

# Hypothesis Testing

## 1. Statistical hypothesis:

A statistical hypothesis is an assertion or conjecture about the distribution of one or more random variables. It is an assertion about the nature of a population.

## 2. Hypothesis testing:

The statistical hypothesis testing is a procedure to determine whether or not an assumption about some parameter of a population is supported by the information obtained from the observed random sample.

## 3. Null hypothesis:

A null hypothesis denoted by  $H_0$ , is that hypothesis which is tested for possible rejection under the assumption that it is true.

## 4. Alternative hypothesis:

An alternative hypothesis denoted by  $H_1$ , is that hypothesis which we are willing to accept when the null hypothesis is rejected.

5. Simple hypothesis:

A statistical hypothesis is said to be a simple hypothesis if it completely specifies the underlying population distribution.

6. Composite hypothesis:

A statistical hypothesis is said to be a composite hypothesis if it does not completely specify the underlying population distribution.

7. Test statistic:

A test statistic is a quantity calculated from the sample that is used to make a decision about the hypothesis of interest. The most commonly used test statistics are  $Z$ ,  $T$ ,  $\chi^2$  etc.

8. Rejection Region:

A rejection region specifies a set of values of the test statistic for which the null hypothesis is rejected. It is also called as critical region.

If the critical region is located equally in both tails of the sampling distribution for which the null hypothesis is not rejected.

7. One tailed test:

If the critical region is located only in one tail of the sampling distribution of the test statistic, the test is called one-tailed or one-sided test.

8. Two tailed test:

If the critical region is located equally in both tails of the sampling distribution of the test statistic, the test is called two-tailed or two-sided test.

9. Type I-error.

Rejection of null hypothesis when it is actually true is called a type-I error.

10. Type II error:

Acceptance of null hypothesis when it is actually false is called a type-II error.

11. Level of Confidence:

The level of confidence is the probability of accepting true null hypothesis.

12. Level of Significance:

The level of significance of a test is the maximum probability with which we are willing to a risk of type I error. It is denoted by  $\alpha$ .

## Hypothesis test:

A hypothesis test is a statistical method that uses sample data to evaluate a hypothesis about a population.

## Research Study:

A hypothesis test is typically used in the context of a research study. That is a researcher completes a research study and then uses a hypothesis test to evaluate the results.

## Steps:

i) State the hypothesis<sup>sis</sup>:

As the name implies, the process of hypothesis testing begins by stating a hypothesis about the unknown population.

We state two hypothesis.

The first and most important of the two hypothesis is called "Null hypothesis." The Null hypothesis is identified by " $H_0$ ".

The second hypothesis is simply the opposite of the null hypothesis, and it is called alternative hypothesis and is denoted by  $H_1$ .

- ii) Level of Significance:  $\alpha \Rightarrow 10\%$ ,  $5\%$ ,  $1\%$ .  
It is a probability value that is used to define the concept of "very Unlikely" in a hypothesis test.
- iii) Test - Statistic:
- iv) Critical region:
- v) Calculation:
- vi) Conclusion:

$\Rightarrow$  T-test:

The t-statistic is used to test hypothesis about an unknown population mean  $\mu$ , when the value of  $\sigma$  is unknown.  
The formula of t-test is

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

$$d.f = n - 1$$

$\Rightarrow$  Degree of freedom:

describe the number of scores in a sample that are independent and free to vary.

$\Rightarrow$  Assumptions:

- i) The values in the sample must be consist of independent observations
- ii) The population sampled must be normal.

Question no: 1.

A psychologist has prepared an "optimism test" that is administered yearly to graduating college seniors. The test measures how each graduating class feels about its future. The higher the score, the more optimistic the class. Last year's class had a mean score of  $\mu = 15$ . A sample of  $n = 9$  seniors from this year was selected. The scores are 7, 12, 11, 15, 7, 8, 15, 9 and 6, with sample mean of 10 and  $s = 1.14$ . Can a psychologist conclude that this year's class has a different level of optimism than last year?

Solution:

1) Null hypothesis  $H_0: \mu = 15$   
Alternative hypothesis  $H_1: \mu \neq 15$ .

2) Level of Significance:  $\alpha = 0.05$   
3) Test Statistic:

$$t = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}}$$

1) Calculations:

$$\bar{x} = \frac{\sum x}{n} = 10 \quad s = 1.14, \quad \mu = 15, \quad n = 9$$

So

$$t = \frac{10 - 15}{\frac{1.14}{\sqrt{9}}} = \frac{-5}{0.38} = -13.15$$

Critical region

$$|t| > t_{\alpha(n-1)}$$

$$n = 9 \Rightarrow n-1$$

$$|-13.15| > t_{0.05(8)}$$

$$t_{\alpha(n-1)} = \pm 2.306.$$

$$\pm 2.306$$

$$|-13.15| > \pm 2.306$$

Decision/Conclusion:

Our  $t$  value falls in critical region so we reject the null hypothesis and conclude that there is significant difference in level of optimism.