System Validation

Lecture 3

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Rules for eliminating implication

- Rule 1: Modus ponens (MP) (Latin name)
- Example:
  - It rained.
  - If it rained, then the street is wet.
  - Therefore
    - The street is wet.

$$\frac{\phi \quad \phi \to \psi}{\psi} \to \mathrm{e}.$$

Rules for eliminating implication

- Rule 2: Modus tollens (MT) (Latin name)
- Example:
  - If it rained, then the street is wet.
  - The street is not wet
  - Therefore
    - It didn't rain.

$$\frac{\phi \rightarrow \psi \qquad \neg\psi}{\neg\phi}\ \mathrm{MT}.$$

Activity

$$p \rightarrow (q \rightarrow r),\ p,\ \neg r \vdash \neg q$$

1	$p \rightarrow (q \rightarrow r)$	premise
2	p	premise
3	$\neg r$	premise
4	$q \rightarrow r$	$\rightarrow$e $1, 2$
5	$\neg q$	MT $4, 3$

Rule of implies introduction

- To Add an implication to your proof
  - You suppose p
  - Verify q based on the given premise or any other already verified result
  - Obtain $p \rightarrow q$
  - This is done in a closed rectangle

$$\frac{\boxed{\begin{array}{c}\phi\\ \vdots\\ \psi\end{array}}}{\phi \longrightarrow \psi} \longrightarrow \mathrm{i}.$$

Example

1	$\neg q \rightarrow \neg p$	premise
2	p	assumption
3	$\neg\neg p$	$\neg\neg$i 2
4	$\neg\neg q$	MT 1, 3
5	$p \rightarrow \neg\neg q$	$\rightarrow$i 2–4

Activity

$$p \rightarrow q \vdash p \wedge r \rightarrow q \wedge r$$

1	$p \rightarrow q$	premise
2	$p \wedge r$	assumption
3	p	$\wedge e_1$ 2
4	r	$\wedge e_2$ 2
5	q	$\rightarrow$e 1, 3
6	$q \wedge r$	$\wedge$i 5, 4
7	$p \wedge r \rightarrow q \wedge r$	$\rightarrow$i 2–6

Activity 1

$$p \wedge q \rightarrow r \vdash p \rightarrow (q \rightarrow r)$$

1	$p \wedge q \rightarrow r$	premise
2	p	assumption
3	q	assumption
4	$p \wedge q$	$\wedge$i 2, 3
5	r	$\rightarrow$e 1, 4
6	$q \rightarrow r$	$\rightarrow$i 3–5
7	$p \rightarrow (q \rightarrow r)$	$\rightarrow$i 2–6

Activity 2

$$p \rightarrow (q \rightarrow r) \vdash p \wedge q \rightarrow r$$

1	$p \rightarrow (q \rightarrow r)$	premise
2	$p \wedge q$	assumption
3	p	$\wedge e_1$ 2
4	q	$\wedge e_2$ 2
5	$q \rightarrow r$	$\rightarrow e$ 1, 3
6	r	$\rightarrow e$ 5, 4
7	$p \wedge q \rightarrow r$	$\rightarrow i$ 2–6

Equivalent Formulas

$$p \wedge q \to r \vdash p \to (q \to r)$$

$$p \to (q \to r) \vdash p \wedge q \to r$$

- The two formulas are equivalent to one another

$$p \wedge q \to r \dashv\vdash p \to (q \to r)$$