UNIVERSITY of SARGODHA

Introductory Lecture

Lecture 1

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Outline

- Introduction to Formal Methods
- Propositional Logic
  - Declarative Sentence
  - Natural Deduction

Formal methods: Mathematically based techniques for the specification, development and verification of software and hardware systems.

Specification: Well formed statements which describes what system/software should do.

Development: The process of developing the system/software formally.

Verification: The act of proving or disproving the correctness of some algorithm of system w.r.t. a certain formal specification.

Logic

- Logic is primarily a language to
  - model the system (program)
  - reason about the correctness or incorrectness of the system's properties

Philosophical Logic

- 500 B.C – 19th Century
- Logic dealt with reasoning of arguments in the natural language used by humans.

- Example: Valid Argument
  - All UOS students are good at studies.
  - Abdullah is a UOS student.
  - Therefore, Abdullah is good at studies.

Philosophical Logic

1. If the train arrives late and there are no taxis at the station, then John is late for his meeting.
2. John is not late for his meeting.
3. The train did arrive late.
4. Therefore, there were taxis at the station.

- Valid or Invalid?
  - **Valid (Combine 1 and 3 and then use 2)**

Philosophical Logic

- Natural languages are very ambiguous.

- Example:
- Tom hates Jim and he likes Mary.
    - Tom likes Mary, or
    - Jim likes Mary

- Thus, we need a more mathematical language for logical reasoning

Propositional Logic

- A proposition – a sentence that can be either true or false.
  - x is greater than y
  - Zohaib is teaching Formal Methods course in this semester

Activity: Identify Propositions

May fortune come your way.

Jane reacted violently to Jack's accusations.

Every even natural number > 2 is the sum of two prime numbers.

Ready, steady, go.

All Martians like pepperoni on their pizza.

Could you please pass me the salt.

Symbols in Propositional Logic

- Each proposition is assigned a symbol
- ‘x is greater than y.’: p
- ‘Zohaib is teaching Formal Methods this term’: q
- ‘I won a gold medal in last years Sports gala.’: r

Connectives in Propositional Logic

- $\wedge$ and (conjunction):
  - **a $\wedge$ b**: Both a and b are true.
- $\vee$ or (disjunction)
  - **a $\vee$ b**: at least one of a or b are true
- $\neg$ not (negation)
  - **$\neg$ a**: a is not true
- $\rightarrow$ implication
  - **a $\rightarrow$ b**: if a then b *(a:assumption, b: conclusion)*
- $\leftrightarrow$ equivalent to
  - **a $\leftrightarrow$ b**: a is equivalent to b, i.e., a $\rightarrow$ b $\wedge$ b $\rightarrow$ a
- $\vdash$ therefore
- $\bot$, T False, True

Activity: Modeling with Propositional Logic

1. If the train arrives late and there are no taxis at the station, then John is late for his meeting.
2. John is not late for his meeting.
3. The train did arrive late.
4. Therefore, there were taxis at the station.

Activity: Modeling with Propositional Logic

1. If the train arrives late (p) and there are no taxis at the station (q), then John is late for his meeting (r).
   - $(p \wedge (\neg q)) \rightarrow r$
2. John is not late for his meeting.
   - $\neg r$
3. The train did arrive late.
   - p
4. Therefore, there were taxis at the station.
   - q

$$((p \wedge (\neg q)) \rightarrow r), (\neg r), p \vdash q$$

Activity: Modeling with Propositional Logic

- If a request occurs, then either it will eventually be acknowledged, or the requesting process won't ever be able to make progress.

p : "A request occurs."

q : "The request will eventually be acknowledged."

r : "The requesting process will eventually make progress."

The formula representing the declarative sentence is then

$$p \to (q \vee \neg r) .$$

Activity: Modeling with Propositional Logic

- Today it will rain and shine, but not either.

r : “Today, it will shine.”

s : “Today, it will rain.”

The resulting formula is (r s) ¬ (r s)

Propositional Logic Formulas

- We omit parenthesis whenever we may restore them through operator precedence:
- $\neg$ binds more strictly than $\wedge$, $\vee$,
- $\wedge$, $\vee$ bind more strictly than $\rightarrow$, $\leftrightarrow$.
- Thus, we write:

$\neg\neg A$	for	$(\neg(\neg A))$,
$\neg A \wedge B$	for	$((\neg A) \wedge B)$
$A \wedge B \rightarrow C$	for	$((A \wedge B) \rightarrow C)$,

Example: Parenthesis

- Reinsert as many brackets as possible

$$(p \longrightarrow q) \longrightarrow (r \longrightarrow s \vee t)$$

$$(p \to q) \to (r \to (s \vee t))$$

Natural Deduction

- A calculus for reasoning about propositional formulas, so that we can establish their validity

- For example, we want to know if
  - $p \wedge \neg q \rightarrow r, \neg r, p \vdash q$
- Is valid.

Natural Deduction

- Terminology:
  - Sequent: $\phi_1, \phi_2, \ldots, \phi_n \vdash \psi$
  - Premises (Formulas): $\phi_1, \phi_2, \ldots, \phi_n$
  - Conclusion (Formula): ψ

- A sequent is valid if it is provable

- A valid sequent is a theorem if it doesn’t have Premises