with CamScanner
Scanned with CamScanner

Postulate 4 :-

If a System is described by a normalized wavefunction ψ. Then The average value of observable corresponding To $\hat{A}$ is.

$$\langle a \rangle = \int \psi^* \hat{A} \psi d\tau$$

Implications and elaborations on Postulates :-

Postulate 1 :-

The physically relevant quantity is $|\psi|^2$

$$\psi^*(r,t)\psi(r,t) = |\psi(r,t)|^2$$

= Probability density at Time t and Position r

$\psi(r,t)$ must be normalized

$$\int \psi^* \psi d\tau = 1$$

$\psi(r,t)$ must be well behaved

Scanned with CamScanner
Scanned with CamScanner

Single valued
ψ and ψ' continuous
Finite.

Postulate 2 :-

Example :- Particle in a box
eigenfunctions of $\hat{H}$

$$\hat{H}(x)\psi_n(x) = E_n \psi_n(x)$$
$$\psi_n(x) = \left[\frac{2}{a}\right]^{1/2} \sin\left[\frac{n\pi x}{a}\right]$$

But if ψ is not an eigenfunction of operator, Then statement is not True.
e.g. $\psi_n(x)$ above with momentum operator.

$$\hat{P}_n\psi_n(x) = -i\hbar\frac{d}{dx}\psi_n(x) = -i\hbar\frac{d}{dx}\left[\left[\frac{2}{a}\right]^{\frac{1}{2}}\sin\left(\frac{n\pi x}{a}\right)\right]$$
$$\neq P_n\left[\left[\frac{2}{a}\right]^{\frac{1}{2}}\sin\left[\frac{n\pi x}{a}\right]\right]$$

In Order To Create a Q.M. operator from a Classical observable, use $\hat{X} = x$ and $\hat{P}_x = -i\hbar\frac{d}{dx}$ and replace in Classical expression.

e.g.

Scanned with CamScanner
Scanned with CamScanner

$$K.E. = \frac{1}{2m}\hat{p}^2 = \frac{1}{2m}(\hat{p})(\hat{p}) = \frac{h^2}{2m}\frac{d^2}{dx^2} \quad (1D)$$

$$= \frac{h^2}{2m}\left[\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}\right] \quad (3D)$$

Postulate 3:-

If Ψ is an eigen function of operator, then it's easy

$\hat{H}\Psi_n = E_n \Psi_n$ measurement of energy yield

Postulate :- 4

This connects to expectation value.

if Ψ_n is an eigen function of $\hat{A}$, then

$\hat{A}\Psi_n = a_n \Psi_n$

$$\langle a \rangle = \int \Psi_n^* \hat{A} \Psi_n d\tau = a_n \int \Psi_n^* \Psi_n d\tau = a_n$$

$\langle a \rangle = a_n$ only value possible

If $\Psi = C_1\theta_1 + C_2\theta_2$ as above

$$\langle a \rangle = \int \Psi^* \hat{A} \Psi d\tau = \int (C_1\theta_1 + C_2\theta_2)^* \hat{A}(C_1\theta_1 + C_2\theta_2) d\tau = C_1^2 a_1 + C_2^2 a_2$$

C_1^2 is probability of measuring a_1 -

Scanned with CamScanner
Scanned with CamScanner

Difference between Classical And Quantum Mechanics

Classical	Quantum
Events are Continuous. They move in Smooth, orderly and predictable patterns	1 It is not so accurate in Quantum Mechanics. e.g H-atom
Macroscopic Level	2 Microscopic Level.
Energy levels are Continuous.	Discrete Energy level.
HUP (X) is absent.	3 HUP is present.
It is based upon Newton's Law of motion.	It takes into account 4 Heisenberg's uncertainty principle and de Broglie concept of dual nature of matter (particle and wave nature).
It is based on Maxwell's electromagnetic wave Theory according To which any amount of energy may be emitted or obserbed continuously	5 It is based on Planck's Quantum

Scanned with CamScanner
Scanned with CamScanner

Classical	Quantum
The state of a system is defined by specifying all forces acting on particles as well as Their position and velocity. The future state Then can be predicted with certainty.	Theory according to which only discrete values of energy are emitted or absorbed 6 It gives probabilities of finding particles at various location in space.
It is an oldest and fundamental Mechanics	7 It is an modern Concept of Mechanics.
$E = \frac{1}{2}mv^2$	8 $E = hF$
Momentum $= mv$	9 Momentum $= h/\lambda$
Orbits.	10 Orbitas
	 a standing wave.

Scanned with CamScanner
Scanned with CamScanner