Activity 2

$$p \rightarrow (q \rightarrow r) \vdash p \wedge q \rightarrow r$$

1		$p \rightarrow (q \rightarrow r)$	premise
2		$p \wedge q$	assumption
3		p	$\wedge e_1$ 2
4		q	$\wedge e_2$ 2
5		$q \rightarrow r$	$\rightarrow e$ 1, 3
6		r	$\rightarrow e$ 5, 4
7		$p \wedge q \rightarrow r$	$\rightarrow i$ 2–6

Activity

$$p \to q \vdash p \wedge r \to q \wedge r$$

1		$p \to q$	premise
2		$p \wedge r$	assumption
3		p	$\wedge e_1$ 2
4		r	$\wedge e_2$ 2
5		q	$\to e$ 1, 3
6		$q \wedge r$	$\wedge i$ 5, 4
7		$p \wedge r \to q \wedge r$	$\to i$ 2–6

Equivalent Formulas

$$p \wedge q \rightarrow r \vdash p \rightarrow (q \rightarrow r)$$

$$p \rightarrow (q \rightarrow r) \vdash p \wedge q \rightarrow r$$

- The two formulas are equivalent to one another

$$p \wedge q \rightarrow r \dashv\vdash p \rightarrow (q \rightarrow r)$$

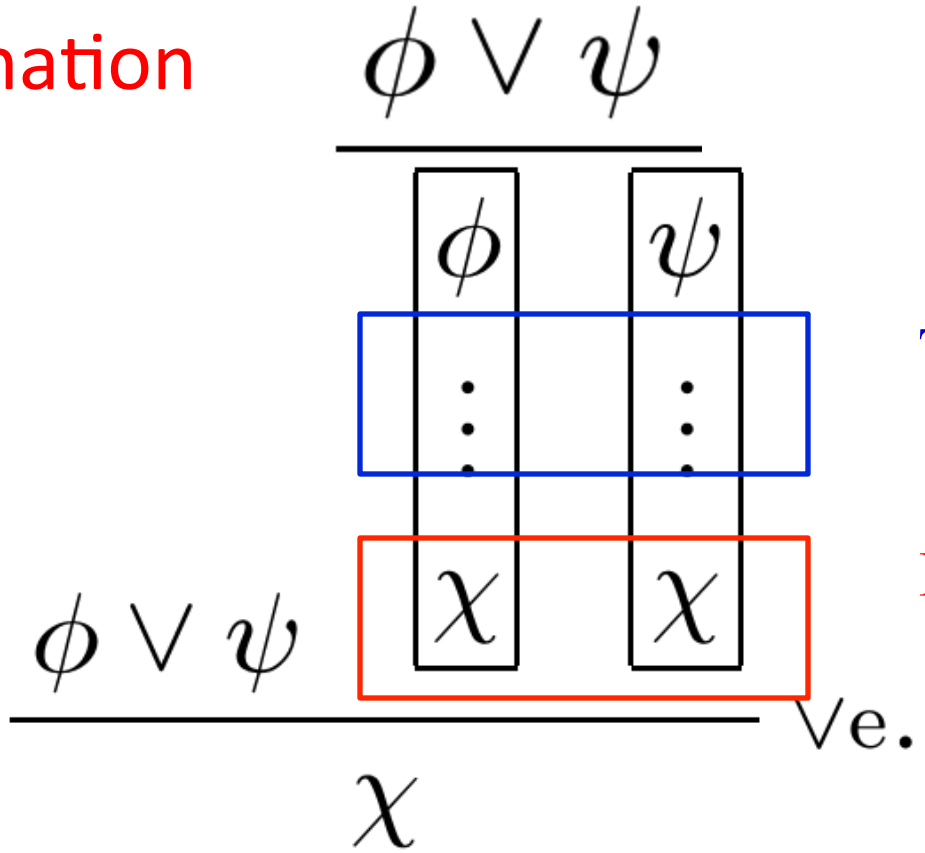
Rules of Disjunction

- OR Introduction

$$\frac{\phi}{\phi \vee \psi} \vee \mathrm{i}_1 \qquad \frac{\psi}{\phi \vee \psi} \vee \mathrm{i}_2 .$$

Rules of Disjunction

- OR Elimination



The proofs are independent

Have to be the same

- IF or CASE statement

Example: Disjunction is Commutative

1	$p \vee q$	premise
2	p	assumption
3	$q \vee p$	$\vee \mathrm{i}_2\ 2$
4	q	assumption
5	$q \vee p$	$\vee \mathrm{i}_1\ 4$
6	$q \vee p$	$\vee \mathrm{e}\ 1, 2{-}3, 4{-}5$

Activity

$$q \rightarrow r \vdash p \vee q \rightarrow p \vee r$$

1		$q \rightarrow r$	premise
2		$p \vee q$	assumption
3		p	assumption
4		$p \vee r$	$\vee \mathrm{i}_1$ 3
5		q	assumption
6		r	$\rightarrow$e 1, 5
7		$p \vee r$	$\vee \mathrm{i}_2$ 6
8		$p \vee r$	$\vee$e 2, 3–4, 5–7
9		$p \vee q \rightarrow p \vee r$	$\rightarrow$i 2–8